Modeling of Mobile Manipulator and Adaptive Super-Twisting Backstepping Control

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Abstract

A mobile manipulator with three wheels and three DOF links is modeled by using Euler-Lagrange method and vector orientated constraint conditions. In this modeling process, the conventional complex nonholonomic constraint transformation need not be considered in mobile robot system and then much simpler dynamic model can be obtained. Next, the super-twisting sliding mode control is combined with nonlinear backstepping control to obtain the systematic nonlinear controller design, fast response speed, and improved robustness to uncertainty due to dynamic coupling and disturbance. Simulation and experiment were carried out to prove the effectiveness of the proposed control methodology.

Keywords: Mobile-manipulator, Super-twisting sliding mode control, Backsteppping control.

1. Introduction

In recent years, there has been a great interest for researches of multi-agent systems, whose applications include spacecraft, mobile robots, sensor networks, etc. Interesting research directions are containment control, consensus, formation, and flocking control [1]. These problems focus on two cases, namely, the case that there does not exist a leader and the case where there exists a leader. The coordinate tracking problems to track a single leader have been investigate for followers with single-integrator, double-integrator,high-order dynamics, nonlinear or Euler-Lagrange dynamics [2-5]. Linear control theory and variable structure control methods in most researches are used. On the other hand, there are few examples that use the backstepping control technique [6] for nonlinear or Euler-Lagrange multi-agent system. In this method, the problem of unmatched uncertainty and neglecting the efficient nonlinearities is overcome via adopting step-by-step recursive process.

However, although a controller designed using this theorem guarantees the infinite-time stability of a closed-loop system, it has drawbacks such as a slow convergence rate and reduced robustness to uncertainty. On the other hand, systems with finite-time settling-time design possess attractive features such as improved robustness and disturbance rejection properties [7], In this paper, terminal backstepping control based multi-agent consensus control for Euler-Lagrange system with one-leader and multi-followers is developed.

2. Background and Preliminaries

2.1. *Description of the Mobile-Manipulator*

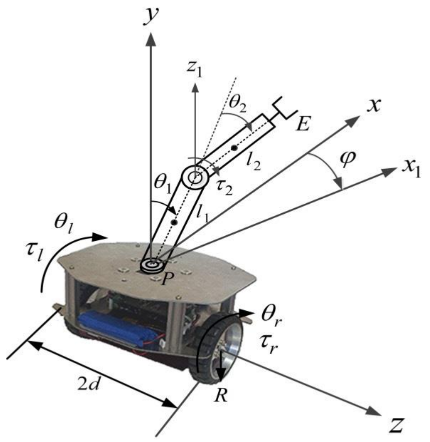


Fig. 1 Schematic diagram of the two-link and two-wheeled mobile manipulator.

The following variables are selected to describe the two-link and two-wheeled mobile manipulator shown in Fig. 1: τ_r, τ_l , and τ_2 are the torques of two wheels and the joint2; θ_r and θ_l are the rotation angles of the right and left wheels of the mobile platform, respectively; x and φ are the forward distance and the rotation angle of the mobile platform, respectively; θ_1 and θ_2 are the rotation angle of the links 1 and 2, respectively; $m_p = 5kg$, $m_w = 0.58kg$, $m_1 = 2kg$, and $m_2 = 0.5kg$ are the masses of the mobile platform, wheel, link 1, and link 2, respectively; I_p, I_1, and I_2 are the moment of inertia of the mobile platform, link 1, and link 2 with respect to y axis, respectively; $I_{\theta_1}, I_{\theta_2}$, and I_w are the moment of inertia of link 1, link 2, and each wheel with respect to the z and z_1 axis, respectively; $d = 0.145m$ is the distance between point P and the wheels; $R = 0.08m$ is the radius of the wheels; $l_1 = 0.25m$ and $l_2 = 0.2m$ are the lengths of link 1 and link 2; and r_1 and r_2 are the distances between the joints and the center of mass of the links.

The generalized coordinated is selected as $q = [x,\ \varphi,\ \theta_1,\ \theta_2,\ \theta_l, \theta_r]^T$. The total kinematic energy is given as,

$$T = \frac{1}{2}(m_p + m_1 + m_2 + 2m_w)\dot{x}^2 + \frac{1}{2}(I_p + I_w + I_1 + I_2)\dot{\varphi}^2 + \frac{1}{2}(m_1 r_1^2 + I_{\theta_1})\dot{\theta}_1^2 + \frac{1}{2}m_2[r_1^2\dot{\theta}_1^2 + r_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + 2r_1 r_2(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2)c\theta_2] + \frac{1}{2}I_{\theta_2}\dot{\theta}_2^2 + \frac{1}{2}\left(m_w + \frac{I_{zw}}{R^2}\right)\dot{x}_r^2 + \frac{1}{2}\left(m_w + \frac{I_{zw}}{R^2}\right)\dot{x}_l^2, \quad (1)$$

where $I_1 = \frac{1}{3}m_1(l_1 s\theta_1)^2$ and $I_2 = \frac{1}{3}m_2(l_1 s\theta_1 + r_2 s\theta_{12})^2$.

The potential energy is obtained as follows:

$$V = m_1 g r_1 c\theta_1 + m_2 g(l_1 c\theta_1 + r_2 c\theta_{12}) . \quad (2)$$

The dynamics are obtained using the Euler-Lagrangian method. From the tire dynamics, we obtain

$$I_{zw}\ddot{\theta}_l = \tau_l , \quad (3)$$

$$I_{zw}\ddot{\theta}_r = \tau_r , \quad (4)$$

with the kinematic relations $x = (x_r + x_l)/2$, $\ddot{x} = (\ddot{x}_r + \ddot{x}_l)/2$, $\theta_r = x_r / R$, $\theta_l = x_l / R$, $\ddot{\theta}_l = \ddot{x}_l / R$, and $\ddot{\theta}_r = \ddot{x}_r / R$, which is obtained from the assumption that a slip does not occur between tire and contact surface. Therefore, we obtain

$$F_l = \frac{\tau_l}{R} - \left(\frac{I_{zw}}{R^2} + M_l\right)\ddot{x}_l , \quad (5)$$

$$F_r = \frac{\tau_r}{R} - \left(\frac{I_{zw}}{R^2} + M_r\right)\ddot{x}_r . \quad (6)$$

Under the assumption of no sideslip, the relationships of $M_r = M_l = M_w$, $\ddot{x} = (\ddot{x}_l + \ddot{x}_r)/2$, $\dot{\varphi} = R(\dot{\theta}_l - \dot{\theta}_r)/2d = (\dot{x}_l - \dot{x}_r)/2d$, and $\ddot{\varphi} = R(\ddot{\theta}_l - \ddot{\theta}_r)/2d \ = (\ddot{x}_l - \ddot{x}_r)/2d$ give

$$F_l + F_r = \frac{\tau_l}{R} + \frac{\tau_r}{R} - \left(\frac{I_{zw}}{R^2} + M_w\right)(\ddot{x}_l + \ddot{x}_r) = \frac{\tau_l}{R} + \frac{\tau_r}{R} - 2\left(\frac{I_{zw}}{R^2} + M_w\right)\ddot{x} , \quad (7)$$

$$F_l - F_r = \frac{\tau_l}{R} - \frac{\tau_r}{R} - \left(\frac{I_{zw}}{R^2} + M_w\right)(\ddot{x}_l - \ddot{x}_r) = \frac{\tau_l}{R} - \frac{\tau_r}{R} - 2d\left(\frac{I_{zw}}{R^2} + M_w\right)\ddot{\varphi} . \quad (8)$$

Finally, the reduced generalized coordinate is given as $q_r = [x,\ \varphi,\ \theta_1,\ \theta_2]^T$. The derived dynamic equation is written as

$$M(q_r)\ddot{q}_r + C(q_r, \dot{q}_r)q_r + G(q_r) = B(q_r)\tau , \quad (9)$$

Where,

$$M(q_r) = \begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} \\ 0 & m_{22} & 0 & 0 \\ m_{31} & 0 & m_{33} & m_{34} \\ m_{41} & 0 & m_{43} & m_{44} \end{bmatrix},$$

$$C(q_r, \dot{q}_r) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & l_{eq1} & l_{eq2} \\ 0 & 0 & -m_2 r_1 r_2 \dot{\theta}_2 s\theta_2 & -m_2 r_1 r_2 \dot{\theta}_2 s\theta_2 \\ 0 & 0 & -m_2 r_1 r_2 \dot{\theta}_1 s\theta_2 & 0 \end{bmatrix},$$

$$B(q_r) = \begin{bmatrix} 1/R & d/R & 0 & 0 \\ 1/R & -d/R & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T , \text{ and } \tau = [\tau_l, \tau_{r,}\ \tau_2]^T .$$

By denoting v and m as suffixes of the mobile platform and manipulator, respectively, the coordinates q_r can be decomposed as $q=[q_v^T, q_m]^T$, $q_v=[x, \varphi, \theta_1]^T$, and $q_m=\theta_2$. Thus, (51) can be written as follows:

$$\begin{bmatrix} M_v & M_{vm} \\ M_{mv} & M_m \end{bmatrix}\begin{bmatrix} \ddot{q}_v \\ \ddot{q}_m \end{bmatrix}+\begin{bmatrix} C_v & C_{vm} \\ C_{mv} & C_m \end{bmatrix}\begin{bmatrix} \dot{q}_v \\ \dot{q}_m \end{bmatrix}+\begin{bmatrix} G_v \\ G_m \end{bmatrix}+\begin{bmatrix} \tau_{dv} \\ \tau_{dm} \end{bmatrix}=\begin{bmatrix} B_v & 0 \\ 0 & B_m \end{bmatrix}\begin{bmatrix} \tau_v \\ \tau_m \end{bmatrix}. \quad (10)$$

The state space model of the mobile platform of (10) can be expressed as follows:

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -M_v^{-1}\left(C_v x_2 + G_v + F_{dv}\right) + M_v^{-1} B_v u_2$$
$$y_v = x_1, \quad (11)$$

where $x_1=q_v$, $x_2=\dot{q}_v$, $F_{dv}=M_{vm}\ddot{q}_m + C_{vm}\dot{q}_m+\tau_{dv}$, and $u_2=\tau_v$. The state space model of the manipulator from (10) can be expressed as follows:

$$\dot{x}_3 = x_4,$$
$$\dot{x}_4 = -M_m^{-1}\left(C_m \dot{q}_m + G_m + F_{dm}\right) + M_m^{-1} B_m u_4,$$
$$y_m = x_3, \quad (12)$$

where, $x_3=\theta_2$, $x_4=\dot{\theta}_2$, $F_{dm}= M_{mv}\ddot{q}_v + C_{mv}\dot{q}_v + \tau_{dm}$, and $u_4=\tau_m$. The state variables of the mobile manipulator system are selected as $x_1=[x_{1,1}, x_{2,1}, x_{3,1}]^T$ $=[x, \varphi, \theta_1]^T$, $x_2=[x_{1,2}, x_{2,2}, x_{3,2}]^T$ $=[\dot{x}, \dot{\varphi}, \dot{\theta}_1]^T$, $x_3=x_{4,1}=\theta_2$, and $x_4=x_{4,2}=\dot{\theta}_2$. The control objective is that the outputs track the desired command in finite-time, and the tilting angle, θ_1, should be maintained stably in the vertical position.

3. Design of Finite-Time Controller Design and Stability Analysis

3.1 *Design of a STA backstepping controller for a mobile platform*

The auxiliary tracking error and new states are defined as

$$z_{v1} = q_v - q_{vd}, \quad (13)$$

$$\zeta_{v1,1} = k_{v1} z_{v1} + |z_{v1}|^{1/2}\, sign(z_{v1}), \quad (14)$$

$$\zeta_{v1,2} = -\beta_{v1,2}\int_0^t [k_{v1}^2 z_{v1} + (3/2)k_{v1}|z_{v1}|^{1/2}\, sign(z_{v1}) + \gamma_{v1} sign(z_{v1})]d\tau. \quad (15)$$

$$\dot{\zeta}_{v1,1} = \xi_{v1}(x_{v2} - \dot{q}_{vd}), \quad (16)$$

$$\dot{\zeta}_{v1,2} = -\xi_{v1}\beta_{v1,2}\zeta_{v1,1}, \quad (17)$$

where $\xi_{v1} = k_{v1} + \gamma_v |z_{v1}|^{\gamma_v - 1}$. Define the virtual error surface and new states as

$$z_{v2} = x_{v2} - \alpha_{v1}, \quad (18)$$

$$\zeta_{v2,1} = k_{v2} z_{v2} + |z_{v2}|^{1/2}\, sign(z_{v2}), \quad (19)$$

$$\zeta_{v2,2} = -\beta_{v2,2}\int_0^t [k_{v2}^2 z_{v2} + (3/2)k_{v2}|z_{v2}|^{1/2}\, sign(z_{v2}) + \gamma_{v2} sign(z_{v2})]d\tau, \quad (20)$$

If we select the virtual control as follows:

$$\alpha_{v1} = -\beta_{v1,1}\zeta_{v1,1} + \dot{q}_{vd} + \zeta_{v1,2}, \quad (21)$$

(21) can be expressed as

$$\dot{\zeta}_{v1,1} = -\beta_{v1,1}\xi_{v1}\zeta_{v1,1} + \xi_{v1}\zeta_{v1,2} + \xi_{v1} z_{v2}. \quad (22)$$

$$\dot{\zeta}_{v1} = [\dot{\zeta}_{v1,1} \quad \dot{\zeta}_{v1,2}]^T = \xi_{v1} A_{v1}\zeta_{v1} + \xi_{v1} B_{z1}, \quad (23)$$

where $A_{v1}=\begin{bmatrix} -\beta_{v1,1} & 1 \\ -\beta_{v1,2} & 0 \end{bmatrix}$ and $B_{z1}=\begin{bmatrix} z_{v2} \\ 0 \end{bmatrix}$. Defining the Lyapunov function candidate $V_{v1}=(1/2)\zeta_{v1}^T P_{v1}\zeta_{v1}$,

$$\dot{V}_{v1} \le -\xi_{v1}\zeta_{v1}^T Q_{v1}\zeta_{v1} + \xi_{v1}\psi_{v1}\zeta_{v1} z_{v2}, \quad (24)$$

where $\psi_{v1}=(c_{v1}+4\varepsilon_{v1}^2)\zeta_{v1,1} - 2\varepsilon_{v1}\zeta_{v1,2}$; P_{v1} is a positive definite matrix defined with constants $c_{v1}>0$, $\varepsilon_{v1}>0$, $P_{v1}=\begin{bmatrix} c_{v1}+4\varepsilon_{v1}^2 & -2\varepsilon_{v1} \\ -2\varepsilon_{v1} & 1 \end{bmatrix}$ and is a solution of an algebraic Lypunov equation (ALE) $A_{v1}^T P_{v1} + P_{v1} A_{v1} = -2Q_{v1}$, and a positive definite symmetric matrix Q_{v1} is defined as

$$Q_{v1}=\begin{bmatrix} \beta_{v1,1}(c_{v1}+4\varepsilon_{v1}^2) - 4\beta_{v1,2}\varepsilon_{v1} & (c_{v1}+4\varepsilon_{v1}^2) - 2\varepsilon_{v1}\beta_{v1,1} + \beta_{v1,2} \\ (c_{v1}+4\varepsilon_{v1}^2) - 2\varepsilon_{v1}\beta_{v1,1} + \beta_{v1,2} & 4\varepsilon_{v1} \end{bmatrix}.$$

Defining the state vector as $\zeta_{v2}=[\zeta_{v2,1} \quad \zeta_{v2,2}]^T$ and one can then obtain that

$$\dot{\zeta}_{v2,1} = \xi_{v2}[-M_v^{-1} C_v \dot{q}_v - \rho_v + \bar{g}_v \tau_v) - \dot{\alpha}_{v1}], \quad (25)$$

$$\dot{\zeta}_{v2,2} = -\xi_{v2}\beta_{v2,2}\zeta_{v2,1}, \quad (26)$$

If we select the control law τ_v as

$$\tau_v = g_v^+\left[-\beta_{v2,1}\zeta_{v2,1} + M_v^{-1}C_v\dot{q}_v + \hat{\rho}_v - \frac{\xi_{v1}\psi_{v1}\zeta_{v1}}{\xi_{v2}\psi_{v2}\zeta_{v2}} z_{v2} + \dot{\alpha}_{v1}\right], \quad (27)$$

Defining the following Lyapunov function candidate as

$$V_{v2} = V_{v1} + (1/2)\zeta_{v2}^T P_{v2}\zeta_{v2} + (1/2\eta_v)\tilde{\rho}_v^T\tilde{\rho}_v, \quad (28)$$

where $\eta_v > 0$ is constant. The time derivative of V_{v2} is written as

$$\dot{V}_{v2} = \dot{V}_{v1} + \zeta_{v2}^T P_{v2}\dot{\zeta}_{v2} + \frac{1}{\eta_{\rho v}}\tilde{\rho}_v^T\dot{\tilde{\rho}}_v$$
$$\le -\sum_{i=1}^{2}\xi_{vi}\zeta_{vi}^T Q_{vi}\zeta_{vi} + \xi_{v1}\psi_{v1} z_{v2} - \xi_{v1}\psi_{v1} z_{v2} + \tilde{\rho}_v^T\left(\xi_{v2}\psi_{v2}\zeta_{v2} - \frac{1}{\eta_v}\dot{\hat{\rho}}_v\right), \quad (29)$$

where $\psi_{v2}=(c_{v2}+4\varepsilon_{v2}^2)\zeta_{v2,1} - 2\varepsilon_{v2}\zeta_{v2,2}$. If we select an adaptive law as

$$\dot{\hat{\rho}}_v = \eta_v \left(\xi_{v2} \psi_{v2} \zeta_{v2} - \eta_v' \hat{\rho}_v \right), \tag{30}$$

where $\eta_{\rho v}' > 0$ are constants, (50) can be written as:

$$\dot{V}_{v2} \le -\sum_{i=1}^{2} \xi_{vi} \zeta_{vi}^T Q_{vi} \zeta_{vi} + \eta_v' \tilde{\rho}_v^T \hat{\rho}_v$$

$$\le -\sum_{i=1}^{2} \xi_{vi} \zeta_{vi}^T Q_{vi} \zeta_{vi} - \frac{\eta_{\rho v}' \tilde{\rho}_v^T \tilde{\rho}_v}{2} + \frac{\eta_{\rho v}' \rho_v^T \rho_v}{2}. \tag{31}$$

3.2. *Design of a STA backstepping controller for a mobile platform*

The auxiliary tracking error and new states are defined as:

$$z_{m1} = q_m - q_{md}, \tag{32}$$

$$z_{m2} = x_{m2} - \alpha_{m1}, \tag{33}$$

$$\zeta_{m1,1} = k_{m1} z_{m1} + |z_{m1}|^{1/2} \, sign(z_{m1}), \tag{34}$$

$$\zeta_{m1,2} = -\beta_{m1,2} \int_0^t [k_{m1}^2 z_{m1} + (3/2) k_{m1} |z_{m1}|^{1/2} sign(z_{m1}) + \gamma_{m1} sign(z_{m1})] d\tau, \tag{35}$$

$$\zeta_{m2,1} = k_{m2} z_{m2} + |z_{m2}|^{1/2} \, sign(z_{m2}), \tag{36}$$

$$\zeta_{m2,2} = -\beta_{m2,2} \int_0^t [k_{m2}^2 z_{m2} + (3/2) k_{m2} |z_{m2}|^{1/2} sign(z_{m2}) + \gamma_{m2} sign(z_{m2})] d\tau. \tag{37}$$

We select the virtual control, the control law, and adaptive law as follows:

$$\alpha_{m1} = -\beta_{m1,1} \zeta_{m1,1} + \dot{q}_{md} + \zeta_{m1,2},$$

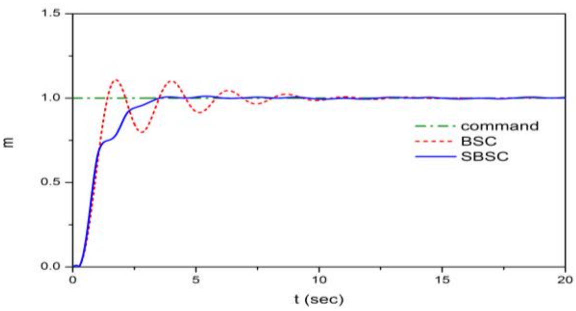
(38)

$$\tau_m = g_m^{-1} \left[M_m^{-1} (C_m \dot{q}_m + G_m) - \beta_{m2,1} \zeta_{m2,1} - \frac{\xi_{m1} \psi_{m1} \zeta_{m1}}{\xi_{m2} \psi_{m2} \zeta_{m2}} z_{m2} + \hat{\rho}_m + \dot{\alpha}_{m1} \right], \tag{39}$$

$$\dot{\hat{\rho}}_m = \eta_{\rho m} \left(\xi_{vm} \psi_{m2} \zeta_{m2} - \eta_{\rho m}' \hat{\rho}_m \right). \tag{69}$$

4. Simulation Example

To validate the proposed control scheme, the following group of one leader indexed by 0 and four followers indexed by 1, 2, 3, and 4, respectively as shown in Fig. 1. The strict feedback state equations of each agent are expressed as:



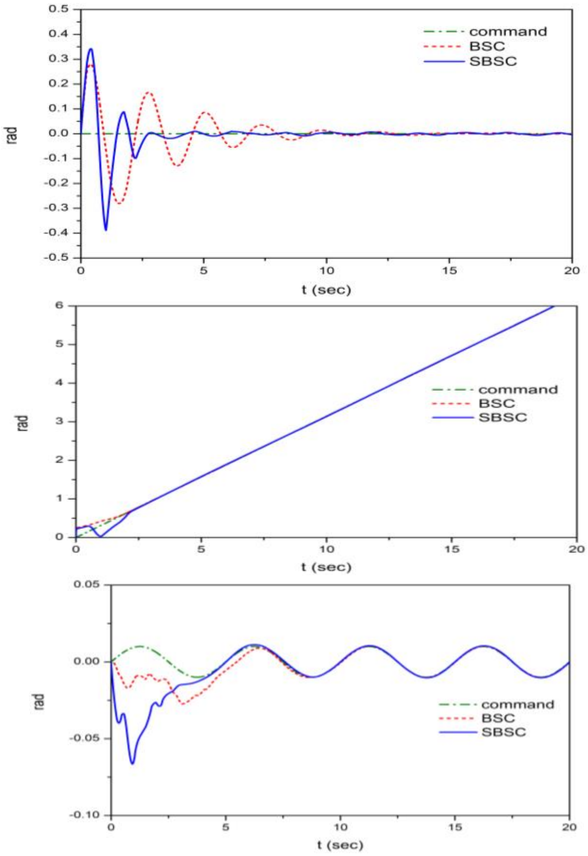


Fig. 2. Simulation result for proposed algorithm. (a) Mobile platforms tracking performance, (b) Pitch control of mobile platform, (c) Yaw control of mobile platform (d) Manipulator's position control(Sinusoidal input).

Above figure 2 illustrates simulation result for vehicle's tracking performance, pitch and yaw control, and manipulator's sinusoidal postion control.

5. Conclusion

A terminal backstepping control scheme to guarantee the fast error convergence and small tracking error performance for a multi-agent Euler-Lagrange system is developed in this paper. A virtual finite-time error surface is defined to design a virtual control. The finite-time convergence is proved by the finite-time stability analysis of Lyapunov function. Simulation for one-link manipulator agents confirms the theoretical proposal.

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