# Adaptive Steering Controller for Vehicles with Driving/Braking Force Distribution

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**Abstract**: In this paper, we propose an adaptive steering controller with driving/braking force distribution of direct yaw-moment. To achieve good steering performance, a design scheme for ideal vehicle model is proposed. To consider the effects of the driver dynamics, in the scheme, numerical simulations including a driver model are carried out and an ideal vehicle model having good steering performance is designed by using a trial and error method. To realize the good steering performance also in the actual vehicles, an adaptive tracking controller is developed so that the behavior of the actual vehicle can track that of the designed ideal vehicle model.

Key word: steering control of vehicle, adaptive tracking control, ideal vehicle model

### **1 INTRODUCTION**

When a vehicle is running on the low-µ road surface, it is very difficult to achieve good steering performance. In recent years, in order to overcome the problem, active Four-Wheel Steering(4WS) schemes have been proposed <sup>[1]-[4]</sup>. In the methods, it is assumed that tire lateral forces are linear with respect to steering angles and saturations are not occurred. If the saturations of tire lateral forces are ignored and a steering controller is designed, good steering performance cannot be expected. To address the problem, the control schemes<sup>[5]-[6]</sup> using Direct Yaw-moment Control(DYC) have been developed. However, in these schemes, the saturations of the tire driving/braking force have not been taken into account. Caused by the influence of the driving/braking force and the lateral forces, the saturations of tire forces may occur. Recently, a scheme<sup>[7]</sup> has been proposed that can prevent steering instability caused by the saturations of tire forces. To achieve the objective, in the scheme, an optimal distribution of driving/braking force was used and a sliding mode controller was employed to guarantee good robust performance. However, chattering of control input will occur. Furthermore, there is the problem that the influence of the driver characteristics has not been considered.

In order to improve the steering stability on the low- $\mu$  road surface, we propose a new adaptive steering control scheme with driving/braking force distribution. By employing an adaptive control scheme, chattering of control input does not occur and good robust performance can be achieved. In the control scheme, the variation of the driver characteristics is taken into consideration. Carried out numerical simulations including the driver characteristics, the effectiveness of the proposed controller is shown.

### **2 VEHICLE MODEL**

In this paper, we use the two degrees of freedom Four-Wheel Steering vehicle model shown in Fig.1. The explanations of parameters are shown in Table 1. The motion equation of the vehicle is given as follows. The state variable  $z(t) = \begin{bmatrix} v_y & \gamma \end{bmatrix}^T$  expresses lateral velocity  $v_y(t)$  and yaw rate  $\gamma(t)$  of vehicle, and  $u(t) = \begin{bmatrix} u_f & u_r & M \end{bmatrix}^T$  is input that includes steering angle of front and rear wheels and yaw-moment.

$$M_{c}\dot{z}(t) = \left(-v_{x}^{-1}HKH^{T} - M_{c}D\right)z(t) + HKu(t)$$

$$K = \begin{bmatrix} 2K_{f} & 0 & 0\\ 0 & 2K_{r} & \frac{1}{I_{z}} \end{bmatrix}, H = \begin{bmatrix} 1 & 1\\ l_{f} & -l_{r} \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & v_{x}\\ 0 & 0 \end{bmatrix}, M_{c} = \begin{bmatrix} m & 0\\ 0 & I_{z} \end{bmatrix}$$
(1)

# **3 CONVENTIONAL METHOD**<sup>[7]</sup>

In this section, we explain a distribution method and problems of conventional control system method.

#### 3.1 Necessity of driving/braking force distribution

If the resultant force vector of driving/braking force X and lateral force Y remains in the circle of radius  $\mu Z$  where Z is the vertical load of the tire and  $\mu$  road surface friction coefficient, the vehicle can be driven stably. The circle of radius  $\mu Z$  is called friction circle(see Fig.2) and represented by the following equation.

$$\sqrt{X^2 + Y^2} \le \mu Z \tag{2}$$

We defined the new signal  $\mu_i$  as the estimated load

$$\sqrt{\frac{X_i^2 + Y_i^2}{Z_i^2}} = \mu_i.$$
 (3)

If one of the estimated load  $\mu_i$  is greater than the road surface friction coefficient  $\mu$ , the saturation of the tire forces occur. Then, the behavior of vehicle will become unstable. Therefore, in order to prevent saturation of the tire force, it is necessary to redistribute the forces of tires.







Fig.2. Friction circle

### 3.2 Optimum minimax distribution

As mentioned in the previous subsection, no saturation of the tire force occurs as long as the following equation holds.

$$\max \mu_i \le \mu \tag{4}$$

To hold the equation, tire forces are redistributed so that the estimated load of the tires may become same as follows.

$$\mu_1 = \mu_3, \mu_2 = \mu_4 \tag{5}$$

### 3.3 Problems of the conventional method

In the paper[7], a sliding mode controller is proposed to achieve good robust tracking performance for the parameter uncertainty of the vehicle(Fig.3). However, in order to prevent chattering in the control input, the controller was approximated as

$$\operatorname{sgn}(\sigma_i) = \frac{\sigma_i}{|\sigma_i|} \approx \frac{\sigma_i}{|\sigma_i| + \tau_i}, \tau_i > 0 (i = 1, 2).$$
(6)

Reducing the value of  $\tau_i$ , control performance can be improved. However, chattering will occur in the control input. In addition, the driver characteristics have not considered in the control system shown in Fig.3.

# **4 DESIGN OF IDEAL VEHICLE MODEL**

### 4.1 Driver model

In this paper, in order to design the ideal vehicle model that takes into account the variation of the driver steering

Table 1. Notation of vehicle			
C.G.	center of gravity of vehicle		
m	mass of vehicle		
$m_s$	sprung mass		
$m_{fs}, m_{rs}$	unsprung mass of front and rear wheels		
$X_i, Y_i$	driving/braking force and lateral force of the tire i		
$Z_i$	vertical load of the tire		
М	yaw moment		
$\beta_i, \beta$	side slip angle of tire and vehicle body center		
γ	yaw rate		
$u_f, u_r$	steering angle of front and rear wheels		
$I_z$	yaw moment of inertia of vehicle		
v	longitudinal velocity		
$a_{y}$	lateral acceleration of vehicle center of gravity		
$a_x$	longitudinal acceleration of vehicle center of gravity		
$l_f, l_r$	distance from C.G to front and rear wheels		
l	wheelbase		
$t_r$	tread		
$K_f, K_r$	cornering stiffness of front and rear wheels		
$h_s$	height of the center of gravity of the sprung weight		

characteristics, we used a predicted driver model<sup>[8]</sup>. Steering dynamics is given as

$$T_s \ddot{\delta}_c(t) + \dot{\delta}_c(t) = -g_n \dot{\tilde{y}}(t) - g_i \tilde{y}(t)$$
(7)

where  $T_s$  is a steering constant including the delay of muscular system and mechanical systems,  $T_p$  is predicted time and  $g_p$ ,  $g_i$  are the gain steering. The predicted deviation  $\tilde{y}(t)$  at the predicted position  $P_p$  is given by

$$\tilde{y}(t) = y_r(t) + T_p \dot{y}_r(t)$$
(8)

where  $y_r(t)$  is relative deviation between target lane and vehicle.

### 4.2 Ideal vehicle model

Based on the design scheme of ideal model proposed in [8], we use the ideal vehicle model given by

$$v_{yd}(t) = L^{-1} \left[ \frac{g_v \omega^2 s}{s^2 + 2\xi \omega s + \omega^2} L[v_x \delta_c(t)] \right]$$
(9)

$$V_d(t) = L^{-1} \left[ \frac{g_{\gamma} \omega^2(s\eta + 1)}{s^2 + 2\xi \omega s + \omega^2} L[v_x \delta_c(t)] \right]$$
(10)

where *s* is Laplace operator, *L*,  $L^{-1}$  represent Laplace transform and Inverse Laplace transform,  $\omega, \xi, \eta, g_{\nu}, g_{\gamma}$  are positive design parameters.

In the case that commanded steering angle  $\delta_c(t)$  is constant(steady-state circular), ideal lateral velocity  $v_{yd}(t)$  is asymptotic to zero, and ideal yaw rate  $\gamma_d(t)$  is converged to a constant value as  $\lim_{t\to\infty} \gamma_d(t) = \overline{\gamma}_d$ . Then, the vehicle will be driven in contact with a circle of radius  $v_x/\overline{\gamma}_d$ . This is one of the advantages of the ideal vehicles model given by (9) and (10).

In below, we show a design method of parameters in the ideal vehicle model mentioned above.

(1) To prevent vibration in the ideal vehicle model,  $\xi$  sets as 1.

(2) In the passive vehicle, when the speed  $v_x$  is constant, the steady gain from the front wheel steering angle to the



Fig.3. Block diagram of the conventional scheme

 Table 2. Nominal values of vehicle parameter

т	1070Kg	$l_{f}$	1.033m
$m_s$	860Kg	$l_r$	1.657m
$m_{fs}$	120 Kg	t <sub>r</sub>	1.540m
$m_{rs}$	80 Kg	$h_{s}$	0.542m
$K_{f}$	29770N/rad	$K_r$	41460N/rad
$I_z$	1507[kgm <sup>2</sup> ]		

yaw rate is give by

$$g_{p\gamma} = \frac{2.10 \times 10^2}{5.25 \times 10^2 + 1.8 v_r(t)^2} \quad . \tag{11}$$

We set the parameter  $g_{\gamma}$  as  $g_{\gamma} = g_{p\gamma}$  so that the steady yaw rate gain of the ideal vehicle model becomes the same as that of the passive vehicle.

(3) Carrying out numerical simulations with the driver model and using a trial and error technology, we set the design parameters  $\omega$ ,  $g_{\nu}$ ,  $\eta$  so that good steering performance can be achieved for any driver's characteristics  $0.05 \text{ s} \le T_s \le 0.4 \text{ s}, 1.5 \text{ s} \le T_p \le 4.5 \text{ s}, 0 \le g_p \le 1$ ,  $0 \le g_i \le 1$ .

$$\omega = 5.4, g_{\nu} = 0.25, \eta = 0.39 \tag{12}$$

## **5 ADAPTIVE TRACKING CONTROLLER**

In order to design the adaptive steering controller which makes behavior of actual vehicle track that of ideal vehicle model, we define the following tracking error  $\tilde{z}(t)$ .

$$\tilde{z}(t) = z(t) - z_d(t) \tag{13}$$

Where  $z_d(t) = \begin{bmatrix} v_{yd} & \gamma_d \end{bmatrix}^T$  represents the state of the ideal vehicle model. If we can realize  $\lim_{t\to\infty} \tilde{z}(t) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ , the behavior of actual vehicle can track that of designed ideal vehicle model.

Error equation of the tracking error  $\tilde{z}(t)$  is given by

$$M_{c}\tilde{z}(t) = -v_{x}^{-1}HKH^{T}\tilde{z}(t) + HK\left[u(t) - v_{x}^{-1}H^{T}z_{d}(t) - \Theta(t)\boldsymbol{\omega}(t)\right], \qquad (14)$$

$$\Theta(t) = \left(M_c^{-1}HK\right)^{-1}, \qquad (15)$$



Fig.4. Block diagram of the proposed scheme

$$\boldsymbol{\omega}(t) = D\boldsymbol{z}(t) + M_c \dot{\boldsymbol{z}}_d(t). \tag{16}$$

The adaptive tracking controller is designed as

$$u(t) = -\beta H^T \tilde{z}(t) + v_x^{-1} H^T z_d(t) + \Theta(t) \boldsymbol{\omega}(t)$$
(17)

$$\hat{\Theta}(t) = -\tilde{z}(t)^T H \overline{K} \boldsymbol{\omega}(t)^T \Gamma, \tilde{\Theta}(t) = \Theta(t) - \hat{\Theta}(t)$$
(18)

where  $\beta$  is positive design parameter,  $\Gamma$  is positive definite matrix,  $\overline{K}$  is the nominal value of conering stiffness *K* and  $\hat{\Theta}(t)$  is the estimated matrix of unknown matrix  $\Theta(t)$ . Fig.4 shows the configuration of the proposed control system.

#### **6 NUMERICAL SIMULATION**

The numerical simulations were carried out in order to show the effectiveness of the proposed controller. In the simulations, the values of parameters of the vehicle are shown in Table 2. The driver parameters are set as  $T_s=0.1$ s,  $T_p=1.5$ s,  $g_p=g_i=0.05$ , and the target lane is a circular lane of radius 400m.

Fig.5(a), (b) show the responses of acceleration and longitudinal velocity of the vehicle. The responses of the controlled vehicle, the passive vehicle and the ideal vehicle model are shown in Fig.5(c)-(f). Fig.5 (c) shows the trajectory of vehicles, Fig.5(d) shows the responses of driver's steering angle, Fig.5(e) shows the responses of yaw rate of vehicles, and Fig.5(f) show responses of the relative deviation between target lane and the vehicle.

As shown in Fig.5(d), in the case of deceleration, vibration occurs in the responses of the steering angle and the yaw rate of the passive vehicle. However, in the controlled vehicle, the steering angle and the yaw rate become smooth. From this, it can be expected that driver's burdens is improved in the controlled vehicle. Moreover, as shown in Fig.5(f), the relative deviation between the passive vehicle and target lane is larger than that of the controlled vehicle. From this, it can be seen that the controlled vehicle is more stable than the passive vehicle.

### 7 CONCLUSION



Fig. 5 Simulation results

In this paper, we propose an adaptive steering controller with driving/braking force distribution to improve the steering stability in low- $\mu$  road surface. Carrying out numerical simulations, it has been shown that the control objective was achieved.

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