Application of variable search space GAs to fine gain tuning of model-based robotic servo controller

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Abstract: In this paper, genetic algorithms with a variable search space function are proposed for fine gain tuning of a resolved acceleration controller which is one of model based robotic servo controllers. Genetic algorithms proposed in this paper have a variable search space function which is activated if the optimal solution is not updated for fixed generations. The function is terminated if the optimal solution is updated, or if the optimal solution is not updated within certain generations. This proposed method is evaluated through a trajectory following control problem in simulation. Simulations for sine curve trajectories are conducted using the dynamic model of PUMA560 manipulator. The result shows the improvement of optimal solution and its convergence.

Keywords: Fine Gain Tuning, Genetic Algorithms, PUMA560.

1 Introduction

At designing a control system, feedback gains tuning is the complicated problem. Conventionally, the gains have been experimentally and instinctively tuned by trial and error based on known model information and an operator's skill. For articulated-type industrial robots, this problem tends to become more complicated. Recently, the computing power is increased significantly, so that the gains in a control system can be numerically tuned through computer simulations by using the target dynamic model. Gain tuning problem is a typical nonconvex optimization problem, so that genetic algorithms have been applied to efficiently solve this problem. For example, Deris et al. [1] considered the stabilization of an inverted pendulum which can be controlled by moving a car in an intelligent way. Ya and Meng [2] proposed a genetic algorithm-based optimized hybrid controller which is suitable for controlling both linear and nonlinear systems. Also, Nagata et al. [3] proposed an effective fine gain tuning method for a computed torque controller, in which genetic algorithms were applied to obtain more suitable feedback gains after manual turning process.

Genetic algorithms proposed in this paper have a variable search space function which is activated if the optimal solution is not updated for fixed generations. The function is terminated if the optimal solution is updated, or if the optimal solution is not updated within certain generations. The propose method is evaluated through a trajectory following control problem using sine curves. Simulations are conducted by using the dynamic model of PUMA560 manipulator. Two types of the desired trajectories are designed with sine curves on x-y plane and x-z plane. The results of the simula-



Fig. 1. Block diagram of resolved acceleration control

tions showed the improvement of optimal solution and its convergence. The simulation environment used is called Robotics Toolbox and was developed by Corke [4].

2 Robotic servo controller

The dynamic model of an industrial manipulator without friction torque is generally given by

$$\boldsymbol{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \boldsymbol{H}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \boldsymbol{G}(\boldsymbol{\theta}) = \boldsymbol{\tau}$$
(1)

where $\boldsymbol{M}(\boldsymbol{\theta}) \in \mathbb{R}^{6 \times 6}$, $\boldsymbol{H}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \in \mathbb{R}^{6 \times 1}$ and $\boldsymbol{G}(\boldsymbol{\theta}) \in \mathbb{R}^{6 \times 1}$ are the inertia term, Coriolis/centrifugal force term and gravity term in joint space, respectively. $\boldsymbol{\theta} = [\theta_1, ..., \theta_6]^T$, $\dot{\boldsymbol{\theta}} \in \mathbb{R}^{6 \times 1}$, $\ddot{\boldsymbol{\theta}} \in \mathbb{R}^{6 \times 1}$, are the vector of joint angular position, velocity and acceleration. $\boldsymbol{\tau} \in \mathbb{R}^{6 \times 1}$ is the joint driven torque vector. Fig. 1. shows the block diagram of the resolved acceleration controller, which is known as one of model based robotic servo controllers. Where $\boldsymbol{F}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \in \mathbb{R}^{6 \times 1}$ is the frictional torque term. Desired position, velocity and acceleration in Cartesian coordinate system must be given to the reference of the servo system in order to apply the controller to a trajectory fol-



Fig. 2. Example of chromosome of an individual

Table 1. Initial parameters of genetic algorithms

Population size	100		
Number of elite	6		
Selection	Tournament selection and elitism		
Crossover	Uniform crossover (rate=87.5%)		
Mutation	Random mutation (rate= 1.0%)		
Search space of K_{vi}	$50 \le K_{vi} \le 220$		
Search space of K_{pi}	$1000 \le K_{pi} \le 35000$		
Maximum generation	200		

lowing control problem. The resolved acceleration controller generates the joint driven torque as given by

$$\tau = \hat{\boldsymbol{M}}(\boldsymbol{\theta})\boldsymbol{J}^{-1}(\boldsymbol{\theta}) \Big[\ddot{\boldsymbol{x}}_r + \boldsymbol{K}_v \left\{ \dot{\boldsymbol{x}}_r - \dot{\boldsymbol{x}} \right\} \\ + \boldsymbol{K}_p \left\{ \boldsymbol{x}_r - \boldsymbol{x} \right\} - \dot{\boldsymbol{J}}(\boldsymbol{\theta}) \dot{\boldsymbol{\theta}} \Big] + \hat{\boldsymbol{H}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \hat{\boldsymbol{G}}(\boldsymbol{\theta})$$
(2)

where the hat symbol means the estimated term obtained by computing inverse dynamics via recursive Newton-Euler formulation. $\boldsymbol{x} \in \Re^{6 \times 1}$ consists of position vector $[x, y, z]^T$ and orientation vector $[\alpha, \beta, \gamma]^T$. $\dot{x} \in \Re^{6 \times 1}$ and $\ddot{x} \in \Re^{6 \times 1}$ are the velocity and acceleration vectors, respectively. \boldsymbol{x}_r , $\dot{\boldsymbol{x}}_r$ and $\ddot{\boldsymbol{x}}_r$ are the desired vectors of position, velocity and acceleration, respectively. θ , $\dot{\theta}$, x and \dot{x} are actual controlled values. $Fkine(\boldsymbol{\theta})$ is the function of forward kinematics given by $\boldsymbol{x} = Fkine(\boldsymbol{\theta})$. $\boldsymbol{K}_v = \text{diag}(K_{v1}, ..., K_{v6})$. $\mathbf{K}_{p} = \operatorname{diag}(K_{p1}, ..., K_{p6})$ are the feedback gains of velocity and position respectively, each of which is set to a positive definite diagonal matrix. $\hat{M}(\theta), \hat{H}(\theta, \theta)$ and $\hat{G}(\theta)$ are called nonlinear compensation terms, which act on the cancellation of nonlinearity of manipulator. The nonlinear compensation terms are effective to achieve a stable trajectory following control. If the estimate error dose not exist at all, then the second order error equation of this system described in Eq. (3) is derived from Eqs. (1) and (2).

$$\ddot{\boldsymbol{e}} + \boldsymbol{K}_v \dot{\boldsymbol{e}} + \boldsymbol{K}_v \boldsymbol{e} = \boldsymbol{O} \tag{3}$$

where $\boldsymbol{e} = \boldsymbol{x}_r - \boldsymbol{x}$, $\dot{\boldsymbol{e}} = \dot{\boldsymbol{x}}_r - \dot{\boldsymbol{x}}$, $\ddot{\boldsymbol{e}} = \ddot{\boldsymbol{x}}_r - \ddot{\boldsymbol{x}}$. Accordingly, a nominal response is obtained by adjusting \boldsymbol{K}_v and \boldsymbol{K}_p suitably considering the following equation based on critically damped condition.

$$K_{vi} = 2\sqrt{K_{pi}}$$
 $(i = 1, ..., 6)$ (4)



Fig. 3. Flowchart of proposed genetic algorithms

If \mathbf{K}_v and \mathbf{K}_p are selected based on Eq. (4), the stability is guaranteed and the errors convergence to zero.

3 Fine gain tuning by genetic algorithms

In this section, genetic algorithms are used to optimize K_v and K_p . In order to apply genetic algorithms, \mathbf{K}_v and \mathbf{K}_p are transformed into a digit binary string as chromosome. Diagonal elements of them are encoded into 8 bits and 16 bits codes respectively, so that an individual chromosome is composed of 144 (=8) \times 6+16 \times 6) bits as shown in Fig. 2.. A population has 100 individuals and each individual in initial population is generated within the expectable values. The parameters of GAs are defined as shown Table 1. In our study, variable search space GAs are proposed and applied. Fig. 3. shows the flowchart of the proposed variable search space GAs, in which if the optimal gain is unchanged for 10 generations then the search space is varied. This means that the search spaces are redefined temporarily based on the optimal gains at the generation.

$$K_{pi\min} = K_{pibest} - 10000 \tag{5}$$

$$K_{pi\max} = K_{pibest} + 10000 \tag{6}$$

Each individual is evaluated through the following evaluation function.

$$E_v = \sqrt{\sum_{k=1}^{500} ||\boldsymbol{e}(k)||}$$
(7)

where k is the sampled discrete-time number in simulation. Note that, this evaluation function also represents the position errors.

4 Simulations

4.1 Desired trajectory

In this section, to evaluate the effectiveness of the proposed method, simulations are carried out by us-



Fig. 4. Illustration of desired trajectory

ing the dynamic model of PUMA560 manipulator on MATLAB system [4]. First of all, in order to apply the resolved acceleration control method to PUMA560 manipulator, the desired trajectory in Cartesian coordinate system must be prepared. Discrete values of desired position, velocity and acceleration are generated for each sampling period in advance. Four type trajectories are prepared to be compared for each and set as below.

Case 1.
$$z = 0.1\sin(10\pi x)$$
 (8)

Case 2.
$$z = 0.1\sin(20\pi x)$$
 (9)

Case 3.
$$y = 0.1\sin(10\pi x) - 0.15$$
 (10)

Case 4.
$$y = 0.1\sin(20\pi x) - 0.15$$
 (11)

Fig. 4. is the overview of the desired manipulator movement. The desired positions given by Eq. (8) and (9) are designed with sinusoidal waves on x-z plane. Also, the desired positions given by Eqs. (10) and (11) are generated with sinusoidal waves on x-y plane. As an example, Fig. 5. illustrates the desired trajectory of Eq. (10) in x and y directions. The position x of desired trajectory is generated with a 4-1-4 order polynomial equation. The initial and final position/orientation vectors are set to $\mathbf{x}_{init} = (0.28, -0.15, 0.00, 0.00, \pi/2, 0.00),$ $\mathbf{x}_{fin} = (0.78, -0.15, 0.00, 0.00, \pi/2, 0.00)$, respectively.

In this simulation, we set them such that the total simulation time is 5.00 sec, the sampling width is 0.01 sec, the acceleration and deceleration time is 1.00 sec. To obtain satisfactory and safe control performance with avoiding singularities, \mathbf{K}_v and \mathbf{K}_p are roughly tuned with trial and error, considering the critically damped condition.

4.2 Dynamic model

In this simulation experiment, it is assumed that the friction is composed of viscous friction and Coulomb friction. Therefore, robotic dynamic model considering friction can be rewritten by

$$\boldsymbol{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \boldsymbol{H}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \boldsymbol{G}(\boldsymbol{\theta}) + \boldsymbol{F}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \boldsymbol{\tau}$$
(12)



 $F(\theta, \dot{\theta})$ is composed of viscous friction and Coulomb friction and given by

$$\boldsymbol{F}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \boldsymbol{B}\boldsymbol{G}_r^2 \dot{\boldsymbol{\theta}} + \boldsymbol{G}_r \boldsymbol{\tau}_c \{ \operatorname{sign}(\dot{\boldsymbol{\theta}}) \}$$
(13)

where $\boldsymbol{B} \in \Re^{6\times 6}$ is the coefficient matrix of viscous friction at each motor, $\boldsymbol{G}_r \in \Re^{6\times 6}$ is the reduction gear ratio matrix which represents the motor speed to joint speed, and $\boldsymbol{\tau}_c\{\operatorname{sign}(\boldsymbol{\dot{\theta}})\}$ is the Coulomb friction torque appeared at each motor. If $\operatorname{sign}(\boldsymbol{\dot{\theta}})$ is positive/negative, then $\boldsymbol{\tau}_c = \boldsymbol{\tau}_c^+/\boldsymbol{\tau}_c = \boldsymbol{\tau}_c^-$, respectively. The friction parameters of PUMA560 manipulator such as $\boldsymbol{B}, \boldsymbol{G}_r, \boldsymbol{\tau}_c^+$ and $\boldsymbol{\tau}_c^-$ are technically available to engineers[4].

It is confirmed through preliminary simulations that this friction term, which works as disturbance as shown in **Fig. 1.**, causes undesirable error even in critically damped condition. Also, in actual robot systems there inevitably exists the modeling error with respect to $\hat{M}(\theta)$, $\hat{H}(\theta,\dot{\theta})$ and $\hat{G}(\theta)$. That is the reason why the fine gain turning after manual gain tuning is important for robotic servo system in order to realize more accurate trajectory following control.

4.3 Result and discussions

Fig. 6. shows the minimum E_v values at every generation searched by the proposed GAs. It is observed that optimal solutions of each case are almost converged before 50 generations. In Fig. 7., the curved line is critical damping condition, and the plotted points are the optimal gains of each case. It is observed that most of gains lie on under-damping condition. Table 2 tabulates each best fitness E_{vbest} which is obtained using the four optimal gains.

At first, E_{vbest} in case of using the trajectories in xy and x-z planes are compared. Table 2 indicates that

 Table 2. Best fitness comparison for each case

	Case 1	Case 2	Case 3	Case 4
Ev_{best}	0.0630	0.0874	0.0672	0.1147

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Fig. 6. Evolutionary histories of best fitness



Fig. 7. Best gains and critical damping condition

the position error of Case 3 was larger than Case 1. The similar tendency was observed from E_{vbest} of Case 2 and Case 4. The reason is supposed that only three joints must be driven to track the desired trajectories given by Eq. (8) and (9), whereas all joints must be driven when Eq. (10) and (11) are applied.

Next, E_{vbest} in case of using two kinds of frequencies are further compared. E_{vbest} of Case 1 is smaller than the one of Case 2. The reason is supposed that the desired velocity of Case 1 is slower than the one of Case 2. In other words, the desired trajectories shown in **Fig. 4**. are calculated within the range $0.28 \le x \le 0.78$, so the higher the frequency becomes, the longer the moved distance becomes. The similar tendency can be observed between Case 3 and Case 4. **Fig. 8**. and **Fig. 9**. are two examples of simulation when E_{vbest} shown in Table 2 are applied.

5 Conclusions

In this paper, genetic algorithms with a variable search space function has been proposed for fine gain tuning of a resolved acceleration controller. The pro-



posed GAs are applied for a trajectory following control based on several sine curves using dynamic model of PUMA560 manipulator. It has been confirmed that the proposed GAs effectively can reduce the generations for convergence and search better optimal gains.

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