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Synthesis of continuous-time dynamic quantizers for LFT type quantized feedback systems

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Abstract: This paper focuses on analysis and synthesis methods of continuous-time dynamic quantizers for LFT type quantized feedback systems. Our aim is to find multiple (decentralized) quantizers such that a given linear system is optimally approximated by the given linear system with the quantizer in terms of invariant set analysis. In the case of minimum phase systems, this paper clarifies that optimal dynamic quantizers and its performance are parameterized by a design parameter. Also, an analytical relation between the static and dynamic quantizers will be presented.

Keywords: Quantizer, Continuous-time, LFT

1. INTRODUCTION

The cyber-physical system connects the physical systems with the information systems. In the research field of control theory, the control problem of such the system is one of the most active topics since the system covers various systems including discrete-valued-signals such as networked systems, hybrid systems, embed devices with D/A-A/D converters [1]. For the above challenging problem, it is important to focus on optimality of systems controlled by the discrete-valued signals. There clearly exits a difference between control performances of the systems controlled by the continuous-valued signal and the discrete-valued signal. Motivated by this, this paper focuses on the quantized feedback systems including the discrete-valued signals.

Considering optimality of quantized feedback systems, some existing results have provided optimal dynamic quantizers for the following problem formulation: When a plant and a controller are given in the usual feedback system, the framework synthesizes a dynamic quantizer that minimizes the maximum output difference between before and after the quantizer insert. In this case, the quantized feedback system with such the quantizer optimally approximates the original feedback system in the sense of the input-output relation. The two main types of dynamic quantizer are discrete-time and continuous-time settings. A number of the dynamic quantizer studies have been done in the discrete-time setting [2-5]. On the other hand, continuous-time dynamic quantizer is a key device for recent broadband wireless communication and mobile systems because of lower power and longer battery life compared with discrete-time ones [6]. Also, it is natural to consider the continuous-time setting in the sense that model uncertainty expressed in continuous-time domain is suitable for robust control of physical model.

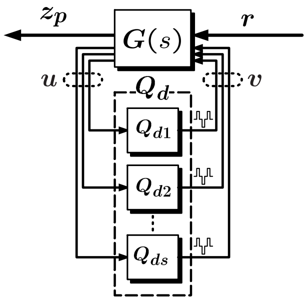
Motivated by the above, this paper considers a continuous-time dynamic quantizer design for quantized feedback systems. In particular, we consider the LFT (linear fractional transformation) type quantizer feedback system. Our early work has provides an optimal dynamic quantizer which is applicable for the centralized control system. Since the sensors and actuators are distributed in the networked control system, it is natural to implement multiple (decentralized) quantizers rather than a centralized quantizer for the I/O quantized feedback system. Focusing on the LFT type system that covers various quantized feedback systems, we will provide extension results of our early work [7]. As space is limited, this paper concentrates on the minimum phase case. We will clarify the effectiveness and the limitation of our proposed quantizers. In the case of minimum phase systems, it is clarified that optimal dynamic quantizers and its performance are parameterized by a design parameter.

Notation: The set of $n \times m$ (positive) real matrices is denoted by $\mathbb{R}^{n\times m}$ ($\mathbb{R}_+^{n\times m}$). The set of $n\times$ (positive) integer matrices is denoted by $\mathbb{N}^{n\times m}$ ($\mathbb{N}_+^{n\times m}$). $0_{n\times m}$ and I_m (or for simplicity of notation, 0 and I) denote the $n\times m$ zero matrix and the $m\times m$ identity matrix, respectively. For a matrix M, M^{T}, $\lambda(M)$, $\lambda_i(M)$ and $\lambda_{\max}(M)$ denote its transpose, its eigenvalue set, the i^{th} element of the set $\lambda(M)$ and its maximum eigenvalue, respectively. For a number $n \in \mathbb{N}_+$, $n!$ denotes its factorial. For a complex number c, $\mathbf{Re}(c)$ is its real part. For a vector x, x_i is the i^{th} entry of x. For a symmetric matrix X, $X > 0$ ($X \geq 0$) means that X is positive (semi) definite. For a full row rank matrix M, $M^\dagger$ denotes its pseudo inverse matrix which is given by $M^\dagger = M^{\mathrm{T}}(MM^{\mathrm{T}})^{-1}$. For a matrix X, $\|X\|_2$ denotes its 2-norms. Finally, we use the "packed" notation: $\left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right) := C(sI-A)^{-1}B+D$.

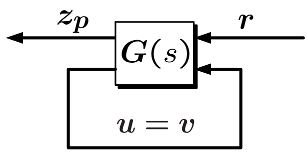
2. PROBLEM FORMULATION

Consider the quantized feedback system which consists of the LTI continuous-time plant $P(s)$ with the state $x_p \in \mathbb{R}^{n_p}$, the LTI continuous-time controller $C(s)$ with the state $x_c \in \mathbb{R}^{n_c}$, and the dynamic quantizers $v_1 = Q_{d_1}(u_1)$, $v_2 = Q_{d_2}(u_2)$. The systems $P(s)$ and $C(s)$ are given by

$$\begin{bmatrix} z_p \\ u_2 \end{bmatrix} = \left(\begin{array}{c|c} A_p & B_p \\ \hline C_{p1} & 0 \\ C_{p2} & 0 \end{array}\right) v_1, \quad u_1 = \left(\begin{array}{c|cc} A_c & B_{c1} & B_{c2} \\ \hline C_c & D_{c1} & D_{c2} \end{array}\right) \begin{bmatrix} v_2 \\ r \end{bmatrix}$$



(a) LFT type quantized feedback system with multiple quantizers



(b) Usual feedback system

Fig. 1. Generalized quantized and unquantized systems

where $z_p \in \mathbb{R}^q$, $r \in \mathbb{R}^p$, $u_1 \in \mathbb{R}^{m_1}$, $u_2 \in \mathbb{R}^{m_2}$, $v_1 \in \mathbb{R}^{m_1}$, and $v_2 \in \mathbb{R}^{m_2}$ denote the measured output, the exogenous input, the controller output, the plant measured output, the plant input, and the controller input, respectively. The continuous-valued signals u_1 and u_2 are quantized into the discrete-valued signals v_1 and v_2 because of $v_1 = Q_{d_1}(u_1)$ and $v_2 = Q_{d_2}(u_2)$. For the above systems, define the following vectors: $x_g := [\ x_p^{\mathrm{T}}\ \ x_c^{\mathrm{T}}\ ]^{\mathrm{T}} \in \mathbb{R}^{n_g} (n_g := n_p + n_c)$, $u := [\ u_1^{\mathrm{T}}\ \ u_2^{\mathrm{T}}\ ]^{\mathrm{T}}$, $v := [\ v_1^{\mathrm{T}}\ \ v_2^{\mathrm{T}}\ ]^{\mathrm{T}}$, and matrices:

$$A := \begin{bmatrix} A_p & 0 \\ 0 & A_c \end{bmatrix},\ B_1 := \begin{bmatrix} 0 \\ B_{c2} \end{bmatrix},\ B_2 := \begin{bmatrix} B_p & 0 \\ 0 & B_{c1} \end{bmatrix},$$
$$C_1 := \begin{bmatrix} C_{p1} & 0 \end{bmatrix},\ D_{11} := 0,\ \ C_2 := \begin{bmatrix} 0 & C_c \\ C_{p2} & 0 \end{bmatrix},$$
$$D_{21} := \begin{bmatrix} D_{c2} \\ 0 \end{bmatrix},\ \ D_{22} := \begin{bmatrix} 0 & D_{c1} \\ 0 & 0 \end{bmatrix},$$

and the quantizer $Q_d = \mathsf{diag}(Q_{d_1}, Q_{d_2})$. In this case, one gets the linear fractional transformation (LFT) type quantized feedback system in **Fig. 1 (a)** where the LTI continuous-time generalized plant $G(s)$ with the state $x_g \in \mathbb{R}^{n_g}$ is represented by

$$\begin{bmatrix} \dot{x}_g \\ z_p \\ y \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & 0 \\ C_2 & D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} x_g \\ r \\ v \end{bmatrix}. \qquad (1)$$

As shown above, the LFT formulation in (1) covers the various systems. Then this paper considers the LFT type quantized feedback systems. Also, we assume that the matrix $A+B_2(I-D_{22})^{-1}C_2$ is Hurwitz, that is, the usual feedback system in **Fig. 1 (b)** is stable in the continuous-time domain.

For the system $G(s)$, we define the discrete-valued vector $v := [\ v_1^{\mathrm{T}}, ..., v_s^{\mathrm{T}}\ ]^{\mathrm{T}} \in \mathbb{R}^m$ and the continuous-valued vector $u := [\ u_1^{\mathrm{T}}, ..., u_s^{\mathrm{T}}\ ]^{\mathrm{T}} \in \mathbb{R}^m$, respectively, and consider the dynamic quantizer $v = Q_d(u)$ which consists of the multiple dynamic quantizers $v_i = Q_{di}(u_i)$ $(i = 1, ..., s)$ with the state vector $x_{qi} \in \mathbb{R}^{n_{qi}}$. The case $s > 1$ implies that the multiple dynamic quantizers Q_{di} are distributedly implemented. The sub-quantizer Q_{di} consists of the static quantizer $q_{st} : \mathbb{R}^{m_i} \to d\mathbb{N}^{m_i}$ with the quantization interval $d \in \mathbb{R}_+$,

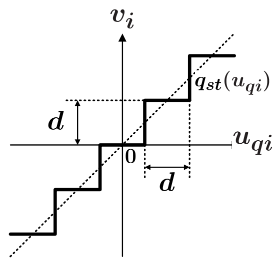


Fig. 2. Midtread type quantization

i.e.,

$$v_i = q_{st}(u_{qi}),\quad u_{qi} := v_{qi} + u_i$$

and the dynamic compensator $Q_i(s)$ given by

$$\begin{bmatrix} \dot{x}_{qi} \\ v_{qi} \end{bmatrix} = \begin{bmatrix} A_{qi} & B_{qi} \\ C_{qi} & 0 \end{bmatrix} \begin{bmatrix} x_{qi} \\ e_{qi} \end{bmatrix},\quad e_{qi} := v_i - u_i$$

where $v_i \in \mathbb{R}^{m_i}$, $u_i \in \mathbb{R}^{m_i}$, and $v_{qi} \in \mathbb{R}^{m_i}$. For $s = 2$, one gets the i/o type dynamic quantizers. Then the quantizer $Q_d := \mathsf{diag}(Q_{d1}, ..., Q_{ds})$ is realized by the static quantizer $Q_{st} := [q_{st}^{\mathrm{T}}, .., q_{st}^{\mathrm{T}}]^{\mathrm{T}} : \mathbb{R}^m \to d\mathbb{N}^m$, i.e.,

$$v = Q_{st}(u_q),\quad u_q := v_q + u \qquad (2)$$

and the compensator $Q(s) := \mathsf{diag}(Q_1(s), ..., Q_s(s))$, i.e.,

$$\begin{bmatrix} \dot{x}_q \\ v_q \end{bmatrix} = \begin{bmatrix} A_q & B_q \\ C_q & 0 \end{bmatrix} \begin{bmatrix} x_q \\ e_q \end{bmatrix}, \qquad (3)$$
$$A_q := \mathsf{diag}(A_{q1}, ..., A_{qs}),\ B_q := \mathsf{diag}(B_{q1}, ..., B_{qs}),$$
$$C_q := \mathsf{diag}(C_{q1}, ..., C_{qs})$$

where $x_q := [\ x_{q1}^{\mathrm{T}}, ..., x_{qs}^{\mathrm{T}}\ ]^{\mathrm{T}} \in \mathbb{R}^{n_q}$, $v_q := [\ v_{q1}^{\mathrm{T}}, ..., v_{qs}^{\mathrm{T}}\ ]^{\mathrm{T}} \in \mathbb{R}^m$, $u_q := [\ u_{q1}^{\mathrm{T}}, ..., u_{qs}^{\mathrm{T}}\ ]^{\mathrm{T}} \in \mathbb{R}^m$ and $e_q := [\ e_{q1}^{\mathrm{T}}, ..., e_{qs}^{\mathrm{T}}\ ]^{\mathrm{T}} \in \mathbb{R}^m$. Note that q_{st} is of the nearest-neighbor type toward $-\infty$ with the quantization interval $d \in \mathbb{R}_+$ and the initial state is given by $x_q(0) = 0$ for the drift-free of Q_d[2, 3] such as the midtread type quantizer in **Fig. 2**.

For the LFT system with $G(s)$ and Q_d in **Fig. 1 (a)** with the initial state $x_0 = x_g(0)$ and the exogenous signal $r \in \mathcal{L}_\infty^p$, $z_p(t, x_0, r)$ denotes the output of z_p at the time t. Also, for the system in **Fig. 1 (b)** without Q_d, $z_p^*(t, x_0, r)$ denotes its output at the time t. This paper considers the following cost function:

$$J(Q_d) := \sup_{(x_0, r) \in \mathbb{R}^{n_g} \times \mathcal{L}_\infty^p} z(x_0, r)$$

where

$$z(x_0, r) := \max_i \sup_t |z_{pi}(t, x_0, r) - z_{pi}^*(t, x_0, r)|$$

and z_{pi}, z_{pi}^* denote the i^{th} entry of z_p and z_p^*, respectively.

If the quantizer Q_d minimizes $J(Q_d)$, the system in **Fig. 1 (a)** optimally approximates the usual system in **Fig. 1 (b)** in the sense of the input-output relation. In this case, we can use the existing continuous-time controller design methods for the system in **Fig. 1 (b)**.

Motivated by the above, our objective is to solve the following continuous-time dynamic quantizer synthesis problem **(E)**: *For the LFT system composed of (1), (2) and (3) with the initial state* $x_0 \in \mathbb{R}^{n_g}$ *and the exogenous signal* $r \in \mathcal{L}_\infty^p$*, suppose that the quantization interval* $d \in \mathbb{R}_+$ *and the performance level* $\gamma \in \mathbb{R}_+$ *are given. Characterize a stable continuous-time dynamic quantizer* Q_d *(i.e., find parameters* $(n_{qi}, A_{qi}, B_{qi}, C_{qi})$*) achieving* $J(Q_d) \le \gamma$.

3. MAIN RESULT

Define the following matrices:

$$\mathsf{D} := (I - D_{22})^{-1}, \quad \mathsf{C} := \mathsf{D}C_2, \quad \mathsf{A} := A + B_2\mathsf{C},$$
$$\mathsf{B} := B_2\mathsf{D}, \ \mathcal{A} := \begin{bmatrix} \mathsf{A} & \mathsf{B}C_q \\ 0 & A_q + B_qC_q \end{bmatrix}, \ \mathcal{B} := \begin{bmatrix} \mathsf{B} \\ B_q \end{bmatrix},$$
$$\mathcal{C} := \begin{bmatrix} C_1 & 0 \end{bmatrix}.$$

For the matrix $\mathsf{B} \in \mathbb{R}^{n_g \times m}$ and $i = 1, ..., s$, $\mathsf{B}_i \in \mathbb{R}^{n_g \times m_i}$ denotes the i^{th} block column of B, i.e., $\mathsf{B} := [\, \mathsf{B}_1, ..., \mathsf{B}_s \,]$.

Assumption 1: For every $i = 1, ..., s$, the matrix $C_1\mathsf{A}^{\tau_i}\mathsf{B}_i$ is full row rank where $\tau_i \in \{0\} \cup \mathbb{N}_+$ is the smallest integer satisfying $C_1\mathsf{A}^{\tau_i}\mathsf{B}_i \neq 0$.

In quantizer analysis, by using our early result [7], we obtain the the optimization problem **(Aop)**:

$$\min_{\mathcal{P}>0, \min_i\{|\mathbf{Re}(2\lambda_i(\mathcal{A}))|\}>\alpha>0, \gamma>0} \gamma \quad \text{s.t.}$$
$$\begin{bmatrix} \mathcal{A}^{\mathrm{T}}\mathcal{P} + \mathcal{P}\mathcal{A} + \alpha\mathcal{P} & \mathcal{P}\mathcal{B} \\ \mathcal{B}^{\mathrm{T}}\mathcal{P} & -\frac{4\alpha}{md^2}I_m \end{bmatrix} \le 0, \tag{4}$$
$$\begin{bmatrix} \mathcal{P} & \mathcal{C}^{\mathrm{T}} \\ \mathcal{C} & \gamma^2 I_q \end{bmatrix} \ge 0. \tag{5}$$

That is, the performance level γ in (5) evaluates the upperbound of the difference between $z_p^*(t, x_0, r)$ and $z_p(t, x_0, r)$ within in invariant set framework, and

$$J(Q_d) \le \gamma$$

holds. Also, the infimum of γ can be expressed by the following lemma [7].

Lemma 1: *Suppose that the quantization interval* $d \in \mathbb{R}_+$ *is given. Consider the problem* **(Aop)**. *The infimum of* γ *is given by*

$$\inf \gamma = \inf_\alpha \frac{d\sqrt{m}}{2\sqrt{\alpha}} \sqrt{\lambda_{\max}\mathcal{D}(\alpha)}, \tag{6}$$
$$\mathcal{D}(\alpha) := \int_0^\infty \mathcal{C}e^{(\mathcal{A}+\alpha/2I)t}\mathcal{B}\mathcal{B}^{\mathrm{T}}e^{(\mathcal{A}+\alpha/2I)^{\mathrm{T}}t}\mathcal{C}^{\mathrm{T}}dt,$$
$$\alpha \in (0, \min_i\{|\mathbf{Re}(2\lambda_i(\mathcal{A}))|\}).$$

The problem **(Aop)** suggests that the quantizer synthesis problem **(E)** reduces to the following non-convex optimization problem **(OP)**:

$$\min_{\mathcal{P}>0, A_q, B_q, C_q, \bar{\alpha}>\alpha>0, \gamma>0} \gamma \quad \text{s.t. (4) and (5)}$$

where $\bar{\alpha} := \min_i\{|\mathbf{Re}(2\lambda_i(\mathcal{A}))|\}$. That is, if **(OP)** is feasible, **(E)** is feasible and the obtained quantizer is stable. Under some circumstances, we obtain an closed form solution from (6) as follow.

Theorem 1: *Consider the non-convex optimization problem* **(OP)**. *Suppose that* $s > 1$ *and Assumption 1 holds. For a given scalar* $f \in \mathbb{R}_+$*, an optimal solution of* $(n_{qi}, A_{qi}, B_{qi}, C_{qi})$ $(i = 1, 2, ..., s)$ *and its infimum of* $\gamma \in \mathbb{R}_+$ *to the problem* **(OP)** *are given by*

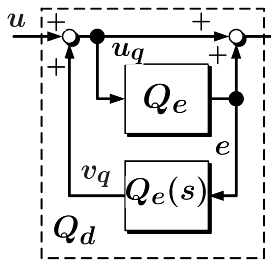
$$\begin{cases} n_{qi} = n_g, \quad A_q = \mathsf{A}, \quad B_{qi} = \mathsf{B}_i \\ C_{qi} = -(C_1\mathsf{A}^{\tau_i}\mathsf{B}_i)^\dagger C_1(\mathsf{A} + fI)^{\tau_i+1} \end{cases} \tag{7}$$

for every $i = 1, 2, ..., s$ *and*

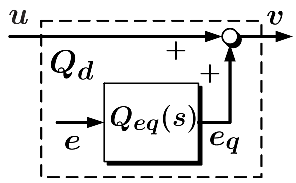
$$\inf \gamma = \frac{d\sqrt{m}}{2\sqrt{\rho}} \| [\, \sigma_{\rho 1} \ \dots \ \sigma_{\rho i} \ \dots \ \sigma_{\rho s} \,] \|_2, \tag{8}$$
$$\sigma_{\rho i} := \sqrt{\frac{(2\tau_i)!}{(\tau_i!)^2(2f-\rho)^{2\tau_i+1}}}\, C_1\mathsf{A}^{\tau_i}\mathsf{B}_i,$$
$$\rho = \min_i\{|\mathbf{Re}(2\lambda_i(\mathsf{A}))|, |\mathbf{Re}(2\lambda_i(A_q + B_qC_q))|\}$$

if the matrix $A_{qi} + B_{qi}C_{qi}$ *defined in (7) is Hurwitz for every* $i = 1, 2, ..., s$.

In this paper, we call the quantizer in (7) the decentralized optimal dynamic quantizer Q_d^{op}. Theorem 1 indicates that Q_d^{op} and its achievable performance are parameterized by the scalar f. Next, this paper considers the relation between the scalar f and the stability of Q_d^{op} for the simple case $q = m$, in addition, presents a tractable adjustable range of f similar to [7]. For the simplicity, we consider the centralized quantizer case $s = 1$ and $\tau_i = \tau$.



(a) Quantizer Q_d with $Q_e(s)$



(b) Quantizer Q_d with $Q_{eq}(s)$

Fig. 3. Equivalent expression of dynamic quantizer Q_d

As shown in **Fig. 3 (a)**, an equivalent expression of the quantizer Q_d is given by the quantization error Q_e and $Q_e(s) := C_q(sI - (A_q + B_qC_q))^{-1}B_q$. The quantizer structure in **Fig. 3 (a)** can be recast as the system with $Q_{eq}(s) := Q_e(s) + I$ in **Fig. 3 (b)** where the signal e is the difference $e := v - u_q$ and $e_q = v_q + e$. For the quantizer Q_d^{op}, then one gets

$$Q_{eq}^{op}(s) := \left(\begin{array}{c|c} (\mathsf{A} - \mathsf{B}(C_1\mathsf{A}^\tau\mathsf{B})^{-1}C_1(\mathsf{A} + fI)^{\tau+1} & \mathsf{B} \\ \hline -(C_1\mathsf{A}^\tau\mathsf{B})^{-1}C_1(\mathsf{A} + fI)^{\tau+1} & I \end{array} \right)$$

which is corresponding to the compensator $Q_{eq}(s)$ in **Fig. 3 (b)**. The inverse system of $Q_{eq}^{op}(s)$ is given by

$$\tilde{Q}_{ep}^{op}(s) = \sum_{k=0}^{\infty} \left(\frac{(C_1\mathsf{A}^\tau\mathsf{B})^{-1}C_1(\mathsf{A} + fI)^{\tau+1}\mathsf{A}^k\mathsf{B}}{s^{k+1}} \right) + I$$
$$= (C_1\mathsf{A}^\tau\mathsf{B})^{-1}(s + f)^\tau\mathsf{G}_e(s)$$

where $\mathsf{G}_e(s) := C_1(sI - \mathsf{A})^{-1}\mathsf{B}$. Then we obtain the stability condition of Q_d^{op}.

Theorem 2: *The following statements hold. (i) The transmission poles of* $Q_{eq}^{op}(s)$ *consist of both "*$-f$*" and the transmission zeros of* $\mathsf{G}_e(s)$. *(ii) The optimal dynamic quantizer* Q_d^{op} *in (7) is stable if and only if the all transmission zeros of* $\mathsf{G}_e(s)$ *are stable (i.e., the system* $G(s)$ *is minimum phase).*

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Denote by $\beta_{\max}(\delta_{\max})$ the maximum real part of the stable transmission poles (zeros) for the system $\mathsf{G}_e(s)$. From the statement (i) of Theorem 2, one gets the range of f as follows:

$$f > \min\{|\beta_{\max}|, |\delta_{\max}|\}.$$

Also, the following theorem derived from (6) in Lemma 1 and (8) in Theorem 1 provides an analytical relation between the quantizers Q_d and Q_{st} in the continuous-time domain.

Theorem 3: *Consider the problem* **(OP)** *and denote by γ_{st} and γ_d upper bounds of $J(Q_{st})$ and $J(Q_d)$, respectively. In the case of $n_g = 1$,*

$$\inf \frac{\gamma_d}{\gamma_{st}} = \frac{\sqrt{|\mathsf{A}|}}{\sqrt{f}}, \quad f > |\mathsf{A}| \tag{9}$$

holds.

Since the matrix A is stable, $\mathsf{A} < 0$ holds in (9). Theorem 3 guarantees that the scalar f of the quantizer Q_d improves $J(Q_d)$ compared with the quantizer Q_{st} in terms of the infimum of the upper bound ratio of the cost functions. Then the larger value the scalar f is set to be, the better approximation the quantizer Q_d^{op} achieves between the minimum phase systems in **Figs. 1 (a)** and **(b)**.

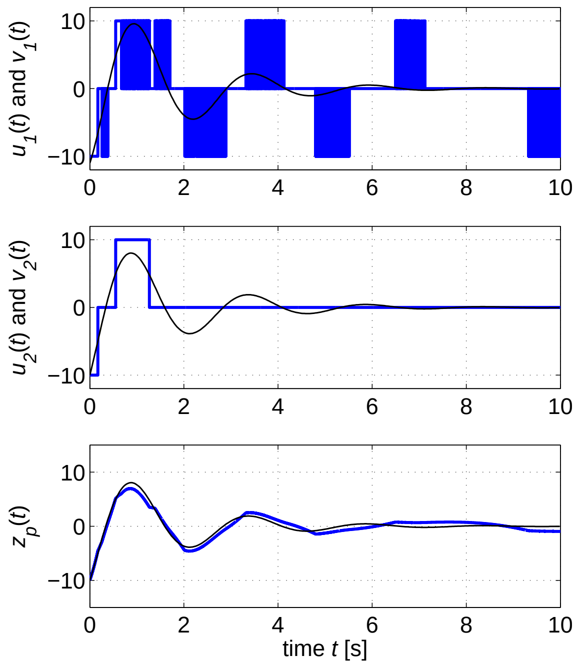

Fig. 4. Time responses with Q_d

Consider the decentralized I/O quantizer system. The plant $P(s)$ is the unstable minimum phase LTI system:

$$\begin{bmatrix} \dot{x}_p \\ u_2 \end{bmatrix} = \left[\begin{array}{cc:c} 0 & 1 & 0 \\ -2 & 1 & -2 \\ \hdashline 4 & 1 & 0 \end{array}\right] \begin{bmatrix} x_p \\ v_1 \end{bmatrix}, \quad z_p = u_2.$$

Its eigenvalues are $0.5 \pm 0.866i$ and its zero is $\{-2\}$. The stabilizing controller $C(s)$ is given by

$$\begin{bmatrix} \dot{x}_c \\ u_1 \end{bmatrix} = \left[\begin{array}{c:c} -3 & 1 \\ \hdashline -1 & 1 \end{array}\right] \begin{bmatrix} x_c \\ v_2 \end{bmatrix}.$$

The resultant zeros of $\mathsf{G}_e(s)$ are -3 and -4. For the quantizer Q_d^{op} in (7), we set $d = 10$ and $f = 10$. The performance is $\inf \gamma = 4.23$. **Fig. 4** shows the time responses of $u_1(t)$, $u_2(t)$, $v_1(t)$, $v_2(t)$ and $z_p(t)$ for the dynamic quantizer in Theorem 1 with the initial state $x_g(0) = [\,-4 \;\; 0 \;\; -1\,]^{\mathrm{T}}$. The thin lines and the thick lines illustrate the time responses of the usual feedback system in **Fig. 1 (b)** and the quantized feedback system in **Fig. 1 (a)**, respectively. The controlled output z_p of the dynamic quantizer does not go to zero. However, we see that the controlled output of **Fig. 1 (a)** approximates that of **Fig. 1 (b)** even if the discrete-valued signals $v_1 \in \{-10, 0, 10\}$ and $v_2 \in \{-10, 0, 10\}$ are applied. Not that the larger value of f not only provides the better approximation performance, but also switches the outputs v_1 and v_2, more quickly.

4. CONCLUSION

Focusing on continuous-time LFT type quantized feedback systems, we have proposed the continuous-time dynamic quantizer analysis and synthesis conditions. This paper has concentrated on the case of minimum phase systems, clarified that optimal dynamic quantizers and their performance are parameterized by the design parameter. Also, an analytical relation between the static and dynamic quantizers has been presented. Finally, it has been pointed out that the proposed method is helpful through the numerical example.

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