Modified Quantum Particle Swarm Optimization for Chaos Synchronization

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Abstract

In this study, a modified quantum-behaved particle swarm optimization (MQPSO) based on hybrid evolution (HEMQPSO) approach is proposed to synchronize chaotic systems, in which the proposed HEMQPSO algorithm combines the conceptions of genetic algorithm (GA) and adaptive annealing learning algorithm with the MQPSO algorithm to search optimal solutions. Simulation results are illustrated to verify the performance of chaos synchronization using the proposed HEMQPSO approach. From the numerical simulations and comparisons with other extant evolutionary methods in chaotic systems, the validity and superiority of the HEMQPSO approach are verified.

Keywords: Quantum-behaved particle swarm optimization, Chaotic system, Genetic algorithm, Chaos synchronization, Hybrid evolution.

1. Introduction

A chaotic system is a nonlinear deterministic system that has some special features of sensitive dependence on initial conditions and unstable bounded trajectories in the phase space. Synchronization in chaotic dynamic systems attains much interest among scientists from various fields. Applications in chaotic systems synchronization are very significant in nonlinear fields. Recently, some researchers have paid much attention to identification and synchronization of chaotic systems.[1-3] Due to their characteristics sensitivity to initial conditions, chaotic systems are not easy to synchronize. Recently, some researchers have endeavored to improve the synchronization of chaotic time series.[4-8] In this article, we propose a scheme to synchronize Chen system and Genesio system with known parameters.[6]

Although the original PSO algorithm possesses the ability of high convergent speed, easily falling in some local optima is its fatal defect. Many researchers[9,10] have presented revised PSO algorithms and obtained good results. Another improvement on traditional PSO algorithm is quantum-behaved particle swarm optimization (QPSO).[11-14] However, in QPSO, particles fall into local optimal state in multimode optimization problems and cannot find any better state, the QPSO algorithm will take on the premature phenomenon.

To overcome the premature phenomenon in QPSO, a modified quantum-behaved particle swarm optimization (MQPSO) based on hybrid evolution (HEMQPSO) algorithm is proposed to synchronize chaotic systems will be proposed to perform the synchronization of chaotic systems in this study. In HEMQPSO, the significant improvement is that the evolutionary algorithm combines the concept of mutation algorithm in GA and adaptive annealing learning similar to SA with QPSO to achieve global search and defeat premature phenomenon in searching optimal solutions. From the illustrated results for three chaotic dynamical systems, the synchronization performance of the proposed HEMQPSO approach is demonstrated.

2. Modified Quantum Particle Swarm Optimization

In the PSO algorithm, each particle keeps trajectory of its coordinates in the problem space. The coordinate of each particle is related to its own best position (local best position) and the global best position achieved so far. The trajectories of particles are updated according to the following equations:

$$v_i(k+1) = w \cdot v_i(k) + c_1 \cdot r_1(p_i^l - p_i) + c_2 \cdot r_2(p^g - p_i),\ i = 1,2,\cdots,n, \tag{1}$$

$$p_i(k+1) = p_i(k) + v_i(k+1),\ i = 1,2,\cdots,n, \tag{2}$$

where n denotes the number of particles in a population; $p_i(k)$ and $v_i(k)$ are position and velocity of the ith particle at generation k in n-dimensional search space; p_i^l and p^g are the best position of the ith particle and the global best position; w is the inertia weight; c_1 and c_2 are cognitive and social constriction coefficients, respectively; r_1 and r_2 are random numbers between 0 and 1.

From the view of classical dynamics, to avoid explosion and guarantee convergence, particles must be bounded and fly in an attractive potential field. Clerc and Kennedy[13] have proved that if these coefficients are properly defined, the particle's position p_i will converge to the center of potential field, $pf^c = [pf_1^c, pf_2^c, \cdots, pf_n^c]$, and is defined as:

$$pf_i^c = \frac{(c_1 \cdot r_1 \cdot p_i^l + c_2 \cdot r_2 \cdot p^g)}{(c_1 \cdot r_1 + c_2 \cdot r_2)},\ i = 1,2,\cdots,n. \tag{3}$$

Inspired by the behavior that particles move in a bounded state and preserve the global search ability, Sun et al.[15] proposed the QPSO algorithm. In the QPSO model, particles move in a quantum multi-dimensional space, the state of particles is usually depicted by a normalized wave function. That is, a single particle with m mass is subjected to the influence of a potential field in quantum space and the wave function of this particle is governed by the Schrödinger equation.[12] The solution of time-independent Schrödinger equation for this system in one dimensional space can be expressed as[12]:

$$p_i = pf_i^c \pm \frac{L}{2} \cdot \ln\left(\frac{1}{\lambda}\right), \tag{4}$$

where λ is a random number uniformly distributed on [0, 1] and L is the characteristic length of delta potential well (called "Creativity" of particles) which specifies the search scope of a particle. In order to improve performance, Sun et al.[15] employ a mainstream thought point to evaluate the parameter. The mainstream thought point and can be expressed as the following forms:

$$mbest = \left[\sum_{i=1}^{n}\frac{p_{i,1}}{n}, \sum_{i=1}^{n}\frac{p_{i,2}}{n}, \cdots, \sum_{i=1}^{n}\frac{p_{i,n}}{n}\right],\ i = 1,2,\cdots,n, \tag{5}$$

$$L = 2 \cdot \beta \left| mbest - p_i \right|, \tag{6}$$

where β is a creative coefficient which is used to adjust the convergence speed of an individual particle and the performance of this algorithm.

Firstly, in order to achieve global searching, β should be set to a large number at the beginning. Then the parameter β is adjusted decreasingly. The decreasing rate of β can be linear, but nonlinear revision according to the convergence of optimization process is more reasonable. The creative coefficient β with adaptive annealing learning mechanism according to the change rate of optimal estimation has the form:

$$\beta = \beta_{\max} - \Delta\beta \cdot (\Delta fit)^{\gamma}, \tag{7}$$

$$\Delta fit = \left| p^g - p_i^l \right|, \tag{8}$$

where $\Delta\beta$ is step length of β, Δfit is the change rate of optimal estimation so far. The mechanism of adaptive annealing learning can overcome the stagnation problem to accelerate the convergent speed. Another improvement of the HEMQPSO is elitist reproduction. The mutation mechanism is usually used for keeping diversity and avoiding premature.

3. Problem Formulation

This section presents two chaos systems of Chen system and Genesio system to synchronize their behavior. The dynamic equation of Genesio system is given by

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2, \\ \dot{\hat{x}}_2 = \hat{x}_3, \\ \dot{\hat{x}}_3 = r_1\hat{x}_1 + r_2\hat{x}_2 + r_3\hat{x}_3 + \hat{x}_1^{\,2}, \end{cases} \tag{9}$$

where $\hat{x}_1$, $\hat{x}_2$, $\hat{x}_3$, are state variables, $r_1 = -6$, $r_2 = -2.92$, $r_3 = -1.2$, for the chaotic system (9). Chen system is described by

$$\begin{cases} \dot{x}_1 = q_1(x_2 - x_1), \\ \dot{x}_2 = (q_2 - q_1)x_1 + q_2x_2 - x_1x_3, \\ \dot{x}_3 = x_1x_2 - q_3x_3, \end{cases} \tag{10}$$

when $q_1 = 35$, $q_2 = 28$, $q_3 = 3$, the system (10) is chaotic.

4. Simulation Results

Consider that Genesio system (9) is the drive system and the controlled Chen system (11) is the response system. The synchronization behavior between Chen system and Genesio system using active control is observed.

$$\begin{cases} \dot{x}_1 = q_1(x_2 - x_1) + u_1, \\ \dot{x}_2 = (q_2 - q_1)x_1 + q_2x_2 - x_1x_3 + u_2, \\ \dot{x}_3 = x_1x_2 - q_3x_3 + u_3, \end{cases} \tag{11}$$

Three control functions u_1, u_2, u_3, are introduced in system (11), in order to determine the control functions to realize synchronization between systems (9) and (11), we subtract (9) from (11) and get the deviation of errors system can be expressed as

$$\begin{cases} \dot{e}_1 = -q_1e_1, \\ \dot{e}_2 = -q_2e_2, \\ \dot{e}_3 = -q_3e_3, \end{cases} \tag{12}$$

where $e_1 = x_1 - \hat{x}_1$, $e_2 = x_2 - \hat{x}_2$, $e_3 = x_3 - \hat{x}_3$, The error system (12) is asymptotically stable by linear control theory.[6]

The sampling time is equal to 0.005 and the number of states is set as 200 for three simulated examples. The initial values of the drive and response systems are $\hat{x}_1(0) = -1$, $\hat{x}_2(0) = 0$, $\hat{x}_3(0) = 1$, $x_1(0) = -5$, $x_2(0) = 10$, $x_3(0) = 5$, respectively. In HEMQPSO, the parameters, $\beta_{\max}$ and γ in Eq. (7), are set as 0.5 and 0.5. Comparisons of HEQPSO[16] and HEMQPSO are shown in Figs. 1 and 2, respectively, in which the superiority of the proposed HEMQPSO is verified.

5. Conclusions

This paper presents the proposed HEMQPSO to synchronize chaotic systems. The evolutionary algorithm can overcome the stagnation in searching global solutions for synchronizing two chaotic systems. From the simulation results, we can conclude that the proposed HEMQPSO method has good performance for chaos synchronization. The future work is to apply HEMQPSO for investigating more complex chaotic systems.

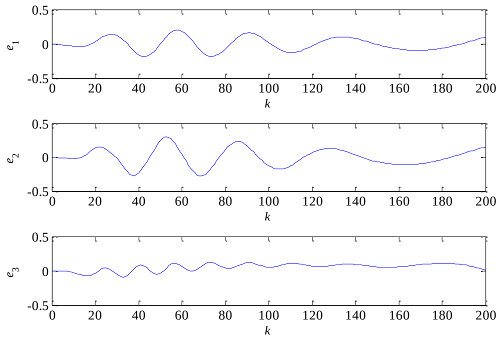

Fig. 1. Synchronization errors between Chen and Genesio systems via active control with HEQPSO.

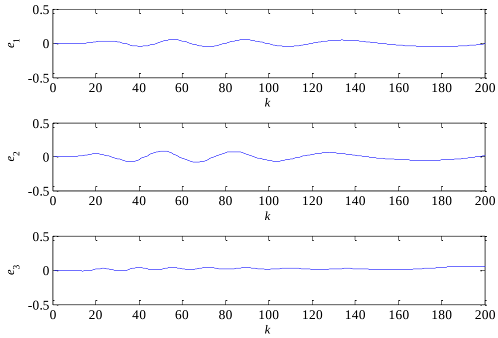

Fig. 2. Synchronization errors between Chen and Genesio systems via active control with HEMQPSO.

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