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Supercomputing of transition to turbulence in pipe with adit flow

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Abstract: Transition from laminar flow to turbulence often occurs in closed pipes such as pathologic blood vessels and artificial systems such as micro-tubes. While varying disturbances entering at pipe inlet or at heart pump, the transition points in space from laminar flow to turbulence in closed pipe are solved by using the weakly-stochastic Navier-Stokes equation and a finite difference method proposed previously by us (Naitoh and Shimiya, 2011), although the previous numerical simulations and instability theories based on the deterministic Navier-Stokes equation could never indicate the transition point in closed tunnel. The most important point of our approach is a philosophical method proposed for determining the stochasticity level, which is deeply related to boundary condition. Here, we qualitatively clarify the relation between the transition point and amount of adit on solid wall, because living systems exchange water and molecules through the wall of blood vessel. A mysterious feature obtained is that a larger amount of additional adit at the inlet may result in laminalization of the boundary layer.

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Keywords: Transition, Turbulence, Drag, Adit.

1. INTRODUCTION

Reynolds experimentally showed the transition to turbulence in pipe flow 100 years ago.[1] However, traditional stability theories based on deterministic continuum mechanics [2, 3, 4, 5] cannot indicate the critical Reynolds number, i.e., the transition points in closed pipe flow for various inlet disturbances, although a great deal of effort has been exerted over the years to reveal the transition points in pipe flow using experimental, theoretical, and computational approaches.

Experimental researches related to puffs and slugs [6, 7, 8, 9] have yielded important information for clarifying the early stage of the transition process around the critical Reynolds number. However, these previous studies could not clarify clearly whether or not the stochasticity coming from molecular fluctuations influence the transition point in pipes, because the spatial resolution was not sufficient.

Recent mathematical and physical theories [10, 11] have revealed important aspects about the early stage of the transition in pipes. Numerical computations of pipe flow have also been tried using the deterministic Navier-Stokes equations [12, 13, 14, 15]. However, the influence of inlet disturbances on the transition point in space and the critical Reynolds number in pipe flow cannot be analyzed by these approaches.

There are also some theoretical researches for the mesoscopic regime lying between molecular dynamics and continuum mechanics. [16, 17, 18] However, the role of stochasticity for the nonlinear unsteady phenomena in multi-dimensional space such as those described by the Navier-Stokes equation are still mysterious.

This has led to the possibility that averaging the phenomenon in a relatively large window for the continuum assumption eliminates the instability driven by small physical fluctuations in the mesoscopic regime. This may be true because our previous computations in a straight and closed pipe using numerical disturbances close to the random number generator have qualitatively showed the transition in space for various Reynolds numbers and inlet disturbances, which suggests the possibility of a stochastic Navier-Stokes equation. [19]

Our previous reports [19-22] showed that weakly stochastic field equations averaged in a mesoscopic window (MW) smaller than that for continuum mechanics will lead to solutions to various transient and critical problems still unsolved even numerically. An important study [19-22] is that we could solve the transition point in space in the transition to turbulence in pipes while varying velocity fluctuations at the inlet. Thus, in this report, we apply the new numerical model based on the weakly-stochastic Navier-Stokes equation to the pipe flow including adit on wall. This is important, because adit on wall appear in various flows such as artifact test inside wind tunnel, blood flow, and fuel cell.

2.METHODOLOGY

It is stressed that actual pipe flow shows a transition to turbulence, although traditional linear instability analysis based on a deterministic governing equation such as the potential equation and Navier-Stokes equation with the divergence of velocity of

$$\hat{D} = \frac{\partial \overline{u}_i}{\partial x_i} = 0$$

shows no transition in closed pipes, even if large disturbances are input.

The early laminar boundary layer in pipes is thinner than the Kolmogorov scale [25]. Then, there are no actual flows without inlet disturbances. Thus, they lead to an inhomogeneous and wavy velocity distribution at the starting point of instability inside the very early laminar boundary layer that is thinner than the Kormogolov scale.

Let us consider the smallest size of vortex in fully-turbulent flow, i.e., the Kolmogorov scale. This is on the order of 1000 times as large as the mean free path of molecules in high Reynolds number flows. [25] Thus, the molecular discontinuity produces stochastic fluctuations of 0.1% in density and velocity in the Kolmogorov-scale vortex. Even if the deterministic Navier-Stokes equation is

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numerically solved using a fine grid system on the Kolmogorov scale, this stochasticity is not evaluated. This stochastic level is also close to the ratio between the inlet disturbance and main flow in wind tunnels. It is also emphasized that stronger inlet disturbances produce shorter transition points, which apparently come from the smaller minimum scale of the fluctuation in the thin boundary layer.

For these reasons, deterministic models such as the deterministic Navier-Stokes equations based on the continuum assumption are essentially defective for solving the transition. Thus, the mesoscopic window size (Dmw) should be identical to this minimum scale of the weak fluctuation in the thin boundary layer.

To say more, an asymmetric distribution of numerical errors in space similar to a random force induced vertical asymmetric flows in many previous computational studies, which include the Karman vortex streets. [4, 12, 13, 29] This fact also supports the inevitability of stochastic terms. It was also confirmed previously that the mean velocity profiles after the transition point computed by the stochastic Navier-Stokes equation [19-22, 26] agreed well with the experimental data in Refs. 27 and 28. The early stage of the transition for a low Reynolds number of 6,000 in our computations also shows a two-dimensional path of particles (corresponding to the well-known Squire theorem), although relatively high Reynolds numbers suddenly generate three-dimensional flow.

In this section, we first show the concrete formulation for the stochastic Navier-Stokes equation related to boundary condition and the computational results performed with a fine grid system having high resolution and accurate random force, while there is no adit on wall.

The definition of physical quantities such as fluid velocity $\vec{u}(=u_1, u_2, u_3)$ and density ρ in molecular velocity $\vec{c}(=c_1, c_2, c_3)$ and physical space $\vec{x}(=x_1, x_2, x_3)$ in Eqs. (3) and (4) leads to the deterministic Navier-Stokes equation, when the window size for averaging is large on the basis of continuum assumption. [2]

$$\rho = m\int f(\vec{c},\vec{x},t)d\vec{c}, \tag{3}$$

$$\vec{u} = \frac{m\int \vec{c} f(\vec{c},\vec{x},t)d\vec{c}}{\rho} \tag{4}$$

where $m, f(\vec{c},\vec{x},t), \text{and } t$ denote molecular weight, probability density function, and time, respectively, while $\vec{c}$ denotes the molecular speed.

Here, we redefine physical quantities such as fluid velocity and density in the mesoscopic window size (D_{mw}) smaller than that for deriving continuum mechanics.

Averaging should be done in the D_{mw}, which is the minimum scale dominating the flow phenomenon, L_{ms}. As a result, we obtain

$$\overline{\rho} = m\int \int f(\vec{c},\vec{x},t)d\vec{c}\, d\vec{x}_{mw}/V_{mw} = \rho - \rho' \text{ and}$$

$$\bar{\vec{u}} = \frac{m\int \int \vec{c} f(\vec{c},\vec{x},t)d\vec{c}\, d\vec{x}_{mw}/V_{mw}}{\overline{\rho}} = \vec{u} - \vec{u}'$$

where dx_{mw} and V_{mw} denote the length scale for integration and control volume with the diameter D_{mw}, respectively, while $\overline{\rho}$ and $\bar{\vec{u}}$ imply the corresponding values averaged in D_{mw} and ρ' and u' are stochastic fluctuations indeterminate. [19,21,22]

A mesoscopic averaging window size, smaller than that for the continuum assumption, yields a stochastic compressible Navier-Stokes equation in a non-conservative form in the case where specific heat and viscosity coefficient have constant values, written as the governing equation below,

$$F \equiv \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} \dfrac{\partial \overline{u}_i}{\partial x_i} + \dfrac{1}{\overline{\rho}}\dfrac{D}{Dt}\overline{\rho} \\ \dfrac{D\overline{u}_i}{Dt} + \dfrac{\partial \overline{p}}{\partial x_i} - \dfrac{1}{\mathrm{Re}}\dfrac{\partial^2 \overline{u}_i}{\partial x_j{}^2} \\ \dfrac{D\overline{T}}{Dt} - \dfrac{D\overline{p}}{Dt} - \dfrac{1}{Pe}\dfrac{\partial^2 \overline{T}}{\partial x_i{}^2} \end{bmatrix} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} \tag{6a}$$

where u_i, p, ε_i $(i=1-3)$, t, Re, and Pe denote the dimensionless quantities of velocity components in the i-direction, pressure, random forces, time, Reynolds number, and Peclet number, respectively.

Equation (6a) is transformed to Eq. (6b), which is a multi-level formulation. [29]

$$F \equiv \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} \hat{D} + \dfrac{1}{\overline{\rho}}\dfrac{D}{Dt}\overline{\rho} \\ \dfrac{D\overline{u}_i}{Dt} + \dfrac{\partial \overline{p}}{\partial x_i} - \dfrac{1}{\mathrm{Re}}\dfrac{\partial^2 \overline{u}_i}{\partial x_j{}^2} \\ \dfrac{\partial^2 \overline{p}}{\partial x_i{}^2} + \overline{\rho}\dfrac{\partial}{\partial t}\hat{D} + \Phi \\ \dfrac{D\overline{T}}{Dt} - \dfrac{D\overline{p}}{Dt} - \dfrac{1}{Pe}\dfrac{\partial^2 \overline{T}}{\partial x_i{}^2} \end{bmatrix} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{bmatrix} \tag{6b}$$

where $\hat{D}(=\dfrac{\partial \overline{u}_i}{\partial x_i})$ denotes the divergence of velocity and also the equation for f_3 is the spatial derivative of that of f_2, while the third term on the left-hand side

$$\Phi = \frac{\partial}{\partial x_i}[\overline{u}_j \frac{\partial \overline{u}_i}{\partial x_j} - \frac{1}{\mathrm{Re}}\frac{\partial^2 \overline{u}_i}{\partial x_j{}^2}]$$

denotes the spatial derivative of the convection and viscosity terms in f_2. Four variables of $\hat{D}, \overline{u}_i, \overline{p}$, and $\overline{T}$ can be solved by the four equations in Eq. (6b).

The second term on the left-hand side of the energy conservation law in Eq. (6b) can approximately be eliminated for low Mach number conditions.

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In this study, the initial velocity distribution is set to be that of the potential flow, i.e.,

$$\overline{u_1}(t = 0) = Uo = 1$$
$$\overline{u_2}(t = 0) = \overline{u_3}(t = 0) = 0 \quad (7)$$

at each point, while the initial pressure and temperature are constants.

The inlet boundary condition is given as

$$\overline{u}_1 = Uo + \delta = 1 + \delta,\ \overline{u}_2 = \overline{u}_3 = 0 \quad (8a)$$

and the outlet boundary condition as

$$\frac{\partial}{\partial n}\overline{u}_i = 0 \quad (8b)$$

where Uo, δ, and n denote the dimensionless inlet velocity, dimensionless inlet disturbance, and the direction normal to the outlet, respectively.

A no-slip boundary condition

$$\overline{u}_i = o \quad (8c)$$

is imposed at the solid walls of the pipe.

Next, let us consider ε_1 in Eq. (6b). Stronger inlet disturbances result in a smaller characteristic scale, because they lead to more inhomogeneous velocity distributions at the starting point of instability inside the laminar boundary layer.

Two types of indeterminacies, in the boundaries and inner regions, should be set to the same level. [19, 20, 21, 22, 26]

Thus, the relation

$$\sum_{n=1}^{N} \varepsilon_1 / N = Ce\ \delta \quad (9)$$

is used with an arbitrary constant Ce and the number of grid points N. In this report, C_e is set to be of the order of 1.0. [Comparisons with computations and experiments on the transition point in space and the theoretical considerations based on statistical mechanics including the Liouville equation brings that C_x should be about 1.0.]

Stochastic terms $\varepsilon_2, \varepsilon_3,$ and ε_4 in Eq. (6b) should also be added to the momentum conservation law, the Poisson equation, and the energy conservation law averaged in the mesoscopic averaging window (MW). However, in this study, $\varepsilon_2, \varepsilon_3,$ and ε_4 are set to be 0.0, because their role is similar to small variations of Reynolds and Peclet numbers in space and time, resulting in less influence on the transition point than in the case of the mass conservation law. This is clear from the fact that even the turbulence viscosity model based on the RNG theory [23] has less influence on the transition point, when the model is used in numerical computations. The stochastic terms $\varepsilon_2, \varepsilon_3,$ and ε_4 have relatively weak influence on the transition phenomenon, because those in momentum and energy conservation laws can also be dissipative, while mass with ε_1 cannot essentially be dissipative.

There are two ways to calculate the random force term of ε_1: One involves the use of a random number generator and the other a special numerical disturbance close to the actual random number.

The second approach uses numerical errors coming from the iteration method for the matrix calculation, because the rounding error due to the iteration method such as the SOR method [30] is close to the random number generator, although truncation errors distorting the phenomenon should be eliminated.

One reason why previous attempts to compute the transition point in space have not succeeded [12, 13] is that the criterion of ε_1 in Eq. (6b) was set to zero based on mathematics [30], not on fluid physics taking into account indeterminacy and molecular discontinuity. Evaluations of ε in previous reports [30] were too small. Most of the previous studies on computational fluid dynamics have controlled only truncation errors, whereas the approach proposed here also provides physical control of rounding error.

The numerical algorithm used here is based on a multi-level formulation [29] that can simulate both incompressible and compressible flows. The momentum conservation law and the second derivative of the pressure equation in the formulation clarify the mathematical structure that the governing equation varies from an elliptic type to a hyperbolic type as the Mach number increases. Thus, the numerical algorithm is extended from the Marker and Cell (MAC) [31], SIMPLE [32], ICE [33], and CUP methods [34]. Details of the numerical discretizations and numerical algorithms are described in Refs. 29 and 34.

Computational results computed on mean velocity, turbulence intensity, and transition point agree well with the experimental data reported by Laufer, while the present approach also reveals the critical Reynolds number around 1,500 - 2,500. [19-22, 26]

4. TRANSITION TO TURBULENCE IN A ST RATGHT PIPE WITH ADIT

Here, we examine the relation between the transition point in space and flow amount injected from solid wall into a straight pipe. Figure 1 qualitatively shows that larger amount of adit from the wall results in later transition point in space. This may be used for reducing the drag force.

9. CONCLUSION

The present result obtained by computation should be checked by comparing with experiments.

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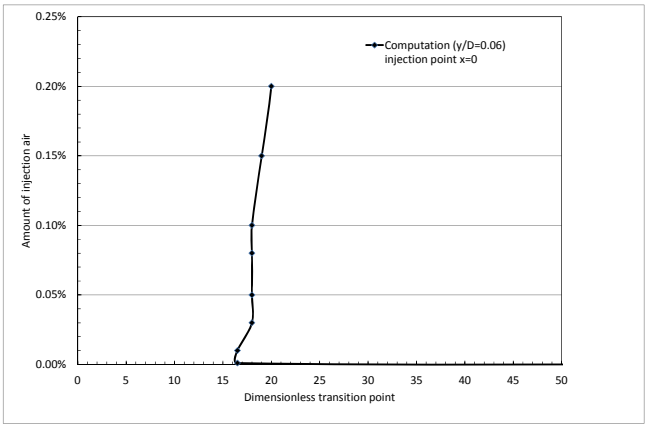

Fig.1 Relation between adit amount and transition point, which is computed by the present approach.