Automated Multiple-Brightness Peak Image Processing Method Using Curvature and Variance Estimation

Yusuke Kawakami
DynaxT Co., Ltd., 2217-6 Hayashi
Takamatsu City, Kagawa 761-0301, Japan

Tetsuo Hattori, Yoshiro Imai, Yo Horikawa
Kagawa University, 2217-20 Hayashi
Takamatsu City, Kagawa 761-0396, Japan

R. P. C. Janaka Rajapakse
Tainan National University of the Arts, 66 Daci
Guantian District, Tainan 72045, Taiwan
E-mail: riverjp2002@gmail.com, {hattori, imai, horikawa}@eng.kagawa-u.ac.jp, janakaraja@gmail.com

Abstract

This paper describes the improvement method for the image which have multiple-brightness peak using Histogram Matching based on Gaussian Distribution (HMGD). The previous papers, we have illustrated that the HMGD is an automated image processing method for improve feeling impression better, through the comparative investigation results of feeling impression among the original image, Histogram Equalization image, and HMGD image. However, the multiple-brightness peak images have been hard to improve using the previous HMGD. In this paper, we propose the processing method of HMGD to correspond image which have multiple-brightness peak, using curvature computation and variance estimation.

Keywords: Image processing, Curvature, Variance estimation, Histogram matching, HMGD

1. Introduction

These days, automated image processing for enhancement of color images has been more familiar to us, for example, Digital Signage, Smart Phone, etc [1-3].

In the previous paper, we have presented that the Histogram Matching based on Gaussian Distribution (HMGD) processing is one of the automated image arrangement method using Elastic Transformation [4-5] based on the brightness axis. And through the comparative investigation, we have illustrated that HMGD processing could improve feeling (or Kansei) impression better than original image[6].

In this paper, we present the improvement HMGD processing method for multiple-brightness peak image using curvature computation and variance estimation of image histogram. We explain the principle of HMGD, and we also explain peak detection and variance estimation of histogram. Last, we illustrate the results of the HMGD processing for multiple-brightness peak image through the experimentation.

2. Principle

2.1. *Histogram Matching based on Gaussian Distribution*

In the section, we explain the principle of HMGD processing.

Fig. 1 shows the conceptual image of HMGD. Let $f(x)$ and $h(y)$ be two probabilistic density functions (PDF) on real variables x and y, respectively. The PDF is corresponding to histogram of image brightness level which is discretely defined.

In addition, let $y=\phi(x)$ be a continuous and monotonic increase function corresponding to cumulative histogram of image brightness level between variables x and y [7-9]. And let $y=\phi(x)$ be defined by Eq. (1).

$$y=\phi(x)=L\int_0^x f(x)dx. \quad (1)$$

At first, we have to expand brightness level of original image histogram and convert into uniform distribution histogram, because we aim to match Gaussian distribution. From Eq. (1) and Fig. 1, we can derive Eq. (2) and (3).

$$f(x)=h(y)\phi'(x)=h(y)Lf(x). \quad (2)$$

$$h(y)=\frac{1}{L}. \quad (3)$$

We understand the histogram of original image $f(x)$ becomes uniform distribution $h(y)$ by Eq. (3). This means that brightness level of original image $f(x)$ is expanded to $h(y)$.

Then, let $Gauss(z)$ and $\gamma(z)$ be the function that is defined by Eq. (4) and (5), respectively.

$$Gauss(z)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(z-\mu)^2}{2\sigma^2}\right). \quad (4)$$

$$\begin{aligned} y=\gamma(z)&=L\int_0^z Gauss(z)dz \\ &=\frac{1}{\sqrt{2\pi\sigma^2}}\int_0^z \exp\left(-\frac{(z-\mu)^2}{2\sigma^2}\right)dz. \end{aligned} \quad (5)$$

Here, Fig.1 shows the relationship between $y=\phi(x)$ and $y=\gamma(z)$. So we can be obtained following Eq. (6).

$$L\int_0^x f(x)dx=\frac{L}{\sqrt{2\pi\sigma^2}}\int_0^z \exp\left(-\frac{(z-\mu)^2}{2\sigma^2}\right)dz. \quad (6)$$

And we can derive Eq. (7) from differential of Eq. (6).

$$\frac{d}{dx}L\int_0^x f(x)dx=\frac{d}{dz}\frac{L}{\sqrt{2\pi\sigma^2}}\int_0^z \exp\left(-\frac{(z-\mu)^2}{2\sigma^2}\right)dz. \quad (7)$$

If we perform Eq. (7),

$$L\phi'(x)=L\gamma'(z), \quad f(x)=Gauss(z). \quad (8)$$

That is, we understand that $f(x)$ becomes Gaussian distribution $Gauss(z)$ when we take the transform function as (1) and (5). Thus, HMGD processing is the function which defined by cumulative histogram transformation the original histogram into Gaussian histogram[9].

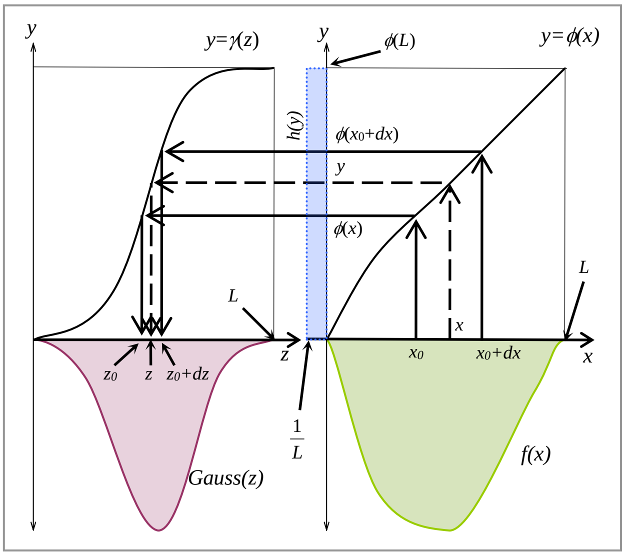


Fig. 1. Conceptual image of HMGD[6, 9].

2.2. *Peak Detection of Histogram*

The HMGD processing method for multiple−brightness peak image need to calculate transforms function for each brightness peak of histogram. And the solution to detect it is curvature computation of the histogram.

Let y be a function with respect to x, the definition curvature R is given by Eq. (9)[6-9].

$$R=\frac{d^2y}{dx^2}\left(1+\left(\frac{dy}{dx}\right)^2\right)^{-\frac{3}{2}}. \quad (9)$$

Let $g(x)$ and K be Gaussian distribution function and a coefficient which is defined by following equation, respectively.

$$\begin{aligned} g(x)&=\frac{K}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(x-a)^2}{2\sigma^2}\right), \\ \frac{K}{\sqrt{2\pi\sigma^2}}&\int_0^L \exp\left(-\frac{(u-a)^2}{2\sigma^2}\right)du=1. \end{aligned} \quad (10)$$

Next, let $y=f(x)$ be a function representing the cumulative histogram which is represented Eq. (11). That is, dy/dx and d^2y/dx^2 be described as Eq. (12) and (13), respectively. From Eq. (12) and (13), we obtain the approximation of curvature R as Eq. (14).

$$f(x)=\int_0^x g(u)du=\frac{K}{\sqrt{2\pi\sigma^2}}\int_0^x \exp\left(-\frac{(u-a)^2}{2\sigma^2}\right)du. \quad (11)$$

$$\frac{dy}{dx}=g(x)\frac{K}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(u-a)^2}{2\sigma^2}\right). \quad (12)$$

$$\frac{d^2y}{dx^2}=\frac{dg(x)}{dx}=\frac{(a-x)}{\sigma^2}g(x) \quad (13)$$

$$R=\frac{\frac{(a-x)}{\sigma^2}g(x)}{\left(1+g(x)^2\right)^{\frac{3}{2}}}\approx\frac{(a-x)}{\sigma^2}g(x). \quad (14)$$

From Eq. (14), we understand that the curvature R varies the sign according to the value of x[9]. That is, if $x<a \rightarrow R>0$ (downward convex shape), and if $x<a \rightarrow R<0$ (upward convex shape).

2.3. *Variance Estimation*

In this section, we propose how to optimize the shape of the reference histogram, which is used in the HMGD processing method for multiple-brightness peak image. Fig. 2 shows the conceptual image of the original image histogram which is variance σ^2 and average a. From Eq. (10), let $g(a)$ be a Gauss density function with variance σ^2 at average a.

$$g(a)=\frac{K}{\sqrt{2\pi\sigma^2}}. \quad (14)$$

Using Eq. (14), we can describe $g\left(a\pm\sqrt{2}\sigma\right)$ and its curvature as Eq. (15) and (16), respectively.

$$g\left(a\pm\sqrt{2}\sigma\right)=\frac{K}{\sigma\sqrt{2\pi}}\exp\left(-\frac{\left(a\pm\sqrt{2}\sigma-a\right)}{2\sigma^2}\right)=g(x)e^{\mp 1}. \quad (15)$$

$$R\left(a\pm\sqrt{2}\sigma\right)=\frac{\left(a-\left(a\pm\sqrt{2}\sigma\right)\right)}{\sigma^2}g\left(a\pm\sqrt{2}\sigma\right)\left(1+g\left(a\pm\sqrt{2}\sigma\right)^2\right)^{-\frac{3}{2}}. \quad (16)$$

From Eq. (15), we can derive Eq. (17) and (18).

$$R\left(a\pm\sqrt{2}\sigma\right)=\left(\mp\frac{\sqrt{2}}{\sigma^2}\right)g\left(a\pm\sqrt{2}\sigma\right)\left(1+g\left(a\pm\sqrt{2}\sigma\right)^2\right)^{-\frac{3}{2}}$$
$$=\left(\mp\frac{\sqrt{2}}{\sigma^2}\right)\frac{g(a)}{e\left(1+g\left(a\pm\sqrt{2}\sigma\right)^2\right)^{-\frac{3}{2}}}\equiv\left(\mp\frac{\sqrt{2}}{\sigma^2}\right)H. \quad (17)$$

$$H=\frac{g(a)}{e\left(1+g\left(a\pm\sqrt{2}\sigma\right)^2\right)^{-\frac{3}{2}}}. \quad (18)$$

Then, we understand that we can obtain reference histogram variance σ^2 from Eq. (17). For example, let $v=\sqrt{2}\sigma$ be the distance from average a,

$$R(a-v)-R(a+v)=\frac{2\sqrt{2}}{\sigma}H=\frac{4}{v}H,\quad \sigma^2=\frac{v^2}{2}. \quad (19)$$

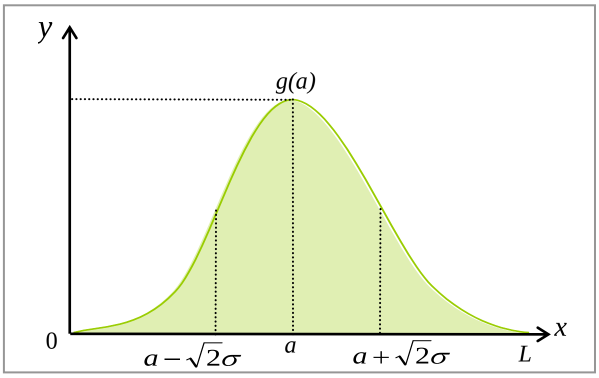


Fig. 2. Conceptual image of the original image histogram[8].

3. HMGD Processing for Multiple-Brightness Peak

In the previous chapter, we have described about the principle for HMGD processing for multiple-brightness peak. In this section, we propose the concrete process of it.

(i) Detect the brightness-peak values x_m (m=1,…,n) of original image histogram.
(ii) Perform variance estimation and generate the reference histogram for brightness-peak value x_m.
(iii) Perform HMGD processing and make an image i_m which has histogram h_m.
(iv) Proceed to (v) if original image has no brightness-peak value. Otherwise, set the brightness-peak value to x_{m+1} and return to (ii).
(v) Calculate the average of the HMGD processed image I_a using Eq. (20).

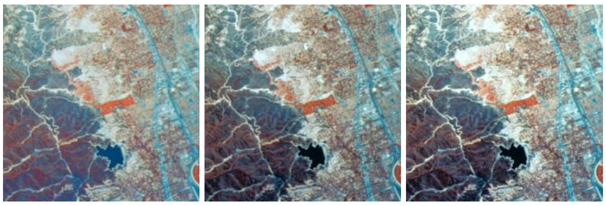
$$I_a=\frac{1}{n}\sum_{m=1}^{n} i_m \quad (20)$$

(vi) Output the processed image I_a.

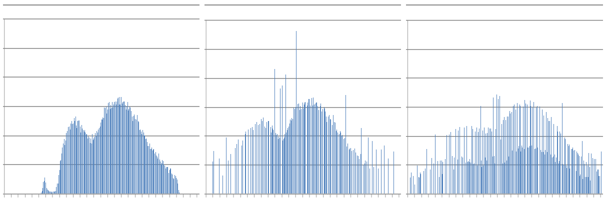
4. Experimentation

Fig. 3(a) shows the example of results for HMGD and HMGD for multiple-brightness peak (HMGD-MBP). And Fig. 3(b) and Fig. 3(c) show the corresponding histogram and cumulative histogram, respectively.

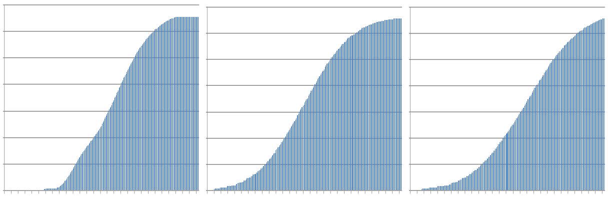
In this case, we understand that HMGD-MBP processing image is enhancing contrast naturally than HMGD processing image. And in the HMGD-MBP image, the edges in the mountain (located left below of image) become clear and detailed.



(a) Image: Original (left), HMGD (center), HMGD-MBP (right)



(b) Histogram: Original (left), HMGD (center), HMGD-MBP (right)



(c) Cumulative histogram: Original (left), HMGD (center), HMGD-MBP (right)

Fig. 3. Example of results (HMGD, HMGD-MBP, and the corresponding histograms).

5. Conclusion

In this paper we have aimed improvement automated processing for multiple-brightness peak image, we have proposed "Histogram Matching based on Gaussian Distribution for multiple-brightness peak (HMGD-MBP)".

As for the concrete processing method, we have used curvature computation and variance estimation for detecting brightness peaks and optimizing the shape of the reference histogram. Then we have taken an average of HMGD processed images.

From the experimentation results, HMGD-MBP processing image is enhancing contrast naturally and the edges of image becomes clear and detailed. That is, we consider that the HMGD-MBP processing method is useful than previous HMGD.

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