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Direct Multivariable PI Controller Tuning from Closed-Loop Response Data

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Abstract: This paper proposes a PI controller tuning method for multivariable plants. The method requires only one set of the input-output transient data of the plant under closed-loop operation to tune the controller. The data is used to obtain an appropriate controller parameter by solving a model matching problem of FRIT (Fictitious Reference Iterative Tuning) in time domain, and the data is also used in frequency domain to confirm if the parameter tuned by FRIT is stable and if the model matching is achieved. The method is applied to a non-interacting control of a gas turbine engine and its effectiveness is shown through simulations.

Keywords: CMA-ES, data driven controller tuning, FRIT, non-interacting control

1 INTRODUCTION

In recent years, in order to save the time and the cost to tune controller parameters for industrial systems, some direct controller parameter tuning methods from the transient data of the plant under closed-loop operation without modeling the plant have been proposed. The FRIT (Fictitious Reference Iterative Tuning) proposed by Souma et al [1] is one of those methods and is expected to be applied to some practical applications. However the FRIT does not show how to specify the reference model for the model matching to tune controller, and the stability of the parameter tuned by the FRIT is no clear. This paper shows how to resolve the problems using the information of the data in frequency domain. The method is applied to a controller tuning for a multivariable plant.

2 Controller tuning for multivariable plant

2.1 Problem setting

This paper deals with the closed-loop system for multivariable plant shown by **Fig.1**. The system consists of a plant given by a $n \times n$ transfer function matrix $\boldsymbol{P}(s)$ and a controller given by a $n \times n$ controller matrix $\boldsymbol{K}(\boldsymbol{\rho}, s)$ with the controller parameter $\boldsymbol{\rho}$. And $\boldsymbol{r}$, $\boldsymbol{e}$, $\boldsymbol{n}$, $\boldsymbol{u}$ and $\boldsymbol{y}$ are the reference vector, the error vector, the observation noise vector, the input and the output vectors of the plant, respectively. In order to make the explanation easier, the closed-loop system for a 2-input 2-output plant shown by **Fig. 2** is used in the following.

The controllers of the system are assumed to be PI controllers given by (1)~(4).

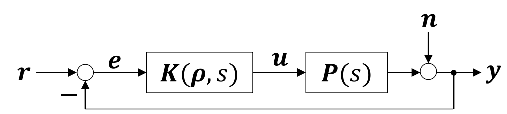

Fig. 1. Closed-loop system for multivariable plant

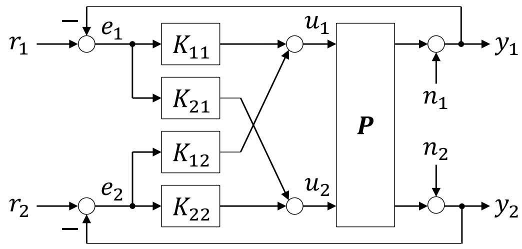

Fig. 2. Closed-loop system for 2-input 2-output plant

$$K_{11}(s) = k_{11}\frac{\tau_{11}s + 1}{\tau_{11}s} \tag{1}$$

$$K_{12}(s) = k_{12}\frac{\tau_{12}s + 1}{\tau_{12}s} \tag{2}$$

$$K_{21}(s) = k_{21}\frac{\tau_{21}s + 1}{\tau_{21}s} \tag{3}$$

$$K_{22}(s) = k_{22}\frac{\tau_{22}s + 1}{\tau_{22}s} \tag{4}$$

The controller matrix consists of (1)~(4) is given by

$$\boldsymbol{K}(\boldsymbol{\rho}, s) = \begin{pmatrix} K_{11}(s) & K_{12}(s) \\ K_{21}(s) & K_{22}(s) \end{pmatrix}. \tag{5}$$

The proportional gains and the integral time constants of the PI controllers are defined as the controller parameter $\boldsymbol{\rho}$ given by

$$\boldsymbol{\rho} = (k_{11}, k_{21}, k_{12}, k_{22}, \tau_{11}, \tau_{21}, \tau_{12}, \tau_{22}). \qquad (6)$$

The goal of this problem is to find the controller parameter such that a non-interacting control with stability is achieved. To achieve this, the loop transfer function matrix $\boldsymbol{P}(s)\boldsymbol{K}(\boldsymbol{\rho}, s)$ must be a diagonal matrix for non-interacting control and its diagonal elements must have appropriate gain crossover frequencies to satisfy the Nyquist stable criterion. Let us assume $\boldsymbol{L}^*(s)$ given by (7) to be one of such the loop transfer function matrix.

$$\boldsymbol{L}^*(s) = \mathrm{diag}[L_{11}^*(s), L_{22}^*(s)] \qquad (7)$$

If $\boldsymbol{P}(s)\boldsymbol{K}(\boldsymbol{\rho}, s)$ becomes $\boldsymbol{L}^*(s)$ by $\boldsymbol{\rho} = \boldsymbol{\rho}^*$, the transfer function matrix from $\boldsymbol{r}$ to $\boldsymbol{y}$ of the closed-loop system is

$$\boldsymbol{T}^*(s) = \mathrm{diag}[T_{11}^*(s), T_{22}^*(s)], \qquad (8)$$

where

$$T_{11}^*(s) = \frac{L_{11}^*(s)}{1 + L_{11}^*(s)} \qquad (9)$$

and

$$T_{22}^*(s) = \frac{L_{22}^*(s)}{1 + L_{22}^*(s)}. \qquad (10)$$

Therefore, the solution of this problem is to find $\boldsymbol{\rho}^*$ such that $\boldsymbol{P}(s)\boldsymbol{K}(\boldsymbol{\rho}^*, s)$ is as similar to $\boldsymbol{L}^*(s)$ as possible.

2.2 Controller tuning by FRIT

In order to find $\boldsymbol{\rho}^*$, the FRIT is employed. The FRIT requires only one set of input-output transient data of the plant under closed-loop operation. The fictitious reference $\tilde{\boldsymbol{r}}(t) = (\tilde{r}_1(t), \tilde{r}_2(t))^T$ given by (11) is used to find $\boldsymbol{\rho}^*$ without additional experiments.

$$\tilde{\boldsymbol{r}}(t) = \tilde{\boldsymbol{e}}(t) + \boldsymbol{y}_0(t), \qquad (11)$$

where $\tilde{\boldsymbol{e}}(t) = (\tilde{e}_1(t), \tilde{e}_2(t))^T$ is the fictitious error and given by

$$\tilde{\boldsymbol{e}}(t) = \boldsymbol{K}^{-1}(\boldsymbol{\rho}, t) * \boldsymbol{u}_0(t). \qquad (12)$$

$\boldsymbol{K}^{-1}(\boldsymbol{\rho}, t)$ shows the impulse response of the inverse matrix of the controller transfer function matrix $\boldsymbol{K}(\boldsymbol{\rho}, s)$. As (11) and (12) show, the fictitious reference $\tilde{\boldsymbol{r}}(t)$ and the fictitious error $\tilde{\boldsymbol{e}}(t)$ can be obtained only from the input data $\boldsymbol{u}_0(t) = (u_{10}(t), u_{20}(t))^T$ and the output data $\boldsymbol{y}_0(t) = (y_{10}(t), y_{20}(t))^T$ of the plant in the closed-loop system with the initial controller parameter $\boldsymbol{\rho_0}$.

The parameter $\boldsymbol{\rho}^*$ which makes the transfer function matrix from $\boldsymbol{r}$ to $\boldsymbol{y}$ of the closed-loop system be similar to $\boldsymbol{T}^*(s)$ can be obtained by (13) and (14) using some non-linear optimization methods.

$$\boldsymbol{\rho}^* = \arg\min_{\rho} \sum_{t}^{N-1} \boldsymbol{\varepsilon}^T(\boldsymbol{\rho}, t)\, \boldsymbol{\varepsilon}(\boldsymbol{\rho}, t) \qquad (13)$$

$$\boldsymbol{\varepsilon}(\boldsymbol{\rho}, t) = \boldsymbol{W}(t)\{\boldsymbol{y}_0(t) - \boldsymbol{T}^*(t) * \tilde{\boldsymbol{r}}(t)\} \qquad (14)$$

Here, N is the data length of the input-output data, $\boldsymbol{W}(s)$ is a weighting function diagonal matrix which specifies the frequency band to emphasis the error vector of the model matching $\boldsymbol{y}_0(t) - \boldsymbol{T}^*(t) * \tilde{\boldsymbol{r}}(t)$ for searching $\boldsymbol{\rho}^*$, and $\boldsymbol{W}(t)$ and $\boldsymbol{T}^*(t)$ are the impulse responses of $\boldsymbol{W}(s)$ and $\boldsymbol{T}^*(s)$, respectively.

2.3 Confirmation in frequency domain

It is difficult to specify $\boldsymbol{T}^*(s)$ or $\boldsymbol{L}^*(s)$ for unknown $\boldsymbol{P}(s)$. Therefore the information in frequency domain of $\boldsymbol{u}_0(t)$ and $\boldsymbol{y}_0(t)$ is also used.

When $\boldsymbol{u}(t) = \boldsymbol{u}_0(t)$ and $\boldsymbol{y}(t) = \boldsymbol{y}_0(t)$ and $\boldsymbol{n}=\boldsymbol{0}$ in **Fig.1**, (15) holds in frequency domain.

$$\begin{aligned} \boldsymbol{y}_0(j\omega) &= \boldsymbol{P}(j\omega)\boldsymbol{u}_0(j\omega) \\ &= \boldsymbol{L}(\boldsymbol{\rho}, j\omega)\tilde{\boldsymbol{e}}(j\omega), \end{aligned} \qquad (15)$$

where

$$\begin{aligned} \boldsymbol{L}(\boldsymbol{\rho}, j\omega) &= \boldsymbol{P}(s)\boldsymbol{K}(\boldsymbol{\rho}, j\omega) \\ &= \begin{pmatrix} L_{11}(\boldsymbol{\rho}, j\omega) & L_{12}(\boldsymbol{\rho}, j\omega) \\ L_{21}(\boldsymbol{\rho}, j\omega) & L_{22}(\boldsymbol{\rho}, j\omega) \end{pmatrix}. \end{aligned} \qquad (16)$$

From (15) and (16), (17) and (18) are derived.

$$\frac{y_{10}(j\omega)}{\tilde{e}_1(j\omega)} = L_{11}(\boldsymbol{\rho}, j\omega) + L_{12}(\boldsymbol{\rho}, j\omega)\frac{\tilde{e}_2(j\omega)}{\tilde{e}_1(j\omega)} \qquad (17)$$

$$\frac{y_{20}(j\omega)}{\tilde{e}_2(j\omega)} = L_{22}(\boldsymbol{\rho}, j\omega) + L_{21}(\boldsymbol{\rho}, j\omega)\frac{\tilde{e}_1(j\omega)}{\tilde{e}_2(j\omega)} \qquad (18)$$

Therefore if we can estimate $y_{10}(j\omega)/\tilde{e}_1(j\omega)$ and $y_{20}(j\omega)/\tilde{e}_2(j\omega)$ with $\boldsymbol{\rho} = \boldsymbol{\rho}^*$, and compare them to $L_{11}^*(j\omega)$ and $L_{22}^*(j\omega)$, respectively, we can investigate to see if $L_{11}(\boldsymbol{\rho}^*, j\omega) \simeq L_{11}^*(j\omega)$, $L_{12}(\boldsymbol{\rho}^*, j\omega) \simeq 0$, $L_{22}(\boldsymbol{\rho}^*, j\omega) \simeq L_{22}^*(j\omega)$ and $L_{21}(\boldsymbol{\rho}^*, j\omega) \simeq 0$. However $y_{01}(j\omega)/\tilde{e}_1(j\omega)$ and $y_{02}(j\omega)/\tilde{e}_2(j\omega)$ cannot be estimated from $y_{10}(j\omega)$, $\tilde{e}_1(j\omega)$, $y_{20}(j\omega)$ and $\tilde{e}_2(j\omega)$ directly since $y_{10}(t)$, $\tilde{e}_1(t)$, $y_{20}(t)$ and $\tilde{e}_2(t)$ are not absolute integrable and the Fourier transform cannot be applied to them. To resolve the problem, a band pass filter proposed by Matsui et al [2] is introduced. The filter is given by

$$F(s) = \frac{100T_s s}{(100T_s s + 1)(10T_s s + 1)}, \qquad (19)$$

where T_s is the sampling period for $\boldsymbol{u}_0(t)$ and $\boldsymbol{y}_0(t)$. The filter and a correlation method are used for the estimation. The estimated frequency responses of $y_{10}(j\omega)/\tilde{e}_1(j\omega)$ and $y_{20}(j\omega)/\tilde{e}_2(j\omega)$ are defined as $\hat{L}_{11}(\boldsymbol{\rho}, j\omega)$ and $\hat{L}_{22}(\boldsymbol{\rho}, j\omega)$, and are estimated by (20) and (21), respectively.

$$\hat{L}_{11}(\boldsymbol{\rho}, j\omega) = \frac{F[R_{ye1}(t)]}{F[R_{ee1}(t)]} \qquad (20)$$

$$\hat{L}_{22}(\boldsymbol{\rho}, j\omega) = \frac{F[R_{ye2}(t)]}{F[R_{ee2}(t)]} \qquad (21)$$

Here, $F[\cdot]$ denotes the Fourier transform, $R_{ye1}(t)$ and $R_{ye2}(t)$ are the cross-correlations, and $R_{ee1}(t)$ and $R_{ee2}(t)$ are the autocorrelations. The correlations are calculated as shown in (22)~(25) using $y_{10f}(t), y_{20f}(t)$, $\tilde{e}_{1f}(t)$ and $\tilde{e}_{2f}(t)$ which are all filtered $y_{10}(t), y_{20}(t)$, $\tilde{e}_1(t)$ and $\tilde{e}_2(t)$ by $F(s)$, respectively.

$$R_{ye1}(t) = \sum_{\tau=0}^{N-1} y_{10f}(\tau)\tilde{e}_{1f}(\tau + N - 1 - t) \qquad (22)$$

$$R_{ye2}(t) = \sum_{\tau=0}^{N-1} y_{20f}(\tau)\tilde{e}_{2f}(\tau + N - 1 - t) \qquad (23)$$

$$R_{ee1}(t) = \sum_{\tau=0}^{N-1} \tilde{e}_{1f}(\tau)\tilde{e}_{1f}(\tau + N - 1 - t) \qquad (24)$$

$$R_{ee2}(t) = \sum_{\tau=0}^{N-1} \tilde{e}_{2f}(\tau)\tilde{e}_{2f}(\tau + N - 1 - t) \qquad (25)$$

3 Numerical example

The LV100, which is taken from Hjalmarsson [3], is a gas turbine engine modeled as a continuous-time linear system with five state variables, two inputs and two outputs. The state variables are the gas generator spool speed, the power output, the temperature, the fuel flow and the variable area turbine nozzle. The inputs are the forth and the fifth state variables. The outputs are the first and the third state variables. The state matrix $\boldsymbol{A}_p$, the input matrix $\boldsymbol{B}_p$ and the output matrix $\boldsymbol{C}_p$ are given by (26), (27) and (28), respectively.

$$\boldsymbol{A}_p = \begin{pmatrix} -1.4122 & -0.0552 & 0 & 42.9536 & 6.3087 \\ 0.0927 & -0.1133 & 0 & 4.2204 & -0.7581 \\ -7.8467 & -0.2555 & -3.333 & 300.4167 & -4.4894 \\ 0 & 0 & 0 & -25.00 & 0 \\ 0 & 0 & 0 & 0 & -33.3333 \end{pmatrix} \qquad (26)$$

$$\boldsymbol{B}_p = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}^T \qquad (27)$$

$$\boldsymbol{C}_p = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \qquad (28)$$

The input-output transient data sets of the plant to find $\boldsymbol{\rho}^*$ were obtained in the simulation for the step reference response of the closed-loop system with $\boldsymbol{\rho_0}$ given by (29), and the data sets were saved as $\boldsymbol{u}_0(t)$ and $\boldsymbol{y}_0(t)$, respectively. The observation noises n_1 and n_2 which were white and independent each other were added in the simulation, and their means and variances were 0 and 0.0025, respectively. **Fig. 3** shows $\boldsymbol{y}_0(t)$.

$$\boldsymbol{\rho_0} = (1, 0.1, -1, 1, 10, 10, 10, 10) \qquad (29)$$

The reference model for the FRIT was given by

$$T_{11}^*(s) = T_{22}^*(s) = \frac{1}{T_d^2 s^2 + 2\zeta_d T_d s + 1}, \qquad (30)$$

where $T_d = 0.1$ and $\zeta_d = 0.7$. Then

$$L_{11}^*(s) = L_{22}^*(s) = \frac{1}{T_d^2 s^2 + 2\zeta_d T_d s}. \qquad (31)$$

The diagonal elements of $\boldsymbol{W}(s)$ were given by

$$W_{11}(s) = W_{22}(s) = \frac{1}{T_w^2 s^2 + 2\zeta_w T_w s + 1}, \qquad (32)$$

where $T_w = 0.05$ and $\zeta_w = 0.7$.

The parameter to achieve the model matching of (13) was obtained by the CMA-ES proposed by Hansen N [4] as given by

$$\boldsymbol{\rho}^* = (0.434, 0.449, 33.3, -4.59, 0.0935, 0.254, 3.70, 0.295)\,. \qquad (33)$$

Figs. 4 and **5** show that although the estimated frequency responses of $\hat{L}_{11}(\boldsymbol{\rho}^*, j\omega)$ and $\hat{L}_{22}(\boldsymbol{\rho}^*, j\omega)$ are contaminated by the noises at the high frequencies, they are very close to the true loop transfer functions $L_{11}^*(s)$ and $L_{22}(\boldsymbol{\rho}^*, s)$, they are similar to the specified reference

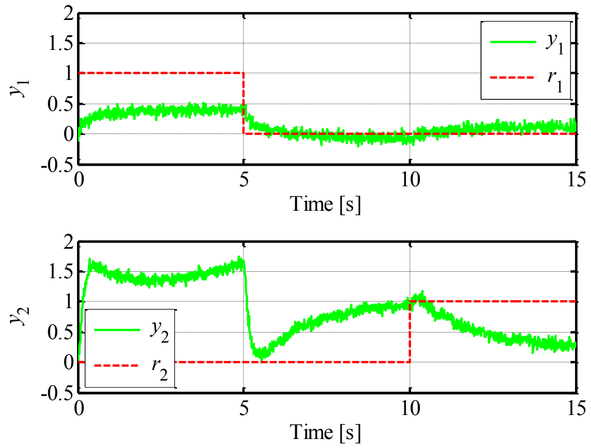

Fig. 3. Step reference responses with $\boldsymbol{\rho}_0$

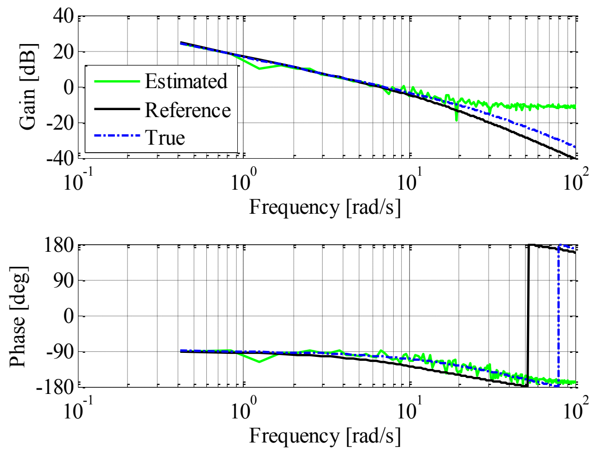

Fig. 4. Bode plots of $\hat{L}_{11}(\boldsymbol{\rho}^*, j\omega)$, $L_{11}^*(s)$ and $L_{11}(\boldsymbol{\rho}^*, s)$

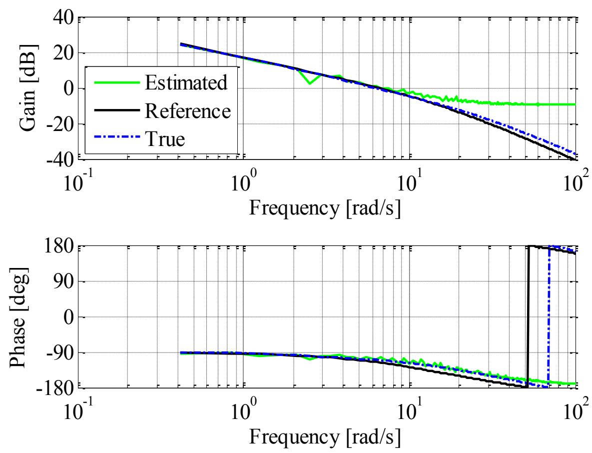

Fig. 5. Bode plots of $\hat{L}_{22}(\boldsymbol{\rho}^*, j\omega)$, $L_{22}^*(s)$ and $L_{22}(\boldsymbol{\rho}^*, s)$

transfer functions $L_{11}^*(s)$ and $L_{22}^*(s)$ at the frequencies less than the gain crossover frequencies, respectively, and they have enough phase margins. Therefore the reference models for the model matching and $\boldsymbol{\rho}^*$ were considered to be chosen and tuned appropriately, respectively. **Fig.6** shows that the step reference responses were improved significantly and a non-interacting control of $y_1(t)$ and $y_2(t)$ was achieved by $\boldsymbol{\rho}^*$.

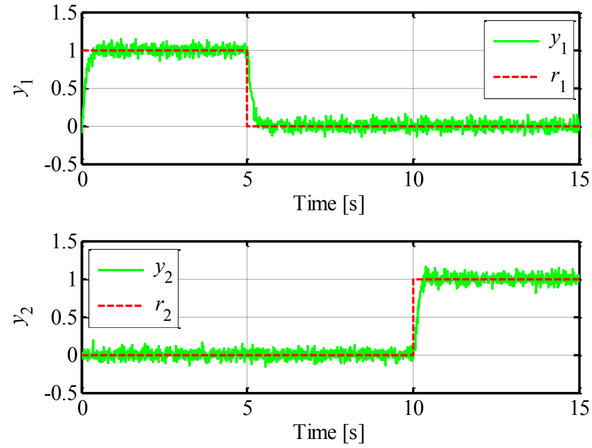

Fig. 6. Step reference responses with $\boldsymbol{\rho}^*$

4 CONCLUSION

A method to improve the weakness of the FRIT using the information of the experimental data in frequency domain was proposed. The method is able to show that the adequacy of the reference model for the model matching of the FRIT and the stability of the controller tuned by the FRIT. The effectiveness of the method was shown by a numerical example of a controller tuning for a multivariable system.

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