

# Propulsive force analysis of a pectoral fin for rajiform type fish robots from fluid dynamic aspects

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**Abstract:** In this paper, we analyze the propulsive force generated from pectoral fins for a rajiform type fish robot from fluid dynamic aspects. A pectoral fin of the rajiform type fish robot is constructed by multiple fin-rays, which move independently, and a film of pushing water. Then, the propulsive force of the fish robot is analyzed from the momentum of the fluid surrounding for every fin between fin-rays. The total propulsive force for one pectoral fin is the sum of these momenta. The propulsion speed of a fish robot is determined from the difference of the propulsive force generated from pectoral fins, and the resistance force that the fish robot receives from the water when moving forward. The effectiveness of the calculated propulsive force is examined through numeric simulation results.

**Keywords:** Fish robot, Rajiform type swimming, Propulsive force analysis

## 1 INTRODUCTION

Oceanographic investigation for investigating mineral resources at sea bottom and an aquatic life is carried out actively. Autonomous (unmanned) underwater vehicles called AUVs are needed for such oceanographic investigation [1] [2]. Furthermore, an application to search activities in the shoal, where the introduction of any AUV is not easy, is also attractive. The conventional AUV has a common mechanism for propulsion using screw propellers. The underwater propulsion using screw propellers is good at performing a long-distance cruise and keeping a maximum speed. On the other hand, any screw propeller is not suitable for sudden acceleration or rapid turning.

When observing underwater living things, they hide themselves from an underwater foreign enemy, or have captured foods in the crevice of reefs, and manipulate sudden acceleration and rapid turning easily [3]. Fishes are in the aquatic life that can manipulate such things. It is known [4] that fishes can produce propulsive forces by oscillating the various kinds of fins and the body. The propulsion induced by oscillating fins and the body can change the propulsive force and direction easily by changing the oscillatory situation. Fishes are divided into some types, depending on every motion that produces propulsive forces. Among such motion types, a ray such as Manta is classified into a rajiform swimming type. Such type fishes can control underwater position and posture by using only one pair of right and left pectoral fins. Therefore, since there are few portions to be operated, compared to other swimming types, and it need not necessarily operate all fins cooperatively, it is easy to realize mechanical reappearance and mimicking in motion.

In this paper, we propose a diving method for a rajiform

type fish robot, and examine the usefulness of the proposal technique by calculating the propulsive force by a pectoral fin. Then, the effectiveness of the diving method proposed here is checked by numeric simulations.

## 2 RAJIFORM TYPE FISH ROBOT

A rajiform type fish robot has a central float and a pectoral fin on each side, as shown in Fig. 1. The float is designed so that the specific gravity of the fish robot may be set to 1. A pectoral fin has six structures, each of which is called a fin-ray. The fin-ray has mounted a fin-ray driving unit, and each fin-ray can be driven independently. The film for pushing out water is mounted between fin-rays. The control circuit and power supply for controlling the pectoral fin of the fish robot are arranged by expanding electric wiring to the outside of water.

### 2.1 Fin-ray driving unit

The fin-ray driving unit is composed of a waterproofed box and a gear part, as shown in Fig. 2. The waterproofed box has one RC servo-motor in inside. The transfer part of driving force to a fin-ray is waterproofed by taking out the driving force of a motor using a stan-tube. A gear part is used to change the rotation axis from a motor 90 deg, and it is realized using a bevel gear.

### 2.2 Waveform with fin-rays

A pectoral fin of the rajiform type fish robot generates a propulsive force by driving the fin as a traveling wave. When the form of the traveling wave formed at a pectoral fin is a sinusoidal wave form, the angular velocity  $\omega_k$  of the  $k$ -th fin

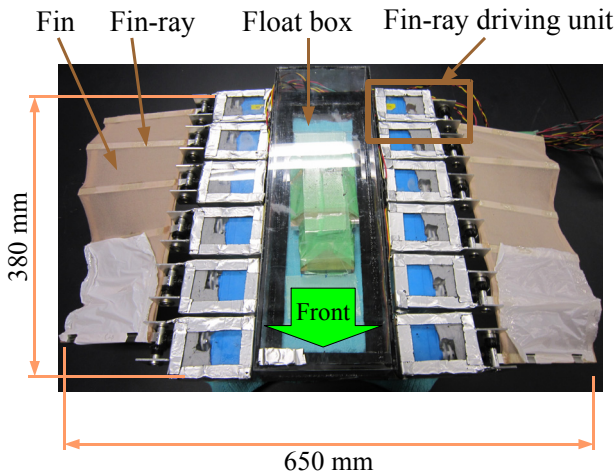


Fig. 1. Rajiform type fish robot

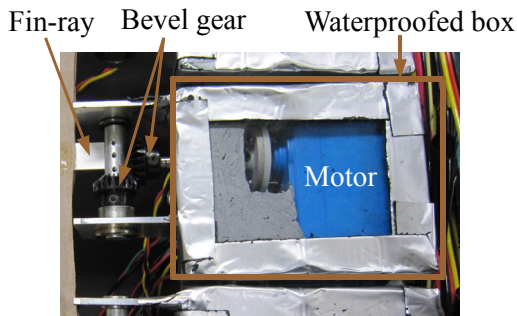


Fig. 2. Fin-ray driving unit

ray from a front side is expressed by the following equation:

$$\omega_k = \frac{2\pi\theta_{\max}}{T} \cos\left(\frac{2\pi t}{T} + (k-1)\phi\right) \quad (1)$$

Here,  $t$  [s] is time,  $\theta_{\max}$  [rad] is amplitude,  $T$  [s] is a period, and  $\phi$  [rad] is the phase difference between adjacent fin-rays. The waveform of a sinusoidal wave is changeable by changing these values.

### 3 DIVING METHOD

A diving method is proposed for a rajiform type fish robot using a forward propulsion speed. As shown in Fig. 3, two front fin-rays and the partial fin between them are fixed, where the fixed partial fin tilts with a fixed angle  $\psi$ . The propulsive force to the forward direction in this fish robot is obtained by making a traveling wave using other remaining fin-rays. When the robot moves forward by the propul-

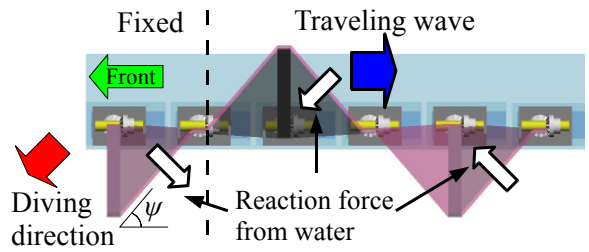


Fig. 3. Diving motion

sive force obtained from the traveling wave, the propulsive force to the diving direction occurs, according to the resistance force that the fixed partial fin receives from water. The propulsive force to the diving direction generated in this way is used in order that this fish robot may dive.

### 4 ANALYSIS OF PROPULSIVE FORCE

In this section, the equation for calculating the forward propulsion speed and diving speed of a rajiform type fish robot is derived. First, the equation for the forward speed of a rajiform type fish robot is derived from a forward propulsive force and a resistance force. Next, the equation for diving speed of a rajiform type fish robot is derived from the resistance added to the fixed partial fin. Two assumptions are set to derive these equations. One assumption is that the whole body of this fish robot is in static water. This means that the wave or the vortex does not occur in the water around this fish robot. When a rajiform type fish robot moves propulsively, the fish robot receives a resistance force by wave or vortex generated from itself, and a profile drag force with water by the profile. However, it is difficult to analyze the resistance force by wave or vortex. Therefore, the resistance force by wave and vortex is included by setting a profile drag coefficient as a larger value. The other assumption is to set the profile drag coefficient for this fish robot to 2.0. This value was decided based on the fact that the profile drag coefficient in a cube was 1.07, and that the maximum of a profile drag coefficient was 2.0.

#### 4.1 Propulsive force between fin-rays

A pectoral fin has several fin-rays. Any two adjacent fin-rays and the fin existing between them are considered. The forces which this partial fin receives from water are shown in Fig. 4 as  $f_x$  and  $f_y$ . The way of the water on the considered partial fin is set as control volume (CV). Assume that the stream of the circumference of CV flows in from  $A_1$  side in Fig. 4, and flows out of  $A_2$  side. The momenta for the inflow and outflow per unit time are calculated from the area and the flow velocity that flows into CV, and the area and the

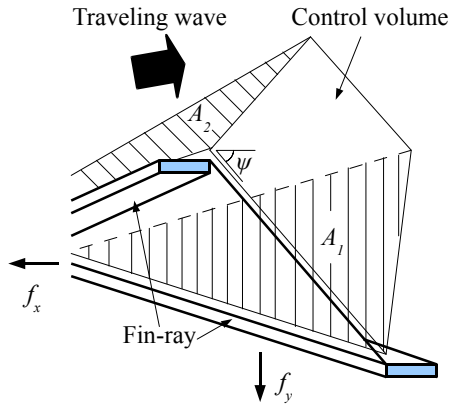


Fig. 4. Generated forces by two fin-rays

flow velocity that flows out of CV. The force that the considered partial fin receives from water is calculated from the difference between the momenta, which are produced when the surrounding water of CV flows in and flows out. The mean angular velocity of a fin-ray  $\omega$ , the speed of a traveling wave  $u$ , the slant of a partial fin  $\psi$ , the inflow area  $A_1$  and the outflow area  $A_2$  of CV are expressed respectively by

$$\omega = \frac{4\theta_{\max}}{T} \quad (2)$$

$$u = \frac{\omega a}{\theta_{\max} \sin \phi} \quad (3)$$

$$\psi = \text{atan} \frac{L \sin(\theta_{\max} \sin \phi)}{a} \quad (4)$$

$$A_1 = L^2 \sin(\theta_{\max} \sin \phi) / 2 \quad (5)$$

$$A_2 = A_1 \cos \psi \quad (6)$$

where  $a$  is a distance between fin-rays and  $L$  is the length of a fin-ray.  $f_x$  which is the force of the forward direction, and  $f_y$  which is the force of the dive direction are expressed by the following equations:

$$f_x = \rho(v_x - u)^2 A_1 (1 - \cos \psi) \quad (7)$$

$$f_y = \rho(v_x - u)^2 A_2 \sin \psi \quad (8)$$

where  $\rho$  is the density of water.

#### 4.2 Forward velocity

A rajiform type fish robot has a pectoral fin in right and left, respectively. Only the surface projected on the direction of motion of a traveling wave pushes water, and generates propulsive force. Therefore, the forward propulsive force  $F_x$  generated from one pectoral fin of a fish robot is  $2\pi/\phi$  times the force to the motion direction  $f_x$  generated from between fin-rays. Thus,  $F_x$  is expressed by the following equation:

$$F_x = \frac{2\pi}{\phi} \rho(v_x - u)^2 A_1 (1 - \cos \psi) \quad (9)$$

However, the forward propulsive force receives a constraint by the number of fin-rays  $n$  for making a traveling wave. Therefore, when  $\pi/\phi > n$ ,  $\pi/\phi$  is set to  $n$ . The resistance force  $D_x$  that a rajiform type fish robot receives from water is the sum of the profile-drag force for the body and the oscillating fin, and the resistance force for the fixed fin.  $D_x$  is expressed by the following equation:

$$D_x = \frac{1}{2} C_D \rho v_x^2 S_1 + 2\rho v_x^2 A_1 (1 - \cos \psi) \quad (10)$$

where the projected area of the propulsion direction is  $S_1$ . When the forward propulsive force  $F_x$  and the resistance force  $D_x$  are equal, the forward propulsion speed  $v_x$  is expressed by the following equation:

$$v_x = \frac{-\beta + \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} \quad (11)$$

where  $\alpha$ ,  $\beta$  and  $\gamma$  are defined respectively as follows:

$$\alpha \triangleq \frac{1}{2} C_D S_1 + 2A_1 (1 - \cos \psi) \left(1 - \frac{\pi}{\phi}\right)$$

$$\beta \triangleq \frac{4\pi}{\phi} A_1 (1 - \cos \psi) u$$

$$\gamma \triangleq -\frac{2\pi}{\phi} A_1 (1 - \cos \psi) u^2$$

#### 4.3 Diving speed

Since fin-rays of a fish robot are oscillated symmetrically with respect to the horizon, it is considered that the average propulsive force to the diving direction generated by the pectoral fin is 0. Therefore, the propulsive force to the diving direction is only a propulsive force to the dive direction generated by the fixed partial fins. In other words, the propulsive force  $F_y$  to the diving direction is the sum of  $f_y$ 's generated by the fixed partial fins. Since the fixed partial fins may not make a traveling wave,  $u$  that is a speed of a traveling wave is 0. From equation (8),  $F_y$  is expressed by the following equation:

$$F_y = 2\rho v_x^2 A_2 \sin \psi \quad (12)$$

When a rajiform type fish robot advances in the diving direction, the profile-drag force  $D_y$  which it receives is expressed by the following equation:

$$D_y = \frac{1}{2} C_D \rho v_y^2 S_2 \quad (13)$$

Here, the projected area to the diving direction is  $S_2$ . When the propulsive force  $F_y$  and the resistance force  $D_y$  to the diving direction are equal, the diving speed  $v_y$  is expressed by the following equation:

$$v_y = \sqrt{\frac{4v_x^2 A_2 \sin \psi}{C_D S_2}} \quad (14)$$

## 5 NUMERICAL SIMULATION

When a rajiform type fish robot is diving by the method proposed in Section 3, the propulsion speed is changed by the phase difference  $\phi$  [rad] between adjacent fin-rays, the amplitude  $\theta_{\max}$  [rad] and the period  $T$  [s], associating with the oscillating fins. Change of a diving speed is examined by a numeric simulation by fixing two design parameters and changing one of the three above parameters using the equation derived in Section 4. Other parameters are set to  $n = 4$ ,  $C_D = 2$ ,  $L = 150$  [mm],  $a = 60$  [mm],  $\rho = 1$  [g/cm<sup>3</sup>],  $S_1 = 28000$  [mm<sup>2</sup>] and  $S_2 = 250000$  [mm<sup>2</sup>].

The result of a numeric simulation is shown in Fig. 5. Fig. 5(a) shows the change of diving speed when changing  $\theta_{\max}$  while  $T = 2$  [s] and  $\phi = 0.52$  [rad] (=30 [deg]). Fig. 5(b) shows the change of a diving speed when changing  $T$  while  $\theta_{\max} = 0.52$  [rad] and  $\phi = 0.52$  [rad]. Fig. 5(c) shows the change of a diving speed when changing  $\phi$  while  $\theta_{\max} = 0.52$  [rad] and  $T = 2$  [s]. Fig. 5(a) shows that the diving speed of a fish robot becomes so speedy that the amplitude of the oscillating fin is large. Fig. 5(b) shows that the diving speed of a fish robot is so fast that the period of the oscillating fin is small. In Fig. 5(c), the diving speed of a fish robot is the fastest, when the phase difference between fin-rays is approximately set around 0.69 to 0.87 [rad] (=40 to 50 [deg]). This is because it becomes  $\pi/\phi = n$  at the time of  $\pi/\phi > n$ , so it is restricted as the forward propulsive force of the fish robot stated in Section 4.2. Since the rajiform type fish robot used in this paper is  $n=4$ , a diving speed displays the maximum at the time of phase difference 0.79 [rad] (=45 [deg]). Therefore, this result is appropriate.

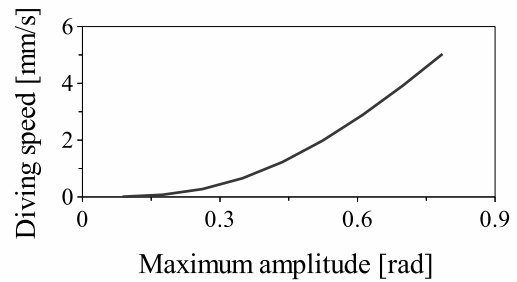
## 6 CONCLUSION

In this paper, we have proposed a diving method for the rajiform type fish robot. The diving speed obtained by the proposed diving method is calculated from the propulsive force obtained from a pectoral fin, and the resistance force that the fish robot receives from water. Furthermore, the effectiveness of the calculated propulsive force is examined by numeric simulation results. In the future, the effectiveness of the proposed technique will be confirmed by experiments using an actual rajiform type fish robot.

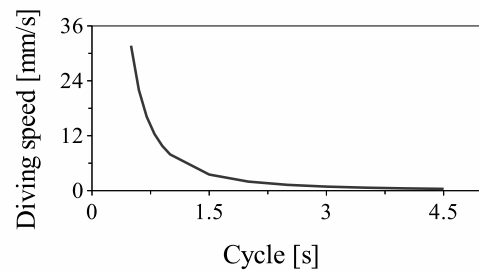
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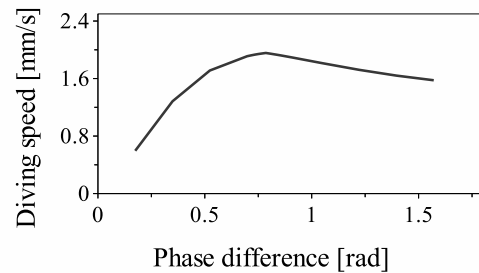
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(a) Diving speed vs. maximum amplitude



(b) Diving speed vs. cycle



(c) Diving speed vs. phase difference between fin-rays

Fig. 5. Diving speed

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