Rollover Prevention Control of Driver-Heavy Vehicle Systems

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In this paper, we propose an adaptive rollover prevention controller. At first, using a evaluation function, we propose a design method for an ideal vehicle model achieving good rollover control performance even if driver's steering characteristics vary. Next, an adaptive tracking controller is developed so that the behavior of the actual heavy vehicle can track that of the ideal vehicle model.

Keywords: Intelligent Truck & Heavy Duty Vehicle Control

1. INTRODUCTION

Recently, to present rollover of heavy vehicles, a lot of rollover prevention control schemes were developed^{[1]-[8]}. The papers [1]-[7] provided the schemes using the yaw moment force due to the torque difference between left and right. In the paper [8], the rollover prevention controller was proposed using the active steering of the front wheel. In the control schemes proposed in [1]-[8], only one control input is used. To prevent the rollover and achieve good steering performance, we have to control four states of the lateral velocity, the yaw rate, roll angle and the roll rate. Therefore, using the control inputs more than one, it is expected that better control performance can be achieve. To address the problem, the papers^{[9]-[10]} provided the control schemes using the lateral force of the front active steering and the yaw moment force, and the scheme using the active steering force of front and rear was proposed in [11]. However, there exist problems in the papers [9]-[11] that the uncertainties of the vehicle system parameters were not considered. In addition, the variation of the driver characteristics was also ignored. For example, there exist large differences in handle operation characteristics between young drivers and elder drivers. Even for the same driver, if the driver becomes tired, the handle operation characteristics may vary. Using the rollover prevention controller ignoring the facts, the designed control performance may not be achieved.

In the paper, to overcome the problems, we propose an adaptive rollover prevention controller. At first, we develop a design method of an ideal vehicle model achieving good rollover control performance even if the handle operation characteristics vary. Next, an adaptive tracking controller is developed so that the behavior of the actual vehicle can track of the ideal vehicle model.

2. DYNAMIC EQUATION OF VEHICLE

Consider the vehicle model shown in Fig.1. It is assumed that the pitch dynamics can be ignored. In addition, it is also assumed that the front steering angle, the rear steering angle and the roll angle are small. The dynamic equation using the new state can be



described as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B\left(\mathbf{b}_{u}\delta_{c}^{*}(t) + \mathbf{u}^{*}(t)\right) \\ \delta_{c}^{*} &= \delta_{c}(t) + \dot{\delta}_{c}(t), \ \mathbf{u}^{*}(t) = \mathbf{u}(t) + \dot{\mathbf{u}}(t) \\ \mathbf{x}(t) &= \left[\ \mathbf{q}_{p}^{T}(t), \mathbf{q}_{s}^{T}(t), \mathbf{\mu}^{T}(t) \ \right]^{T}, \ \mathbf{\mu} = T^{-1}\mathbf{\mu}_{1}(t) \\ \mathbf{\mu}_{1}(t) &= M_{\mu}^{-1}H_{T}^{T}K_{u}\left\{-v_{x}H_{T}\mathbf{q}_{v}(t) + \mathbf{b}_{u}\delta_{c}(t) + \mathbf{u}(t)\right\} \\ &- \frac{m_{s}h_{s}}{m(I_{x} + m_{s}h_{s}^{2}) - m_{s}^{2}h_{s}^{2}} \mathbf{b}_{u}\mathbf{g}_{s}^{T}\mathbf{q}_{s}(t) \\ M_{\mu} &= \operatorname{diag}\left[\frac{I_{x} + m_{s}h^{2}}{m(I_{x} + m_{s}^{2}h_{s}^{2})}, I_{z}\right], \ H = \left[\begin{array}{c} 1 & l_{f} \\ 1 & -l_{r}\end{array}\right] \\ H_{T} &= HT^{-1} \\ \mathbf{q}_{s}(t) &= \left[\phi(t) \ , \ \dot{\phi}(t)\right]^{T} \ , \ \mathbf{q}_{p}(t) = \left[v_{p}(t) \ , \ \psi(t)\right]^{T} \\ A_{1} &= -v_{x}D \ , \ A_{2} &= D - \frac{1}{I_{x} + m_{s}h_{s}^{2}} \mathbf{b}_{s}\mathbf{g}_{s}^{T} \\ A_{3} &= \frac{1}{I_{x} + m_{s}h_{s}^{2}} \mathbf{b}_{s}\mathbf{b}_{u}^{T}T \\ A_{4} &= -v_{x}^{-1}M_{p}^{-1}H^{T}K_{u}H(I + A_{1}) \\ A_{5} &= -m_{m}T^{-1}\mathbf{b}_{u}\mathbf{g}_{s}^{T}(I + A_{2}) \\ A_{6} &= -v_{x}^{-1}M_{p}^{-1}H^{T}K_{u}H - m_{m}T^{-1}\mathbf{b}_{u}\mathbf{g}_{s}^{T}A_{3} - I \\ M_{p} &= T^{T}M_{\mu}T \ , \ K_{u} &= \operatorname{diag}\left[k_{f}, k_{r}\right] \\ D &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 \end{bmatrix}, \ \mathbf{b}_{u} &= \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \ \mathbf{b}_{s} &= \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \\ \mathbf{g}_{s}^{T} &= \left[k_{\phi}l_{y}^{2} - m_{s}gh_{s}, d_{\phi}l_{y}^{2}\right], \ T &= I + l_{p}D \end{array} \right\}$$

In the dynamic equation, $\delta_c(t)$ is the driver's steering angle input

The Eighteenth International Symposium on Artificial Life and Robotics 2013 (AROB 18th '13), Daejeon Convention Center, Daejeon, Korea, January 30-February 1, 2013

Table 1 Meaning of Symbols					
v_x Longitudinal velocit	y h_u	Height of roll axis over ground			
v_p Lateral velocity	h_s	Height of C.G. over roll axis			
ψ Yaw rate	l_p	Distance C.G. to P_f			
ϕ Roll angle	l_f	Distance front axle to P_f			
$\dot{\phi}$ Roll rate	l_r	Distance rear axle to P_f			
g acceleration of gravi	ty l_y	Track width			
<i>m</i> Vehicle mass	k_{f}	Front cornering stiffness			
m_s Sprung mass	k_r^{j}	Rear cornering stiffness			
k_{ϕ} Spring constant	δ_c	Driver's steering input			
d_{ϕ} Damper constant	δ_f	Control input for front wheel			
I'_x Roll moment of iner	tia δ'_r	Control input for rear wheel			
I_z Yaw moment of iner	tia P_L	Vertical load for left wheel			
C.G.Center of gravity	P_R^-	Vertical load for right wheel			
P _f Reference point	10				

Table 2 Nominal value

m	14300	[kg]	k_{ϕ}	132100	[N/m]
m_s	12487	[kg]	d_{ϕ}	28905	[N·s/m]
h_u	0.68	[m]	I_x	24201	[kg·m ²]
h_s	1.15	[m]	I_z	34917	[kg⋅m ²]
l_f	3.95	[m]	k_f	582	[kN/rad]
l_r	4.54	[m]	k_r	783	[kN/rad]
l_y	1.86	[m]	g	9.81	$[m/s^2]$
l_p	0	[m]			

and $\boldsymbol{u}(t) = [\delta_f(t), \delta_r(t)]^T$ is the control steering input for the front wheel and the rear wheel. Definitions for the parameters of heavy vehicles are shown in Table 1.

The rollover index R(t) for the roll dynamics is defined as follows.

$$R(t) = \frac{P_L - P_R}{P_L + P_R} = \frac{2m_s \left(ha_{ys}(t) + gh_s \phi(t)\right)}{mgl_y} \\ = c_{\dot{v}p}^T \boldsymbol{x}(t) + c_o^T \boldsymbol{x}(t) \\ \dot{a}_{ys}(t) = \dot{v}_p(t) + v_x \psi(t) + l_p \dot{\psi}(t) - h_s \ddot{\phi}(t) \\ h = h_u + h_s \\ c_{\dot{v}p}^T \boldsymbol{x}(t) = \frac{2I_x m_s h}{mgl_y (I_x + m_s h_s^2)} ([1 \ 0] \boldsymbol{\mu} - v_x \psi(t)) \\ c_o^T \boldsymbol{x}(t) = \frac{2m_s}{mgl_y (I_x + m_s h_s^2)} (I_x h v_x \psi(t) \\ + (m_s gh_s^2 (h_s - h) + h_s (hk_\phi l_y^2 + gI_x))\phi(t) \\ + h_s hd_\phi l_y^2 \dot{\phi}(t) + [0 \ hl_p (I_x + m_s h_s^2)] \boldsymbol{\mu}(t)) \end{cases}$$
(3)

To represent the rollover index by using the state x(t), we introduce the state including the signal $\mu(t)$. The range of R(t) is $0 \le |R(t)| < 1$. If |R(t)| = 1, the total vertical loads is loaded on right wheel or left wheel. Then, we can see that vehicles were rollover.

When the values of system parameters are nominal values, the vehicle is called the nominal vehicle. Especially, in case of u(t) = 0, the vehicle is called the passive vehicle. Nominal values are shown in Table 2.

For the heavy vehicle (1), we make the following assumptions.

- A1 The lateral velocity $v_p(t)$ of vehicle, the yaw rate $\psi(t)$, the roll angle $\phi(t)$ and the roll rate $\dot{\phi}(t)$ are measured.
- A2 The distances l_f , l_r from the reference point P_f to the front axle and the rear axle are known.
- A3 The vehicle parameters except l_f , l_r include uncertainties.
- A4 The longitudinal velocity v_x is a bounded constant and measured.
- A5 The matrix A_2 is asymptotically stable.



Fig.2 Predictive driver model



Fig.3 Target lane

3. IDEAL VEHICLE MODEL

In order to design an ideal vehicle model in which the variations of the driver's characteristics is considered, we employ the predictive driver model given by

$$T_s \ddot{\delta}_c(t) + \dot{\delta}_c(t) = -g_p \dot{\tilde{y}}(t) - g_i \tilde{y}(t)$$
(4)

where g_p, g_i are the steering gains, T_s is the steering time constant and denotes the reaction delay of drivers and mechanical systems. To explain the predicted distance $\tilde{y}(t)$, the predictive driver model is shown in Fig.2. The solid line represents a state of the vehicle and the dashed line represents the ideal state where the vehicle tracks a target lane. $y_r(t)$ is the lateral distance between P_f and the target lane , $\varepsilon_r(t) = \varepsilon(t) - \varepsilon_d$ is the relative yaw angle between the vehicle and the target lane, ε_d is the yaw angle of the target lane and ρ is the curvature of target lane. The predicted distance $\tilde{y}(t)$ is the lateral distance at the preview point P_p and given by

$$\tilde{y}(t) = y_r(t) + T_p \dot{y_r}(t) \tag{5}$$

where T_p is the preview time constant. Drivers steer so that the predicted distance $\tilde{y}(t)$ can converge to zero.

Using the driver model, and carrying out numerical simulations, we propose the following design scheme for an ideal vehicle.

(1) We determine the values of weighting coefficients of the following evaluation function.

$$J(t) = \int \left(q_1 y_r^2(t) + q_2 \ddot{y}_r^2(t) + q_3 \varepsilon_r^2(t) \right) dt + q_4 \max |y_r^2(t)| + q_5 \max |\ddot{y}_r^2(t)| + q_6 \max |R^2(t)|$$
(6)

(2) We design the ideal vehicle model with the control input $\boldsymbol{u}_{M}^{*}(t)$ given by



$$\begin{aligned} \dot{\boldsymbol{x}}_{M}(t) &= \overline{A} \boldsymbol{x}_{M}(t) + \overline{B} \left(\boldsymbol{b}_{u} \boldsymbol{\delta}_{cM}^{*}(t) + \boldsymbol{u}_{M}^{*}(t) \right) \\ R_{M}(t) &= \overline{\boldsymbol{c}}_{ipp}^{T} \boldsymbol{x}_{M} + \overline{\boldsymbol{c}}_{o}^{T} \boldsymbol{x}_{M} \\ \boldsymbol{x}_{M}(t) &= \left[\begin{array}{c} \boldsymbol{q}_{pM}^{T}(t) , \ \boldsymbol{q}_{sM}^{T}(t) , \ \boldsymbol{\mu}_{M}^{T}(t) \end{array} \right]^{T} \end{aligned} \right\}$$
(7)

where denotes matrixes and vectors in which the values of elements of are nominal values and $\delta_{cM}^*(t)$ is the steering input $\delta_{cM}^*(t) = \delta_{cM}(t) + \dot{\delta}_{cM}(t)$ generated by $T_s \ddot{\delta}_{cM}(t) + \dot{\delta}_{cM} = -g_p \tilde{y}_M - g_i \tilde{y}_M$. The controller $u_M^*(t)$ is designed so that the values of the evaluation function (7) become less than that of the passive vehicle.

(3) We check the region of driver's parameters in which the following conditions are satisfied. In the following equations, $y_{rM}(t)$, $\varepsilon_{rM}(t)$ denote the lateral distance and the relative yaw angle in the case when the driver is driving the ideal vehicle model.

$$\begin{cases} |y_{rM}(t)| < 0.6[\text{m}], |\ddot{y}_{rM}(t)| < 3[\text{m/s}^2] \\ R_M(t)| < 0.85, \varepsilon_{\text{rM}} < 0.2[\text{rad}] \end{cases}$$
(8)

When $|y_{rM}(t)| < 0.6[m]$, the heavy vehicle doesn't stray from a driving lane. Moreover, when $|\ddot{y}_{rM}(t)| < 3[m/s^2]$, we consider that burdens hardly move.

(4) If the obtained region in (3) is large enough compared with that in the passive vehicle, the design of the ideal vehicle model is completed. Otherwise, we return to step (1) ~ (3).

Using a concrete example and carrying out numerical simulations, we show detail explanations of the design steps stated above . Fig.3 shows the target lane used in the example. Fig.3 (a) shows the target lane with the curvature ρ (driving maneuver 1). Fig.3 (b) shows the target lane with the curvature ρ and the angle θ_1 (driving maneuver 2). In the driving maneuver 1, the ideal model is checked for $\rho = 1/300[\text{m}] \sim 1/500[\text{m}]$. In the driving maneuver 2, the ideal vehicle model is checked for $\rho = 1/300[\text{m}] \sim 1/500[\text{m}]$ and $\theta_1 = 0[\text{rad}] \sim \pi/2[\text{rad}]$. For the driver parameters, we check the ideal vehicle model in the region of $T_s = 0.1[\text{s}] \sim 0.4[\text{s}], \text{T}_p = 1.5[\text{s}] \sim 4.5[\text{s}], \text{g}_p = 0 \sim 1, \text{g}_i = 0 \sim 1$ and the vehicle speed $v_x = 120[\text{Km/h}]$.

1) Determination of the weighting coefficients

 $40, q_4 = 10, q_5 = 5, q_6 = 20$ by the trial-and-error approach.

(2) Design of an ideal vehicle model

The controller for the ideal vehicle model (7) is developed as

$$\boldsymbol{u}_{M}^{*}(t) = -\frac{1}{2\varepsilon_{1}}\overline{B}^{T}P\boldsymbol{x}_{M}(t)$$
(9)

$$\overline{A}^{T}P + P\overline{A} - \frac{1}{\varepsilon_{1}}P\overline{B}\,\overline{B}^{T}P + \varepsilon_{2}\overline{c}_{\dot{v}p}\overline{c}_{\dot{v}p}^{T} + \varepsilon_{3}I = 0 \varepsilon_{1} = 0.1, \varepsilon_{2} = 0.2, \varepsilon_{3} = 0.01$$
 $\left. \right\}.$ (10)

Fig.4 shows the values of the evaluation function J of the ideal vehicle model and the passive vehicle model. The vertical axis is the values of J, x axis and y axis are steering gains g_i, g_p . In the case where J exceed 62, the values of J are fixed as 62.In Fig.4, v_x , T_s , T_p are set as $v_x = 120[\text{Km/h}]$, $T_s = 0.4[\text{s}]$ and $T_p = 1.5[\text{s}]$, and the values of the evaluation function J are shown for the driving maneuver 1 with $\rho = 1/300[\text{m}]$ and the driving maneuver 2 with $\rho = 1/300[\text{m}]$ and $\theta_1 = \pi/2[\text{rad}]$. It can be seen that the region of the ideal vehicle model where the values of J become less than 62 is larger than that of the passive vehicle. For lack of space, the simulation results for the other values of ρ, θ_1, v_x, T_s and T_p are not shown. However, it was ascertained that similar results to Fig.4 could be obtained.

(3) Evaluation of conditions

The conditions (8) are checked. As a result of the check, in the case where the values of J are less than 62, the conditions (8) are satisfied in the designed ideal vehicle.

(4) Dicision to end of the iteration

As shown in Fig.4(a) and (b), the obtained region where the conditions (8) are satisfied is large enough compared with that in the passive vehicle. Then the design of the ideal model is completed.

4. ADAPTIVE TRACKING CONTROLLER

In order to develop an adaptive tracking controller so that the behavior of the actual vehicle tracks that of the ideal vehicle model, the tracking errors are defined as

$$\left. \begin{aligned} \widetilde{\boldsymbol{q}}_p(t) &= \boldsymbol{q}_p(t) - \boldsymbol{q}_{pM}(t), \\ \widetilde{\boldsymbol{\mu}}(t) &= \boldsymbol{\mu}(t) - \boldsymbol{\mu}_M(t) \end{aligned} \right\}.$$
(11)

Moreover, the new tracking error signal $z(t) = \tilde{q}_p(t) + \tilde{\mu}(t)$ is also defined. Then, we have

$$\dot{\boldsymbol{z}}(t) = -d\boldsymbol{z}(t) + M_p^{-1} H^T K_u H \Big(H^{-1} \boldsymbol{u}^*(t) \\ -v_x^{-1} (A_1 + I) \boldsymbol{q}_p(t) + \Theta \boldsymbol{\omega}(t) \Big),$$
(12)

$$\Theta = \left(M_p^{-1}H^T K_u H\right)^{-1} \left[A_5, A_6, I\right]$$

$$\omega(t) = \left[q_s^T(t), \mu^T(t), \xi^T(t)\right]^T$$

$$\xi(t) = (dI + A_1)\tilde{q}_p(t) + (d+1)I\tilde{\mu}(t) - \dot{\mu}_M(t)$$
(13)

$$\left. \begin{array}{l} \dot{\boldsymbol{q}}_{s}(t) = A_{2} \boldsymbol{\tilde{q}}_{s}(t) + A_{3} \boldsymbol{\tilde{\mu}}(t) + \boldsymbol{\tilde{A}}_{2} \boldsymbol{q}_{sM}(t) + \boldsymbol{\tilde{A}}_{3} \boldsymbol{\mu}_{M}(t) \\ \boldsymbol{\tilde{A}}_{2} = A_{2} - \boldsymbol{\overline{A}}_{2} , \ \boldsymbol{\tilde{A}}_{3} = A_{3} - \boldsymbol{\overline{A}}_{3} \end{array} \right\},$$
(14)

Weighting coefficients are set as $q_1 = 26, q_2 = 10, q_3 = \frac{10}{166}$ where d is a positive design parameter, Θ is the unknown constant \odot ISAROB 2013

The Eighteenth International Symposium on Artificial Life and Robotics 2013 (AROB 18th '13), Daejeon Convention Center, Daejeon, Korea, January 30-February 1, 2013





matrix and $\boldsymbol{\omega}(t)$ is the known signal vector.

If the tracking error $\boldsymbol{z}(t)$ becomes asymptotically stable, it can be seen that the tracking errors $\tilde{\boldsymbol{q}}_p(t)$ and $\tilde{\boldsymbol{\mu}}(t)$ become asymptotically stable . Then, from (14) and the assumption, it can be also seen that the tracking error $\tilde{\boldsymbol{q}}_s(t)$ also becomes stable . Especially, in the case where \tilde{A}_2 and \tilde{A}_3 are zero matrixes , the tracking error $\tilde{\boldsymbol{q}}_s(t)$ becomes asymptotically stable .

Using the adaptive tracking controller given by

$$\dot{\boldsymbol{u}}(t) = -\boldsymbol{u}(t) - \beta H \boldsymbol{z}(t) + v_x^{-1} H (A_1 + I) \boldsymbol{q}_p(t) - H \hat{\Theta}(t) \boldsymbol{\omega}(t) , \quad \beta > 0 \dot{\hat{\Theta}}(t) = \gamma \boldsymbol{z}(t) \boldsymbol{\omega}^T(t) , \quad \gamma > 0$$

$$\left. \right\}, \quad (15)$$

the tracking errors become stable and the tracking error $\boldsymbol{z}(t)$ converges to zero. For lack of space, the proof is omitted.

5. NUMERICAL SIMULATION RESULTS

The numerical simulation results are shown to demonstrate the effectiveness of the proposed adaptive tracking controller. The driver parameters T_s , T_p , g_p , g_i are set as $T_s = 0.4$ [s], $T_p = 1.5$ [s], $g_p = 0.3$, $g_i = 0.5$. The target trajectory shown in Fig.3(b) is used. The parameters ρ and θ , are set as $\rho = 1/300$ [m], $\theta_1 = \pi/2$ [rad] and the longitudinal velocity is $v_x = 120$ [Km/h]. In the passive vehicle, if the driver parameters stated above are used, the rollover occurs and it very difficult to stabilize the vehicle.

Fig.5 shows the responses of the ideal vehicle, the controlled vehicle and the passive vehicle. Fig.5 (a) shows the responses of the rollover index, Fig.5 (b) shows the responses of the lateral deviation and Fig.5 (c) shows the responses of the relative yaw angle.

From Fig.5, it can be seen that the rollover occurs in the passive vehicle. On the other hand, the rollover didn't occur in the controlled vehicle. For lack of space, we couldn't show responses for the variations of the driver parameters and the vehicle system parameters. However, it was ascertained that the good control performance could be achieved. It can be concluded that for the variations of system parameters, the proposed rollover prevention controller has good robust performance.

6. CONCLUSIONS

In this paper, a new design method for an ideal vehicle model was proposed. It has been shown by carrying out numerical simulation that in the ideal vehicle, the good rollover control performance can be achieved. Moreover, we proposed an adaptive tracking controller so that the behavior of the actual heavy vehicle can track that of the ideal vehicle model even if the actual vehicle system parameters vary. Carrying out numerical simulations, the effectiveness of proposed rollover prevention controller was shown.

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