

# Feature Linking by Synchronized Response in Chaotic Cellular Neural Network for Visual Stimulus of Moving Objects

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## Abstract

A feature linking mechanism by the synchronized response of neural assemblies was studied for the chaotic cellular neural network (Chaotic-CNN). The Chaotic-CNN consists of chaotic spike response neurons that show the chaotic inter-spike-interval dynamics. In our scheme of feature linking, the features of the target objects are linked by the synchronized spike responses that are characterized by the temporal chaotic pattern of spike sequence. In this paper, we analyzed the synchronized spike responses that invoked by the visual stimulus of moving bars. As a result, neural assemblies have higher correlation for the visual stimulus of moving two bars in the same direction than the opposite direction. Then we discussed a possibility of feature linking by the chaotic synchronized response in the view point of neural coding.

*Keywords:* chaotic synchronization, neural coding, spike response model, feature linking

## 1. Introduction

For the brain system, feature linking is one of the principal functions to realize the recognition of visual objects. From the physiological experiments, the correlated firing among neurons is regarded as a candidate of the mechanism of such feature linking<sup>1,2</sup>. In the neural coding scheme using the synchronization of neurons, the feature linking information is represented by the synchronized firing among the corresponding neurons such that the features detected by neurons are linked if their firing pattern are synchronized.

On the other hand, the chaotic system is well known for its complex behavior. Comparing with the periodic spike pattern, the chaotic spike pattern is expected to have an advantage for the variety of represented

information. Therefore, there is a possibility to increase the performance of information representation by using a chaotic synchronization of spike responses<sup>3,4</sup>.

In the view point of neural coding, authors have studied the formation of the chaotic cell assemblies in the chaotic cellular neural network (Chaotic-CNN) that is a two dimensional coupled network of chaotic neurons<sup>5-7</sup>. The chaotic neuron is modeled by the chaotic spike response model (Chaotic-SRM) that is an extended SRM to show chaotic inter-spike interval by adding the background sinusoidal oscillation<sup>5-7</sup>.

In our previous study, chaotic cell assemblies were formed in the Chaotic-CNN for the visual stimulus when the stationary image was inputted<sup>6,7</sup>. In this research, we analyze the formation of chaotic cell assemblies for the

visual stimulus of moving objects and the feature linking property of them.

## 2. Chaotic Cellular Neural Network

The Chaotic-CNN is defined as a two dimensional coupled system of the Chaotic-SRM<sup>5-7</sup>. In the following, the definition of the neuron model and the network is described, respectively.

### 2.1 Neuron Model

As a neuron model, we use the Chaotic-SRM that is an extended spike response model (SRM). The spike response model (SRM) was introduced by Gerstner and Kistler<sup>8</sup>. The definition of Chaotic-SRM is as follows.

The membrane potential  $u(t)$  of the neuron at the time  $t$  is defined as

$$u(t) = u_{rest} + \eta(t - t^*) + \beta, \quad (1)$$

where  $u_{rest}$ ,  $t^*$ , and  $\beta$  denote the resting potential, the last firing time, and the external input, respectively. The kernel function  $\eta$  is defined as

$$\eta(t - t^*) = -\eta_{init} \exp\left(\frac{t - t^*}{\tau_{\eta_0}}\right) \Theta(t - t^*), \quad (2)$$

where  $\tau_{\eta_0}$  is the time constant of the spike response, and  $\Theta$  is a step function such that  $\Theta(s)$  is 1 for  $s \geq 0$  and 0 for the other values.

In this model, when the membrane potential exceeds the threshold value  $\theta$ , the neuron is firing and the membrane potential is reset by the update of the last firing time  $t^*$ . The term  $-\eta_{init}$  is an initial value of the kernel function  $\eta$  after firing.

We extended the original SRM to show the chaotic response by adding a background sinusoidal oscillation in the same way as the bifurcating neuron<sup>9</sup> and the chaotic pulse coupled neural network<sup>4</sup>. In our model, the background oscillation is added to the term  $\eta_{init}$  such that

$$\eta_{init} = \eta_0 - A_{\eta_0} \sin(2\pi\omega_{\eta_0} t^* + \phi), \quad (3)$$

where  $A_{\eta_0}$ ,  $\omega_{\eta_0}$ , and  $\phi$  denote the amplitude, the frequency, and the phase shift of the background oscillation, and  $\eta_0$  is constant. The Chaotic-SRM shows various chaotic behaviors depending on the parameters  $A_{\eta_0}$  and  $\beta$ .

### 2.2 Definition of Network

In the Chaotic-CNN, each neuron is located in the  $N \times M$  lattice and connected to neighbors. Let  $n_{x,y}$  be a neuron located at the position  $(x, y)$  where  $x \in \{0, 1, \dots, N-1\}$

and  $y \in \{0, 1, \dots, M-1\}$ . The membrane potential  $u_{x,y}$  of the neuron  $n_{x,y}$  is defined as

$$u_{x,y}(t) = u_{rest} + \eta(t - t_{x,y}^*) + \beta_{x,y} + \xi \times \sum_{n_{x',y'} \in B(x,y;r)} o_{x',y'}(t) \quad (4)$$

where  $u_{rest}$ ,  $t_{x,y}^*$ ,  $\beta_{x,y}$ ,  $\xi$  and  $B(x, y)$  denote the resting potential, the last firing time, the external input, the coupling weight, and the set of connected neurons, respectively. A set of connected neurons is defined as

$$B(x, y; r) = \{n_{x',y'} \mid (x', y') \neq (x, y), \max(|x - x'|, |y - y'|) \leq r, 0 \leq x' \leq N-1, 0 \leq y' \leq M-1\}. \quad (5)$$

The function  $o_{x',y'}$  is the output from the connected neuron  $n_{x',y'}$  and it is also defined as

$$o_{x',y'}(t) = \sum_{t_{x',y'}^{(k)} < t_{x',y'}^* + \Delta\epsilon < t} \epsilon(t - t_{x',y'}^{(k)} - \Delta\epsilon) \quad (6)$$

where  $t_{x',y'}^{(k)}$  denotes the  $k$ -th firing time of the neuron  $n_{x',y'}$  and  $\Delta\epsilon$  is the time delay of synaptic connection. The kernel function  $\epsilon$  describes the response of the synaptic connection. The definition of the kernel function  $\epsilon$  is

$$\epsilon(s) = \frac{s}{\tau_\epsilon} \exp\left(-\frac{s}{\tau_\epsilon}\right) \Theta(s), \quad (7)$$

where  $\tau_\epsilon$  is the time constant of the synaptic connection.

### 2.3 Gradient Field of Phase Shift

In this research, to emphasize the synchronized response in one direction, a gradient field of the phase shift is introduced in Eq.(3). The gradient field of the phase shift is defined as,

$$\phi_{x,y}(\alpha) = D(\cos(\alpha)x + \sin(\alpha)y) \bmod 2\pi, \quad (8)$$

where  $D$  is the constant of gradient and  $\alpha$  is the direction of gradient. Therefore  $\phi_{x,y}$  is the same value in the orthogonal direction of  $\alpha$  in the Chaotic-CNN.

## 3. Numerical Analysis of Coupled two Neurons

As a preliminary numerical experiment, we examined the synchronizing property of the coupled two neurons that corresponds to the Chaotic-CNN  $2 \times 1$ . In this case, the neuron  $n_{0,0}$  and  $n_{1,0}$  are coupled to each other. In the numerical simulation, the parameter values of the Chaotic-SRM are set as follows:  $u_{rest} = -70$  mv,  $\theta =$

$-35 \text{ mv}$ ,  $\eta_0 = 55$ ,  $\tau_\eta = 10$ ,  $\omega_{\eta_0} = \frac{0.75}{2\pi}$ ,  $A_{\eta_0} = 10.9$ ,  $\tau_\varepsilon = 1.5$  and  $\Delta\varepsilon = 0.1$ . The external input is chosen as  $\beta_{0,0} = \beta_{1,0} = 52.5$  such that the single neuron exhibits chaotic spike response.

As an index of the synchronization, the cross-correlation between two spike sequences is analyzed. Let  $S_{x,y}$  be a set of the firing times of the neuron  $n_{x,y}$ . The cross-correlation between  $S_{x,y}$  and  $S_{x',y'}$  is defined as

$$CC(S_{x,y}, S_{x',y'}; \Delta t) = \#(\{t_{x,y}^{(k)} \in S_{x,y} | \exists t_{x',y'}^{(l)} \in S_{x',y'}, |t_{x,y}^{(k)} - t_{x',y'}^{(l)} - \Delta t| \leq \Delta s\}) / \#(S_{x,y}) \quad (9)$$

where  $\#(X)$  denotes the number of elements of  $X$ , and  $\Delta s$  is the time resolution of the coincident firing. In this research,  $\Delta s$  is set to 0.5 ms. The auto-correlation is also defined as  $AC(S_{x,y}; \Delta t) = CC(S_{x,y}, S_{x,y}; \Delta t)$ . In the

case of  $\Delta t = 0$ , the cross-correlation  $CC(S_{x,y}, S_{x',y'}; 0)$  corresponds to the ratio of synchronized spikes.

We simulated these coupled two neurons and calculated the cross-correlation  $CC(S_{0,0}, S_{1,0}; 0)$  for generated spike sequences. Results are shown in Fig. 1 and Fig. 2. Fig. 1 shows the dependency of the cross-correlation to the coupling weight  $\xi$ . In the case that the phase shift is equivalent for two neurons ( $\phi_{0,0} = \phi_{1,0}$ ), the complete synchronization such that  $CC(S_{0,0}, S_{1,0}; 0) \simeq 1$  is achieved as increasing the coupling weight  $\xi$ . Fig. 2 shows the dependency of the cross-correlation to the difference of phase shifts ( $\phi_{1,0} - \phi_{0,0}$ ). The cross-correlation exhibits a selective preference for the case that the difference of phase shifts is 0. In this research, we apply this selective preference of coupled system to the feature linking.

## 4. Numerical Experiment

### 4.1 Experimental Model

In order to examine the feature linking property by the synchronized response, we construct the simulation model of the Chaotic-CNN  $40 \times 40$  and the motion picture of moving two bars is inputted as visual stimulus like the physiological experiments of cat visual cortex<sup>1</sup>. In this research, we also make two motion patterns. One is the moving two bars in the same direction (Fig. 3(a)) and the other is in the opposite direction (Fig. 3(b)).

The image of bar is represented by a rectangle of  $5 \times 12$  pixels. The velocity of bar is 0.07 pixels /msec. The external input  $\beta_{x,y}$  is 52.5 for the neurons inputted the image of the rectangle and 0 for the others. The parameter of the connection  $r$  is 2. The gradient field of the phase shift is set as  $D = 0.353$  and  $\alpha = 0$ .

### 4.2 Cross-correlation analysis

Numerical simulations are performed for the spike responses during 1,000 ms. Then, the cross-correlation is calculated for the neuron  $n_{20,10}$  and the others. The neuron  $n_{20,10}$  locates at the center of the way of the bar 1 in Fig. 3.

The cross-correlation for the moving two bar in the same direction (Fig. 3(a)) is shown in Fig. 4(a). Fig. 4(a) shows the formation of neural assemblies in which neurons are synchronized. Furthermore, the neural assembly corresponds to the bar 1 has high cross-correlation with the assembly of the bar 2. On the other hand, the cross-correlation for the moving two bar in the opposite direction (Fig. 3(b)) is shown in Fig. 4(b). Although Fig. 4(b) also shows the formation of neural

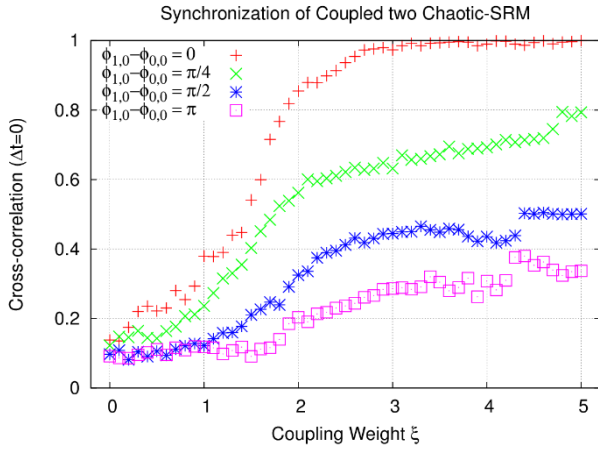


Fig. 1. The cross-correlation vs. the coupling weight  $\xi$  for the coupled two neurons.

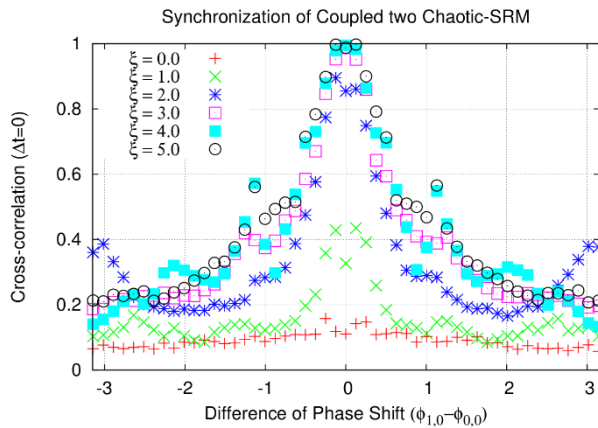


Fig. 2. The cross-correlation vs. the difference of the phase shift ( $\phi_{1,0} - \phi_{0,0}$ ) for the coupled two neurons.

assembly, the cross-correlation between the assembly of the bar 1 and the bar 3 is low.

## 5. Discussions

For the visual stimulus of moving objects, the formation of neural assemblies were observed in the Chaotic-CNN.

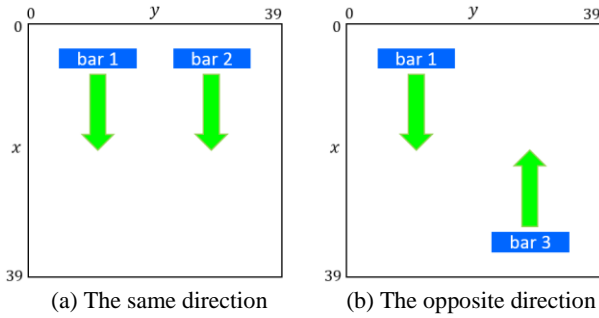


Fig. 3. Visual stimuli for the numerical experiment. The motion of moving two bars in the same direction (a) and the opposite direction (b).

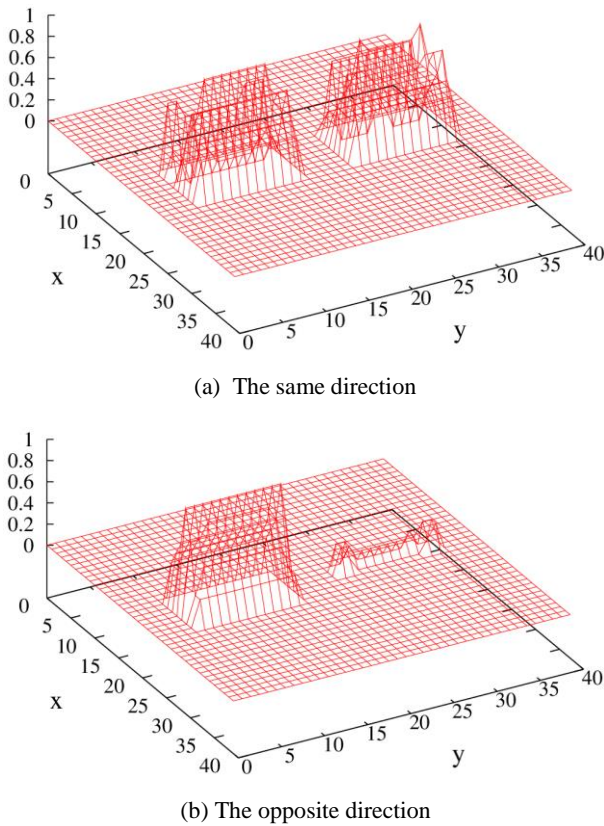


Fig. 4. The cross-correlation between the neuron  $n_{20,10}$  and the others when the visual stimuli as shown in Fig. 3 were inputted.

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By the cross-correlation analysis, the invoked neural assemblies have high correlation for the moving objects in the same direction and low correlation for the opposite direction. These results indicate a possibility of the feature linking by the synchronized spike responses in the Chaotic-CNN in terms of the cross-correlation. In our model, this linking property might be achieved by the preference for the difference of phase shifts mentioned in Sec. 3. Analysis of the dynamics of invoked spike sequences along the moving assembly and the application to the real image are our future works.

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