

Fig. 2. Example of chromosome of an individual

Table 1. Initial parameters of genetic algorithms

Population size	100
Number of elite	6
Selection	Tournament selection and elitism
Crossover	Uniform crossover (rate=87.5%)
Mutation	Random mutation (rate=1.0 %)
Search space of K_{vi}	$50 \leq K_{vi} \leq 220$
Search space of K_{pi}	$1000 \leq K_{pi} \leq 35000$
Maximum generation	200

lowing control problem. The resolved acceleration controller generates the joint driven torque as given by

$$\tau = \hat{M}(\theta)J^{-1}(\theta) [\ddot{x}_r + K_v \{\dot{x}_r - \dot{x}\} + K_p \{x_r - x\} - \dot{J}(\theta)\dot{\theta}] + \hat{H}(\theta, \dot{\theta}) + \hat{G}(\theta) \quad (2)$$

where the hat symbol means the estimated term obtained by computing inverse dynamics via recursive Newton-Euler formulation. $x \in \mathbb{R}^{6 \times 1}$ consists of position vector $[x, y, z]^T$ and orientation vector $[\alpha, \beta, \gamma]^T$. $\dot{x} \in \mathbb{R}^{6 \times 1}$ and $\ddot{x} \in \mathbb{R}^{6 \times 1}$ are the velocity and acceleration vectors, respectively. x_r , \dot{x}_r and \ddot{x}_r are the desired vectors of position, velocity and acceleration, respectively. θ , $\dot{\theta}$, x and \dot{x} are actual controlled values. $Fkine(\theta)$ is the function of forward kinematics given by $x = Fkine(\theta)$. $K_v = \text{diag}(K_{v1}, \dots, K_{v6})$. $K_p = \text{diag}(K_{p1}, \dots, K_{p6})$ are the feedback gains of velocity and position respectively, each of which is set to a positive definite diagonal matrix. $\hat{M}(\theta)$, $\hat{H}(\theta, \dot{\theta})$ and $\hat{G}(\theta)$ are called nonlinear compensation terms, which act on the cancellation of nonlinearity of manipulator. The nonlinear compensation terms are effective to achieve a stable trajectory following control. If the estimate error dose not exist at all, then the second order error equation of this system described in Eq. (3) is derived from Eqs. (1) and (2).

$$\ddot{e} + K_v \dot{e} + K_p e = 0 \quad (3)$$

where $e = x_r - x$, $\dot{e} = \dot{x}_r - \dot{x}$, $\ddot{e} = \ddot{x}_r - \ddot{x}$. Accordingly, a nominal response is obtained by adjusting K_v and K_p suitably considering the following equation based on critically damped condition.

$$K_{vi} = 2\sqrt{K_{pi}} \quad (i = 1, \dots, 6) \quad (4)$$

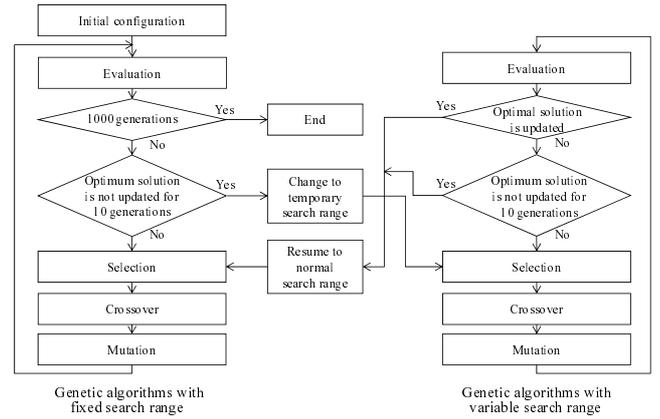


Fig. 3. Flowchart of proposed genetic algorithms

If K_v and K_p are selected based on Eq. (4), the stability is guaranteed and the errors convergence to zero.

3 Fine gain tuning by genetic algorithms

In this section, genetic algorithms are used to optimize K_v and K_p . In order to apply genetic algorithms, K_v and K_p are transformed into a digit binary string as chromosome. Diagonal elements of them are encoded into 8 bits and 16 bits codes respectively, so that an individual chromosome is composed of 144 (=8 × 6+16 × 6) bits as shown in Fig. 2.. A population has 100 individuals and each individual in initial population is generated within the expectable values. The parameters of GAs are defined as shown Table 1. In our study, variable search space GAs are proposed and applied. Fig. 3. shows the flowchart of the proposed variable search space GAs, in which if the optimal gain is unchanged for 10 generations then the search space is varied. This means that the search spaces are redefined temporarily based on the optimal gains at the generation.

$$K_{pimin} = K_{pi\text{best}} - 10000 \quad (5)$$

$$K_{pimax} = K_{pi\text{best}} + 10000 \quad (6)$$

Each individual is evaluated through the following evaluation function.

$$E_v = \sqrt{\sum_{k=1}^{500} \|e(k)\|} \quad (7)$$

where k is the sampled discrete-time number in simulation. Note that, this evaluation function also represents the position errors.

4 Simulations

4.1 Desired trajectory

In this section, to evaluate the effectiveness of the proposed method, simulations are carried out by us-

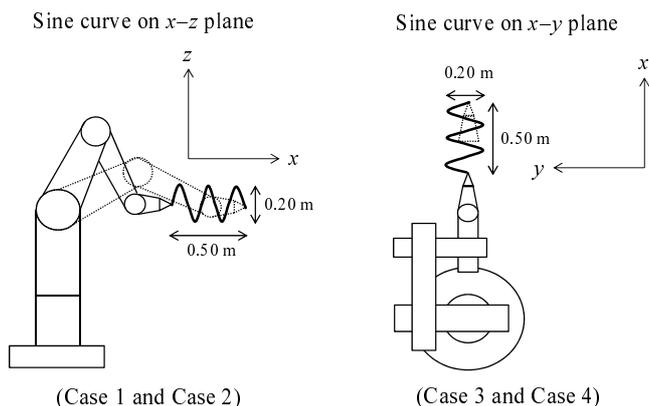


Fig. 4. Illustration of desired trajectory

ing the dynamic model of PUMA560 manipulator on MATLAB system [4]. First of all, in order to apply the resolved acceleration control method to PUMA560 manipulator, the desired trajectory in Cartesian coordinate system must be prepared. Discrete values of desired position, velocity and acceleration are generated for each sampling period in advance. Four type trajectories are prepared to be compared for each and set as below.

$$\text{Case 1.} \quad z = 0.1\sin(10\pi x) \quad (8)$$

$$\text{Case 2.} \quad z = 0.1\sin(20\pi x) \quad (9)$$

$$\text{Case 3.} \quad y = 0.1\sin(10\pi x) - 0.15 \quad (10)$$

$$\text{Case 4.} \quad y = 0.1\sin(20\pi x) - 0.15 \quad (11)$$

Fig. 4. is the overview of the desired manipulator movement. The desired positions given by Eq. (8) and (9) are designed with sinusoidal waves on x - z plane. Also, the desired positions given by Eqs. (10) and (11) are generated with sinusoidal waves on x - y plane. As an example, Fig. 5. illustrates the desired trajectory of Eq. (10) in x and y directions. The position x of desired trajectory is generated with a 4-1-4 order polynomial equation. The initial and final position/orientation vectors are set to $\mathbf{x}_{init} = (0.28, -0.15, 0.00, 0.00, \pi/2, 0.00)$, $\mathbf{x}_{fin} = (0.78, -0.15, 0.00, 0.00, \pi/2, 0.00)$, respectively.

In this simulation, we set them such that the total simulation time is 5.00 sec, the sampling width is 0.01 sec, the acceleration and deceleration time is 1.00 sec. To obtain satisfactory and safe control performance with avoiding singularities, \mathbf{K}_v and \mathbf{K}_p are roughly tuned with trial and error, considering the critically damped condition.

4.2 Dynamic model

In this simulation experiment, it is assumed that the friction is composed of viscous friction and Coulomb friction. Therefore, robotic dynamic model considering friction can be rewritten by

$$\mathbf{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{H}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \mathbf{G}(\boldsymbol{\theta}) + \mathbf{F}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \boldsymbol{\tau} \quad (12)$$

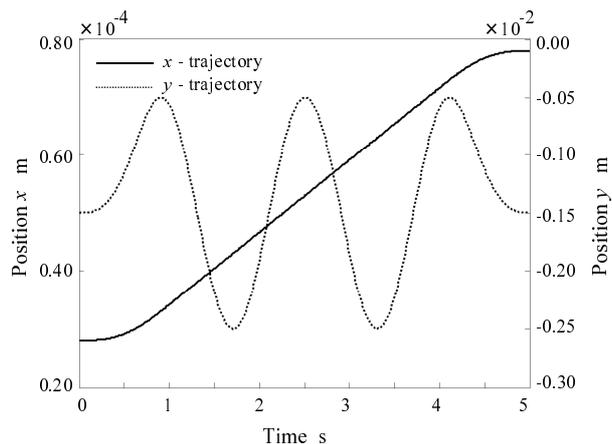


Fig. 5. Desired trajectory using Eq. (8)

$\mathbf{F}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ is composed of viscous friction and Coulomb friction and given by

$$\mathbf{F}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \mathbf{B}\mathbf{G}_r^2\dot{\boldsymbol{\theta}} + \mathbf{G}_r\boldsymbol{\tau}_c\{\text{sign}(\dot{\boldsymbol{\theta}})\} \quad (13)$$

where $\mathbf{B} \in \mathbb{R}^{6 \times 6}$ is the coefficient matrix of viscous friction at each motor, $\mathbf{G}_r \in \mathbb{R}^{6 \times 6}$ is the reduction gear ratio matrix which represents the motor speed to joint speed, and $\boldsymbol{\tau}_c\{\text{sign}(\dot{\boldsymbol{\theta}})\}$ is the Coulomb friction torque appeared at each motor. If $\text{sign}(\dot{\boldsymbol{\theta}})$ is positive/negative, then $\boldsymbol{\tau}_c = \boldsymbol{\tau}_c^+/\boldsymbol{\tau}_c^- = \boldsymbol{\tau}_c^-$, respectively. The friction parameters of PUMA560 manipulator such as \mathbf{B} , \mathbf{G}_r , $\boldsymbol{\tau}_c^+$ and $\boldsymbol{\tau}_c^-$ are technically available to engineers[4].

It is confirmed through preliminary simulations that this friction term, which works as disturbance as shown in Fig. 1., causes undesirable error even in critically damped condition. Also, in actual robot systems there inevitably exists the modeling error with respect to $\hat{\mathbf{M}}(\boldsymbol{\theta})$, $\hat{\mathbf{H}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ and $\hat{\mathbf{G}}(\boldsymbol{\theta})$. That is the reason why the fine gain turning after manual gain tuning is important for robotic servo system in order to realize more accurate trajectory following control.

4.3 Result and discussions

Fig. 6. shows the minimum E_v values at every generation searched by the proposed GAs. It is observed that optimal solutions of each case are almost converged before 50 generations. In Fig. 7., the curved line is critical damping condition, and the plotted points are the optimal gains of each case. It is observed that most of gains lie on under-damping condition. Table 2 tabulates each best fitness $E_{v_{best}}$ which is obtained using the four optimal gains.

At first, $E_{v_{best}}$ in case of using the trajectories in x - y and x - z planes are compared. Table 2 indicates that

Table 2. Best fitness comparison for each case

	Case 1	Case 2	Case 3	Case 4
$E_{v_{best}}$	0.0630	0.0874	0.0672	0.1147

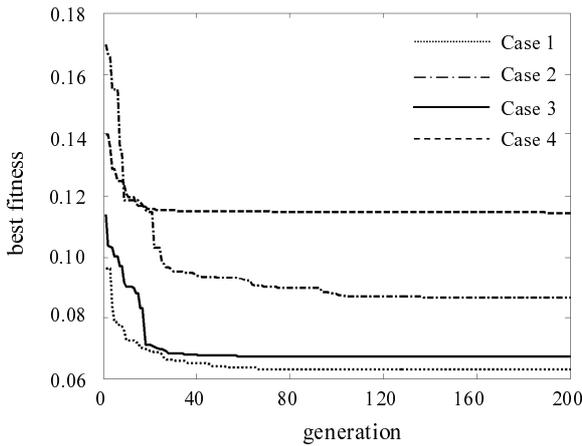


Fig. 6. Evolutionary histories of best fitness

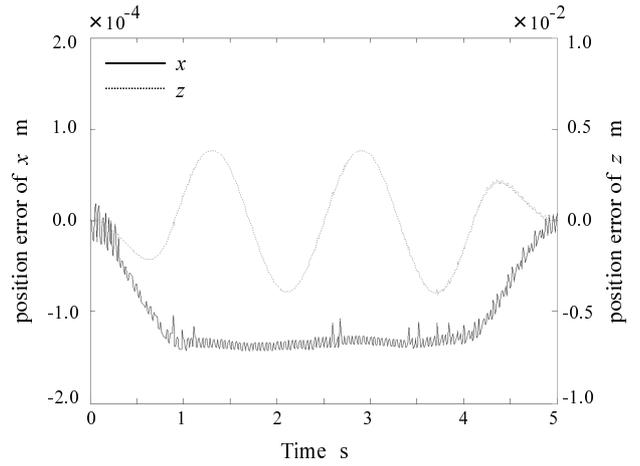


Fig. 8. Position error in Case 1

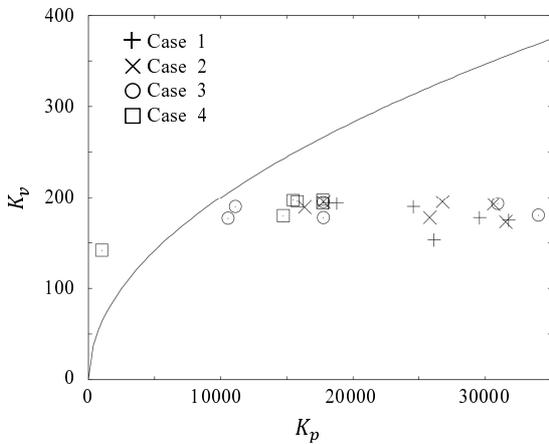


Fig. 7. Best gains and critical damping condition

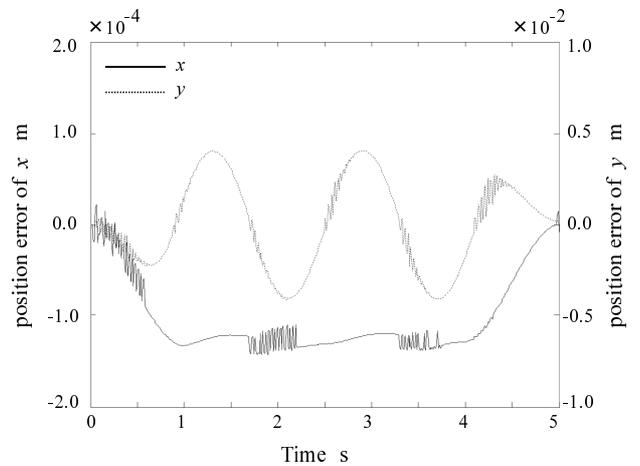


Fig. 9. Position error in Case 3

the position error of Case 3 was larger than Case 1. The similar tendency was observed from E_{vbest} of Case 2 and Case 4. The reason is supposed that only three joints must be driven to track the desired trajectories given by Eq. (8) and (9), whereas all joints must be driven when Eq. (10) and (11) are applied.

Next, E_{vbest} in case of using two kinds of frequencies are further compared. E_{vbest} of Case 1 is smaller than the one of Case 2. The reason is supposed that the desired velocity of Case 1 is slower than the one of Case 2. In other words, the desired trajectories shown in Fig. 4. are calculated within the range $0.28 \leq x \leq 0.78$, so the higher the frequency becomes, the longer the moved distance becomes. The similar tendency can be observed between Case 3 and Case 4. Fig. 8. and Fig. 9. are two examples of simulation when E_{vbest} shown in Table 2 are applied.

5 Conclusions

In this paper, genetic algorithms with a variable search space function has been proposed for fine gain tuning of a resolved acceleration controller. The pro-

posed GAs are applied for a trajectory following control based on several sine curves using dynamic model of PUMA560 manipulator. It has been confirmed that the proposed GAs effectively can reduce the generations for convergence and search better optimal gains.

References

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