Kinodynamic motion planning and control using anisotropic damping forces

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Abstract: We deal with gain optimization in kinodynamic motion planning by using NADFs (Nonlinear, Anisotropic, Damping Forces) and HPF (Harmonic Potential Field). The NADFs are a kind of controller which has been proposed by Masoud, and it has the effect of canceling the force in unnecessary direction (e.g. any external force or the inertial force). The NADFs have two gains, but optimizing them was not mentioned at all. In this paper, it is assumed that a mass with Kinodynamic Motion Planning is controlled in simulation. The effectiveness of NADFs is verified by comparing the control that uses only the gradient of HPF with the control that introduces the NADFs. At the same time, we apply a method called "Clamping Control" to the aforementioned situation, and accomplish a reliable convergence. It is reported through experiments that there exists a possibility of accomplishing more accurate motion planning by optimizing the gains of NADFs.

Keywords: Motion planning, Kinodynamic planning, Potential field

1 INTRODUCTION

Kinodynamics is a discipline dealing with the kinematics and dynamics simultaneously. Kinodynamic motion planning aims at synthesizing the kinematic motion planning like obstacle avoidance, and the dynamic motion planning which is needed in actual control. In this research, we deal with gain optimization in kinodynamic motion planning by using NADFs (Nonlinear, Anisotropic, Damping Forces) and HPF (Harmonic Potential Field). An HPF is a smooth potential field which has no stationary points [1]. It is guaranteed that a kinematic trajectory, which is generated by using the gradient of HPF, can reliably reach a target point. Thus, if a controlled object moves along the gradient of HPF, then such an object can reliably reach the target point. However, its trajectory deviates from the kinematic trajectory in the consequence of the inertia or an external force, even if a controller directly gives the controlled object the gradient of HPF as the control input. Then, we apply the NADFs to a controller. The NADFs is a kind of controller which has been proposed by Masoud [2], and it has the effect of canceling the force in unnecessary direction (e.g. any external force or the inertial force). By applying NADFs, the controlled object can reach the target point quickly without being affected by external forces or inertial forces. The NADFs introduced by Masoud has two gains, but he did not at all mention optimizing these gains. In this paper, it is assumed that a point mass with Kinodynamic Motion Planning is controlled in simulation. The effectiveness of NADFs is verified by comparing the control that uses only the gradient of HPF with the control that introduces the NADFs. At the same time, by changing the gains manually, the trend in the behavior of the controlled object is confirmed according to the change of the gains. We also prove the efficacy of introducing the NADFs in the environment where an external force exists. It is found in simulations that the NADFs are able to cope with the applied external force, but its posture diverges sometimes after the controlled object reaches the target point. Therefore, we apply a method called "Clamping ontrol" to the aforementioned situation, and accomplish a reliable convergence. "Clamping control" is a control method proposed by Masoud, which makes an attractive force in around the target point to confine the controlled object to the target point. It is reported through the experiments that the NADFs can be applied in an environment where external forces exist; the convergence time and trajectory are changed by tuning the gains; and there exists a possibility of accomplishing accurate motion planning by optimizing the gains.

2 MOTION PLANNING USING HPF

Kinodynamic motion planning using NADFs is based on the motion planning using HPF. In the motion planning using HPF, a controlled object is controlled by control input based on the gradient of HPF. HPF is generated from the boundary information on the environment. In what follows, a generation method of HPF and the motion planning using only the gradient of HPF are described.

2.1 Generation method of HPF and the calculation of gradient vector

An HPF is a smooth potential field which has no stationary points. For generating an HPF, an environment is at first segmented into small grids, and the potential at each grid (i, j) is calculated. In initial states, potential is set to 1 for the bound



Fig. 1. Normalized gradi-Fig. 2. Path generation by ent field using gradient of HPF

of obstacles and the environment, and set to 0 for an intended target point. Then, to compute the potential in each grid, the following equation is calculated iteratively:

$$U(i,j) = \frac{1}{4}(U_{i,j-1} + U_{i,j+1} + U_{i-1,j} + U_{i+1,j})$$
(1)

Note that, U(i, j) is the value of the potential at an (i, j) grid in the environment field. By calculating Eq. (1) iteratively, the surface of the generated HPF becomes smooth, because the average of potentials in the four neighborhood grids is saved as the potential at the center grid.

Then, a gradient in each position is calculated by using the generated HPF. Fig. 1 shows gradient vectors calculated from HPF. Note that these vectors have been normalized by the grid size. The position circled in Fig. 1 shows the configured target point. Using this gradient vector, a kinematic trajectory can reliably reach a target point from anywhere in the field while avoiding obstacles. Letting V(x) be a potential value in position $x = [x \ y]^{T}$, a gradient on the surface of HPF in position x can write $-\nabla V(x)$. Using gradient $-\nabla V(x)$, a kinematic trajectory can be generated from the following equation:

$$\dot{\boldsymbol{x}} = -\nabla V(\boldsymbol{x}) \tag{2}$$

As an example, Fig. 2 shows the calculated kinematic trajectories from four configured start positions to the target point. Configured start positions A to D and the target point were set as shown in Fig. 2. It is confirmed from Fig. 2 that a kinematic trajectory using the gradient of HPF can reach a target point from anywhere in the field while avoiding obstacles.

2.2 Control of a point mass based on the gradient of HPF

When controlling an object having a mass, dynamics including object mass, inertial force and external force should be considered. Koditschek et al. [3] proposed a method for designing a control input that considers dynamics by adding both the gradient of HPF and a damping force linearly proportional to the velocity. In this method, when controlling a point mass with 1 [kg], the control input u is given as follow:

$$\boldsymbol{u} = -\boldsymbol{b} \cdot \dot{\boldsymbol{x}} - \nabla V(\boldsymbol{x}) \tag{3}$$

where b denotes a damping coefficient and also is positive constant. In Eq. (3), control input consists of the gradient



Fig. 3. The clamping control

of the current position, $-\nabla V(x)$, and the damping force linearly proportional to the velocity, $-b \cdot \dot{x}$. This damping force increases in proportion to increasing of the damping coefficient or velocity, and suppresses the acceleration of a point mass.

3 MOTION PLANNING USING NADFS

The method described in previous section, which is based on only the gradient of HPF, does not work well under an environment where an external force exists. Masoud [2] proposed a control method that can cancel out the effect of external forces by combining the gradient of HPF with NADFs. The following equation is the controller proposed by Masoud [2]:

$$\boldsymbol{u} = -b_d \cdot h(\boldsymbol{x}, \dot{\boldsymbol{x}}) - k \cdot \nabla V(\boldsymbol{x}) \tag{4}$$

where b_d and k denote positive constant gains, and $h(x, \dot{x})$ is the proposed NADFs, which is given by

$$h(\boldsymbol{x}, \dot{\boldsymbol{x}}) = \left[\boldsymbol{n}^{\mathrm{T}} \dot{\boldsymbol{x}} \boldsymbol{n} + \left(\frac{V(\boldsymbol{x})^{\mathrm{T}}}{\|\nabla V(\boldsymbol{x})\|} \cdot \dot{\boldsymbol{x}} \cdot \Phi(\nabla V(\boldsymbol{x})^{\mathrm{T}} \dot{\boldsymbol{x}})\right) \frac{V(\boldsymbol{x})^{\mathrm{T}}}{\|\nabla V(\boldsymbol{x})\|}\right]$$
(5)

Here, *n* is a unit vector orthogonal to ∇V and $\Phi(\nabla V(\boldsymbol{x})^{\mathrm{T}} \dot{\boldsymbol{x}})$ is the unit step function. The function Φ takes 0 if $\nabla V(\boldsymbol{x})^{\mathrm{T}} \dot{\boldsymbol{x}}$ is negative, and otherwise takes 1. The function $\Phi(\nabla V(\boldsymbol{x})^{\mathrm{T}} \dot{\boldsymbol{x}})$ is prepared to check the consistency between the direction of current velocity of the controlled onject and the direction of gradient of HPF. If the direction of the current velocity matches the gradient direction, then $\nabla V(\boldsymbol{x})^{\mathrm{T}} \dot{\boldsymbol{x}}$ takes a positive value and $\Phi(\nabla V(\boldsymbol{x})^{\mathrm{T}} \dot{\boldsymbol{x}})$ takes 1, thus, a force toward the current velocity direction is increased. By contrast, if their directions are not matched, then $\nabla V(\boldsymbol{x})^{\mathrm{T}} \dot{\boldsymbol{x}}$ takes a negative value and $\Phi(\nabla V(\boldsymbol{x})^{\mathrm{T}} \dot{\boldsymbol{x}})$ takes 0, thus, a force canceling the current velocity is added. At the same time, a force toward the gradient direction is added, and the controlled object is returned to the kinematic trajectory. Thus, this method is also applicable to the control of an object in an environment where external forces exist.

4 CLAMPING CONTROL

The control using NADFs can guide a controlled object to a target point while cancelling external forces. However,



Fig. 4. Simulation environment

after reaching to the target point, the controlled object sometimes cannot maintain any states and therefore it oscillates and/or diverges. Therefore, we apply "clamping control". Fig. 3 shows the schematic structure of the clamping control. The clamping control is applied in a circle area as shown in Fig. 3. The center point of the region is a target point and its radius is σ . In this area, if a controlled object moves toward the target point, then the clamping control is not applicable. By contrast, if a controlled object moves against the target point in the area, then the clamping control adds a force $(x_{\rm T} - x)$ in a direction toward the target point, and converges a controlled object on the target point. Appending the clamping control to the control input based on NADFs given in Eq. (4), it follows that

$$\boldsymbol{u} = -b_d \cdot h(\boldsymbol{x}, \dot{\boldsymbol{x}}) - k \cdot \nabla V(\boldsymbol{x}) - k_{\rm C} \cdot \boldsymbol{F}_{\rm C}(\boldsymbol{x}, \dot{\boldsymbol{x}})$$
(6)

where $k_{\rm C}$ denotes a positive constant gain related to the clamping control and $F_{\rm C}$ is clamping control given by

$$\boldsymbol{F}_{\mathrm{C}}(\boldsymbol{x}, \dot{\boldsymbol{x}}) = (\mathbf{x}_{\mathrm{T}} - \mathbf{x}) \cdot \Phi(\sigma - |\boldsymbol{x}_{\mathrm{T}} - \boldsymbol{x}|) \cdot \Phi(\dot{\boldsymbol{x}}^{\mathrm{T}}(\boldsymbol{x}_{\mathrm{T}} - \boldsymbol{x}) \quad (7)$$

Here, the unit step function $\Phi(\sigma - |x_{\rm T} - x|)$ is set to check whether the controlled object is in the region, and $\Phi(\dot{x}^{\rm T}(x_{\rm T} - x))$ is prepared to check whether the direction of current velocity of the controlled object consists with the direction of the target point.

5 SIMULATION EXPERIMENT

In this section, we made a simulation experiment for comparing three methods as described above (using gradient of HPF only, applying NADFs, and combining NADFs and clamping control). Fig. 4 shows an experimental environment, where there are two obstacles. In this experiment, it is aimed at guiding a point mass with 1 [kg] from the initial position $x_0(40, 40)$ to the target point $x_T(10, 10)$. The trajectory drawn in Fig. 4 is a kinematic trajectory calculated from the gradient of HPF. This kinematic trajectory is set to an ideal trajectory.

5.1 A point mass control in the absence of external forces

At first, we compared three different types control in the absence of external forces. The point mass has the mass of 1



Fig. 5. Simulation results using formula (8) in static environment (gains b = 0.1, 1.0 and 10.0)

[kg], so that the equation of motion with the controller based on only using the gradient of HPF is given by

$$\ddot{\boldsymbol{x}} = -\boldsymbol{b} \cdot \dot{\boldsymbol{x}} - \nabla V(\boldsymbol{x}) \tag{8}$$

Similarly, one based on using the gradient and NADFs and that based on additionally applying clamping control are given as follows:

$$\ddot{\boldsymbol{x}} = -b_d \cdot h(\boldsymbol{x}, \dot{\boldsymbol{x}}) - k \cdot \nabla V(\boldsymbol{x})$$
(9)

$$\ddot{\boldsymbol{x}} = -b_d \cdot h(\boldsymbol{x}, \dot{\boldsymbol{x}}) - k \cdot \nabla V(\boldsymbol{x}) - k_{\rm C} \cdot \boldsymbol{F}_{\rm C}(\boldsymbol{x}, \dot{\boldsymbol{x}}) \quad (10)$$

In this simulation, the gains k and $k_{\rm C}$ are set to k = 1 and $k_{\rm C} = 10$ constantly, and we compared the results when changing the gains b and b_d .

Figs. 5-7 show the trajectory of the point mass and the convergence time with each controller when the gains b and b_d are set to 0.1, 1.0 and 10.0. The trajectory of the point mass came closer to the kinematic trajectory (Fig. 4) in proportion to increasing gains. This is because the controller based on only the gradient of HPF suppresses the velocity with an increase in gain b. On the other hand, in the case of the controller including NADFs, a force canceling the velocity except the matched direction of the gradient is increased with an increase in gain. As a result, the both controllers can guide the mass point to a target point with high accuracy. As long as looking at the trajectory, there exist no big difference between the controller using NADFs and that not using NADFs, but there is a big difference in convergence time. In Fig. 5, the convergence time is increased particularly with an increase in gain b. By contrast, in Fig. 6, convergence time is shorter with an increase in gain b_d . This is because the NADFs consider the direction of the velocity and suppress unnecessary force only. In addition, when using the controller including the clamping control (Fig. 7), the trajectory and the convergence time are almost the same as those using NADFs, but the overshoot was suppressed after the point mass reached the target point.

In this experiment, it is confirmed that all controllers can guide the point mass to the target point by tuning the gains b and b_d . In particular, the controller using NADFs can guide the point mass quickly with a high accuracy. Moreover, we also confirmed that the overshoot is suppressed by applying the clamping control.



Fig. 6. Simulation results using formula (9) in static environment (gains $b_d = 0.1, 1.0$ and 10.0)



Fig. 7. Simulation results using formula (10) in static environment (gains $b_d = 0.1, 1.0$ and 10.0)



(c) using formula (13)

Fig. 8. Simulation results where external forces exist (gains $b, b_d = 1.0, 5.0$ and 10.0)

5.2 A point mass control in the environment where external forces exist

Next, assume that external forces $f_g = [4 \ 4]^T$ exist in the environment. Behavioral models of the point mass with each controller are as follows, respectively:

$$\ddot{\boldsymbol{x}} = -\boldsymbol{b} \cdot \dot{\boldsymbol{x}} - \nabla V(\boldsymbol{x}) - \boldsymbol{f}_{g}$$
(11)

$$\ddot{\boldsymbol{x}} = -b_d \cdot h(\boldsymbol{x}, \dot{\boldsymbol{x}}) - k \cdot \nabla V(\boldsymbol{x}) - \boldsymbol{f}_{g}$$
(12)

$$\ddot{\boldsymbol{x}} = -b_d \cdot h(\boldsymbol{x}, \dot{\boldsymbol{x}}) - k \cdot \nabla V(\boldsymbol{x}) - k_{\mathrm{C}} \cdot \boldsymbol{F}_{\mathrm{C}}(\boldsymbol{x}, \dot{\boldsymbol{x}}) - \boldsymbol{f}_{\mathrm{g}}$$
 (13)

In this simulation, the gains k and $k_{\rm C}$ are set to k = 1 and $k_{\rm C} = 10$ constantly, and we compared the results of when changing the gains b and b_d .

Fig. 8 shows the trajectory of the point mass using each controller when the gains b and b_d are set to 1.0, 5.0 and 10.0. As shown in Fig. 8(a), the controller using only the gradient of HPF cannot suppress the external forces, and the point mass entirely deviated from the route. By contrast, the controller including NADFs suppressed the external forces, and guided the point mass to the target point by amplifying the gains (Fig. 8(b)). However, the point mass could not keep the state at the target point. The controller including NADFs and clamping control can guide the point mass, and kept the state at the target point (Fig. 8(c)). In this experiment, it is confirmed that the controller based on NADFs can also apply in the environment where external forces exist, and a controlled object can keep its state at the target point by adding the clamping control.

6 CONCLUSIONS

In this paper, we have compared three controllers: the controller by using only the gradient of HPF; the controller including NADFs; and the controller including NADFs and clamping control. At the same time, by changing the gains manually, the trend in the behavior of the controlled object is confirmed according to the change of the gains. As a result, kinodynamic motion planning by using NADFs can guide the controlled object to the target point quickly, and it is robust against external forces. Moreover, by applying the clamping control, the secure convergence to the target point was able to be realized. In future work, we will optimize gains for NADFs by using GA and validate the associated behavior.

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