Elitist Differential Evolution for solving Numerical Optimization Problems

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Abstract:
In this paper, an elitist strategy is proposed for enhancing solution searching performance of Differential Evolution (DE). Also, a new variant of mutation for DE is proposed to improved population's exploration and prevent particles form fall into local optimum. In the experiments, 10 hybrid composition functions of CEC 2005 test functions are selected for testing performance of proposed method and compare it with 4 DE variants. From the results, it can be observed that the proposed method exhibits better than related works.

Keywords: Differential Evolution, elitist, optimization, population.

1 INTRODUCTION

In last four decades, more and more heuristic-based algorithms were proposed for solving numerical optimization, such as genetic algorithm (GA) [1] and particle swarm optimizer (PSO) [2], etc. In 1996, the concept of original differential evolution (DE) was proposed by Storn and Price [3]-[6]. The DE algorithm is a powerful population-based optimizer with simply concept to be implemented.

Due to there are many parameters in DE need to be decide before apply it on solving optimization problems. These parameters are very sensitive to different type of problems. Thus, in order to increase robust of DE, researchers are aiming to fine tune these parameters in last decade [8]-[12]. Similar to PSO, particles in DE will be moved according new moving vector, which is generated by variety mutation and crossover strategies. The selection will then be performed to preserve better particles for next generations.

In 2006, Brest *et al*. proposed self-adaptive differential evolution (jDE) [13], which is focus on adjust control parameters, such as crossover rate and scale factor, etc. In 2009, Huang and Suganthan proposed a DE variant named SaDE [14]. The trail vector generating strategies in SADE will change according to current solution search status for dealing with different kinds of problems. It is archived by analyzing the better and worse vectors to provide useful clue for selecting mutation strategy in next generation. Furthermore, the related parameters such as scale factor, crossover and population size, etc. will also self-adjust according current status. In the same year, Zhang and Sanderson proposed adaptive differential evolution named JADE [15]. The mutation strategy in JADE is refer to DE/current to best/1. Due to the particles will toward to around the global best particle due to the natural of current to best/1 mutation. It will make particles' convergence in early stage. Thus, in JADE, DE/current to best/1 is modified as DE/current to *pbest*. The *pbest* is selected form several better target vectors randomly. It will keep particles' diversity and improved DE's robust. Besides, the selected and unselected trail vector will also be recorded as a reference for parameters adjustment in next generation.

Recently, Islam et al. proposed a new variant of DE named MDE_pBX [16]. They proposed a new mutation strategy named DE/current to gr_best/1 for solving early convergence. The gr_gebest is a better target vector form *q* randomly selected target vectors. It can also prevent particles be clustered around the global best particle and avoid particle fall into local optimal. On the other hand, the crossover in MDE_Pbx is to random select two target vectors from *p* better vectors for information exchange. The *p*% value will decrease linearly by generations. It will enhance DE's convergence (deep search) in last stage of solution search process. However, a smaller population will decrease solution searching ability and diversity of DE/current to gr_best/1.

In this paper, a new variant of DE is proposed for solving numerical optimization problems. The proposed elitist strategy is referred to the DE/current to best/1 mutation, and to replace the *best particle* with *elitist particles*. It's an efficient way to solving early convergence problem and can avoid particles from fall into local optimum. Besides, the two of elitist particles will be randomly selected for crossover to discover more potential solution space. Finally, the scale factor is adjusted by Gaussian distribution.

This paper is consisted of five sections. The basic concept of differential evolution (DE) is described in section 2. The detail of proposed method is introduced in section 3. The experiments are presented in section 4. Finally, the conclusions are described in section 5.

2 Differential Evolution

Differential Evolution (DE), in recent years, is one of the popular optimizers. Its main advantages include have a few parameters, simply structure and fast convergence, etc. The DE is a population-based optimization algorithm. The members of population in DE are called parameter vectors. Particles' movement is according to trial vectors. The five common mutation strategies are listed as follows。

1. DE/rand/1

$$V_{i,G} = X_{r1,G} + F(X_{r2,G} - X_{r3,G}) \tag{1}$$

2. DE/best/1

$$V_{\mathrm{i,G}} = X_{\mathrm{best,G}} + F(X_{r1,G} - X_{r2,G}) \tag{2}$$

3. DE/current to best/1

$$V_{\mathrm{i,G}} = X_{i,G} + F\left(X_{best,G} - X_{i,G}\right) + F(X_{r1,G} - X_{\mathrm{r2,G}}) \tag{3}$$

4. DE/rand/2

$$V_{\mathrm{i,G}} = X_{r1,\mathrm{G}} + F(X_{r2,G} - X_{r3,G}) + F(X_{r4,G} - X_{r5,G}) \tag{4}$$

5. DE/best/2

$$V_{\mathrm{i,G}} = X_{\mathrm{best,G}} + F(X_{r1,G} - X_{r2,G}) + F(X_{r3,G} - X_{r4,G}) \tag{5}$$

where i, r_1, r_2, r_3, r_4, r_5 denote current selected particle and five random selected particles of population, respectively. And $\mathrm{i} \neq r_1 \neq r_2 \neq r_3 \neq r_4 \neq r_5$. The G represent current generation, X_{best} denote best particle of population and F denote scale factor where $F \in (0,1)$.

Unlike GA, the mutation process is executed before crossover. Due to the mutation in DE is main approach to explore better solutions in solution space. Thus, either mutation is adopted for generating new trail vector; the crossover will be performed by following equation.

$$\mathrm{u_{j,i,G}} = \begin{cases} \mathrm{V_{j,i,G}}\ , \mathrm{if}\ (\mathrm{rand_{i,j}}[0,1) \leq \mathrm{CR\ or\ j = j_{rand}}) \\ \mathrm{X_{j,i,G}}\ , \mathrm{otherwise} \end{cases} \tag{6}$$

where $\mathrm{CR} \in (0,1)$ denotes crossover rate, j denotes dimension. While the random number is smaller than or equal to CR, the trial vector ($\mathrm{u_{j,i,G}}$) will inherit mutated vector ($\mathrm{V_{i,G}}$). Otherwise, the trial vector ($\mathrm{u_{j,i,G}}$) will duplicate current particle's moving vector ($\mathrm{X_{i,G}}$). Finally, the selection in DE is performed by following equation.

$$X_{\mathrm{i,G+1}} = \begin{cases} u_{i,G}\ , \mathrm{if}\ f(u_{i,G}) \leq X_{\mathrm{i,G}} \\ X_{\mathrm{i,G}}\ , otherwise \end{cases} \tag{7}$$

3 PROPOSED METHOD

3.1 DE/current to elitist/1

In this paper, a new mutation structure is proposed, named DE/current to elitist/1. The vector update equation is listed as follows.

$$\mathrm{V_{i,G}} = \mathrm{X_{i,G}} + \mathrm{F}\left(\mathrm{X_{Elitist,G}} - \mathrm{X_{i,G}}\right) + \mathrm{F}\left(\mathrm{X_{r1,G}} - \mathrm{X_{r2,G}}\right) \tag{8}$$

where $\mathrm{X_{i,G}}$ denotes target vector, $\mathrm{V_{i,G}}$ represents donor vector, scale factor must be a positive integer between 0 and 1. The $\mathrm{X_{r1,G}}$ and $\mathrm{X_{r2,G}}$ are two random selected particles, and $\mathrm{X_{Elitist,G}}$ is the first E% better particles of population. Note that the $\mathrm{X_{i,G}}$ must not be the same as $\mathrm{X_{Elitist,G}}$. Thus, the selected elitist particle will be different for current selected particles. It can avoid particles are guided to the same position (global best particle) and can prevent particles fall into local optimum or early convergence.

3.2. Current to real-random

In this paper, there are two mutation strategies are proposed for deal with complex problems, especially the problems contain many local optimum solutions in solution space. In previous sub-section, the elitist mutation is introduced. In order to prove particles higher ability to jump out from local optimum, the DE/current to real-rand/1 is then proposed.

In traditional DE/current to rand/1, the random selected particle is belonging to current population. It seems a reasonable way to generate random vector. In fact, after several generations, particles will get closer to search specific area. The random selected particles may not be able to provide useful information to help other particles, which could trap in local optimum, to jump out to unsearched solution space. The proposed DE/current to real-rand/1 is listed as follows.

$$V_{i,G} = X_{i,G} + F\left(X_{T,G} - X_{Elitist,G}\right) + F\left(X_{r1,G} - X_{r2,G}\right) \tag{9}$$

where $\mathrm{X_{T,G}}$ is a random generated particle, which is not belonging to current. If particles can find one or more solutions to update the global best solution in k consecutive generations, the DE/current to real-rand/1 will be activated to generate potential moving vectors. It will allow more potential solutions to be found during the solution exploration, and find unsearched solution space. In this paper, the *k* is set as 50.

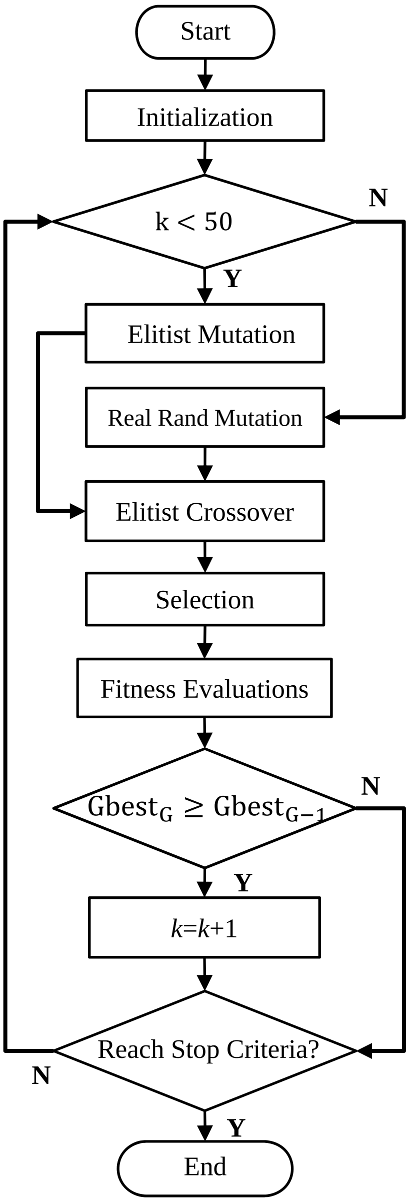

Fig. 1. Flowchart of this paper

3.3. Elitist crossover

In original DE, crossover is to combine mutated particle and random selected particle to produce new particles. In fact, the random selected particle could contain poor information and may not be able to generate better particle. In this paper, one of the *C*% better particles will be selected for join crossover with the mutated particle. It will increase particles' deep searching ability and speed up convergence.

3.4 Parameter adjustment

In this paper, the scale factor will be adjusted in each generation and will be generated by following equation.

$$\mathrm{F} = \mathrm{Gaussian}(\mathrm{F_u}, 0.1) \tag{10}$$

where the $\mathrm{F_u}$ is set as 0.5 in initial stage. After that, the $\mathrm{F_u}$ will be generated according to following equation.

$$\mathrm{F_u} = (1 - \mathrm{u_F}) * \mathrm{F_u} + \mathrm{F_u} * \mathrm{mean_L}(\mathrm{R}) \tag{11}$$

where $\mathrm{u_F}$ is a random generated number between 0.5 and 1. And the $mean_L(R)$ [17] is used to adjust *F* value, where *R* is survival rate in current generation.

3.5 Flowchart of proposed method

The flowchart of proposed is presented in figure 1 and the procedure of proposed method is listed as follows.

Step 1：Initialization and set the generation number G=0.

Step 2：If *k* >= 50 then execute step 3.1, else execute step 3.2

Step 3.1：DE/current to elitist/1, to generate mutated vector by (8)

Step 3.2：DE/current to real-rand/1, to generate mutated vector by (9)

Step 4：Elitist Crossover

Step 5：Selection by (7)

Step 6: Fitness Evaluations.

Step 7：Repeat step 2 to step 6, until meet the stop criteria.

4 EXPERIMANTS RESULTS

4.1. Test functions

In order to test the performance of proposed method and compare it with other variants of DE, ten hybrid composition functions, which are f_{16} ~ f_{25} of CEC 2005 [18], are selected for experiments. All the test functions are set as 30 dimensions. The global optimum, initialization range and search range of ten test functions are listed in table 1.

TABLE 1. GLOBAL OPTIMUM AND SEARCH RANGE OF TEST FUNCTIONS

Functions	Global Optimum	Initialization Range	Search Range
f_1~ f_2	0	$[-5, 5]^D$	$[-5, 5]^D$
f_3~ f_5	0	$[-5, 5]^D$	$[-5, 5]^D$
f_6~ f_8	0	$[-5, 5]^D$	$[-5, 5]^D$
f_9	0	$[-5, 5]^D$	$[-5, 5]^D$
f_{10}	0	$[2, 5]^D$	No Boundary

4.2. Parameters Settings

In the experiments, the population size for all DE variants are set as 100, each approaches are executed for 25 independent runs, and the maximum fitness evaluations (FEs) are set as 300,000. For fair comparison, the original parameters setting of jDE variants are adopted. For the basic DEs, which include DE/Rand/1, DE/best/1, and DE/Current to best/1, both the mutation and crossover rate are set as 0.9. For the proposed method, the F_u is set as 0.5, *CR* is set as 0.9, *E* is set as 15 and *C* is set as 25.

4.3. Experimental Results

The experiment results are listed in table 2 which presents the mean, standard deviation, and average computation time of 25 runs of the five variants of DE approaches on the 10 test functions with 30 dimensions. The best results among the five methods are shown in bold.

From the results, the proposed method performed with better results on most test functions can be observed. The jDE [13] performed best result on function 1 and 2.

TABLE 2. EXPERIMENT RESULTS

Methods	Results	f_1	f_2	f_3	f_4	f_5
Rand/1	Mean	2.8143E+02	3.4813E+02	8.3249E+02	8.3518E+02	8.3087E+02
	Std.	3.1917E+01	5.6843E+01	5.2431E+00	7.4596E+00	4.6088E+00
Best/1	Mean	9.1785E+02	9.6942E+02	1.3100E+03	1.3252E+03	1.3051E+03
	Std.	4.0104E+01	5.7750E+01	2.8452E+01	1.9686E+01	3.0350E+01
Current to best/1	Mean	1.4592E+02	1.8311E+02	8.1645E+02	8.1639E+02	8.1642E+02
	Std.	4.6975E+01	4.8953E+01	2.5121E-01	4.3215E-01	3.5078E-01
jDE	Mean	**1.0541E+02**	**2.0837E+02**	8.1738E+02	8.1738E+02	8.1740E+02
	Std.	**2.4564E+01**	**4.8086E+01**	5.9315E-01	5.2053E-01	5.2154E-01
Proposed Method	Mean	1.8257E+02	2.5625E+02	**8.1600E+02**	**8.1605E+02**	**8.1606E+02**
	Std.	1.9297E+01	3.6249E+01	**2.9137E-02**	**7.4758E-02**	**5.6439E-02**
Methods	**Results**	f_6	f_7	f_8	f_9	f_{10}
Rand/1	Mean	8.6260E+02	7.8090E+02	8.6636E+02	2.1275E+02	2.1236E+02
	Std.	1.0110E+00	1.7051E+02	1.3069E+00	1.6850E+00	1.2230E+00
Best/1	Mean	1.4328E+03	1.5227E+03	1.4392E+03	1.4446E+03	1.4577E+03
	Std.	2.6894E+01	1.0783E+02	2.9659E+01	1.7320E+01	1.4929E+01
Current to best/1	Mean	8.6570E+02	5.0009E+02	8.7159E+02	2.1098E+02	2.1086E+02
	Std.	4.7043E+00	2.6955E-01	2.7118E+00	7.7643E-01	5.4458E-01
jDE	Mean	8.5873E+02	5.0109E+02	8.6621E+02	2.1080E+02	2.1118E+02
	Std.	9.7320E-01	1.5504E+00	1.4459E+00	6.6982E-01	7.4161E-01
Proposed Method	Mean	**8.2198E+02**	**5.0001E+02**	**8.3194E+02**	**2.0925E+02**	**2.0904E+02**
	Std.	**1.8666E+01**	**1.2824E-02**	**1.4056E+01**	**1.7458E-01**	**3.4315E-01**

5 CONCLUSION

In this paper, two mutation strategies are proposed, which are DE/current to elitist/1 and DE/current to real-rand/1, to replace DE's mutation process. The DE/current to elitist/1 can produce more potential particles for improving particles' searching abilities, and can easier to find the global optimal solution. The DE/current to real-rand/1 can prevent particles from fall into the local optimum.

The experimental results proved that the proposed method can find better solutions than four DE variants. Ten hybrid composition functions were adopted for testing through a reasonable average. Form the results; it can be observed that the proposed has high ability to solve complex numerical optimization problems.

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