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Homogeneous Systolic Pyramid Automata with n-Dimensional Layers

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Abstract

Cellular automata were investigated not only in the viewpoint of formal language theory, but also in the viewpoint of pattern recognition. Cellular automata can be classified into some types. A systolic pyramid automata is also one parallel model of various cellular automata. A homogeneous systolic pyramid automaton with n-dimensional layers (n-$HSPA$) is a pyramid stack of n-dimensional arrays of cells in which the bottom n-dimensional layer (level 0) has size a^n ($a \geq 1$), the next lowest $(a-1)^n$, and so forth, the (a-1)st n-dimensional layer (level (a-1)) consisting of a single cell, called the root. Each cell means an identical finite-state machine. The input is accepted if and only if the root cell ever enters an accepting state. An n-$HSPA$ is said to be a real-time n-$HSPA$ if for every n-dimensional tape of size an ($a \geq 1$) it accepts the n-dimensional tape in time a-1. Moreover, a 1-way n-dimensional cellular automaton (1-nCA) can be considered as a natural extension of the 1-way two-dimensional cellular automaton to n-dimension. The initial configuration is accepted if the last special cell reaches a final state. A 1-nCA is said to be a real-time 1-nCA if when started with n-dimensional array of cells in nonquiescent state, the special cell reaches a final state. In this paper, we propose a homogeneous systolic automaton with n-dimensional layers (n-$HSPA$), and investigate some properties of real-time n-$HSPA$. Specifically, we first investigate a relationship between the accepting powers of real-time n-$HSPA$'s and real-time 1-nCA's. We next show the recognizability of n-dimensional connected tapes by real-time n-$HSPA$'s.

Key Words **:** cellular automaton, diameter, finite automaton, n-dimension, parallelism, pattern recognition, real time.

1 Introduction and Preliminaries

The question of whether processing n-dimentional digital patterns is much more difficult than (n-1) dimentional ones is of great in terest from the theoretical and practical standpoints. Thus, the study of n-dimensional automata as a computational model of n-dimensional pattern processing has been meaningful[4-23]. Cellular automata were investigated not only in the viewpoint of formal language theory, but also in the viewpoint of pattern recognition. Cellular automata can be classified into some types [2]. A systolic pyramid automaton is also one parallel model of various cellular automata. In this paper, we propose a homogeneous systolic automaton with n-dimensional layers (n-*HSPA*), and investigate some properties of real-time n-*HSPA*.

Let Σ be a finite set of symbols. An *n-dimensional tape* over Σ is an $(n-1)$-dimensional array of elements of Σ. The set of all n-dimensional tapes over Σ is denoted by $\Sigma^{(n)}$. Given a tape $x \in \Sigma^{(n)}$, for each $j (1 \leq j \leq n)$, we let $l_j(x)$ be the length of x along the jth axis. When $1 \leq i_j \leq l_j(x)$ for each $j (1 \leq j \leq n)$, let $x(i_1, i_2, \ldots, i_n)$ denote the symbol in x with coordinates $(i_1, i_2, \ldots, i_n)$. We concentrate on the input tape x with $l_1(x) = l_2(x) = l_3(x) = \cdots = l_n(x)$.

A *homogeneous systolic pyramid automaton with n-dimensional layers* (n-*HSPA*) is a pyramidal stack of n-dimensional arrays of cells in which the bottom n-dimensional layer (level 0) has size a^n $(a \geq 1)$, the next lowest $(a-1)^n$, and so forth, the $(a-1)$st n-dimensional layer (level $(a-1)$) consisting of a single cell, called the *root*. Each cell means an identical finite-state machine, $M = (Q, \Sigma, \delta, \#, F)$, where Q is a finite set of *states*, $\Sigma \subseteq Q$ is a finite set of *input states*, $\# \in Q - \Sigma$ is the *quiescent state*, $F \subseteq Q$ is the set of *accepting states*, and $\delta : Q^{2^n+1} \to Q$ is the *state transition function*, mapping the current states of M and its 2^n son cells in a $2 \times 2 \times \cdots \times 2$ block on the n-dimensional layer below into M's next state. The input is *accepted* if and only if the root cell ever enters an accepting state. An n-*HSPA* is said to be a *real-time n-HSPA* if for every n-dimensional tape of size a^n $(a \geq 1)$ it accepts the n-dimensional tape in time $a-1$. By $\pounds^R[n\text{-}HSPA]$ we denote the class of the sets of all the n-dimensional tapes accepted by a real-time n-*HSPA*[1].

A 1-*way n-dimensional cellular automaton* (1-nCA) can be considered as a natural extension of the

1-way two-dimensional cellular automaton to n dimensions [3]. The initial configuration of the cellular automaton is taken to be an $l_1(x) \times l_2(x) \times \cdots \times l_n(x)$ array of cells in the nonquiescent state. The initial configuration is *accepted* if the last special cell reaches a final state. A 1-nCA is said to be a *real-time* 1-nCA if when started with an $l_1(x) \times l_2(x) \times \cdots \times l_n(x)$ array of cells in the nonquiescent state, the special cell reaches a final state in time $l_1(x)+l_2(x)+\cdots+l_n(x)-1$. By $\pounds^R[1\text{-}nCA]$ we denote the class of the sets of all the n-dimensional tapes accepted by a real-time 1-nCA [3].

2 Main Results

We mainly investigate a relationship between the accepting powers of real-time n-$HSPA$'s and real-time 1-nCA's. The following theorem implies that real-time n-$HSPA$'s are less powerful than real-time 1-nCA's.

Theorem 2.1. $\pounds^R[n\text{-}HSPA] \subsetneq \pounds^R[1\text{-}nCA]$.

Proof : Let V=$\{x \in \{0,1\}^{(n)} | l_1(x) = l_2(x) = \cdots = l_n(x) \& [\forall_{i_1}, \forall_{i_2}, \ldots, \forall_{i_{n-1}} \ (1 \le i_1 \le l_1(x), 1 \le i_2 \le l_2(x), \ldots, 1 \le i_{n-1} \le l_{n-1}(x))[x(i_1, i_2, \ldots, i_{n-1}, 1) = x(i_1, i_2, \ldots, i_{n-1}, l_n(x))]]\}$.

It is easily shown that $V_1 \in \pounds^R[1\text{-}nCA]$. Below, we show that $V \notin \pounds^R[n\text{-}HSPA]$. Suppose that there exists a real-time n-$HSPA(n = 3)$ accepting V. For each $t \ge 4$, let

$W(n) = \{x \in \{0,1\}^{(3)} | l_1(x) = l_2(x) = \cdots = l_n(x)$ & $[x\ (1,2,1),(t,t-1,t)] \in 0^{(3)}\}$.

Eight sons of the root cell $A_{(t-1,1,1,1)}$ of M $A_{(t-2,1,1,2)}$, $A_{(t-2,1,2,2)}$, $A_{(t-2,2,1,2)}$, $A_{(t-2,2,2,2)}$, $A_{(t-2,1,1,3)}$, $A_{(t-2,1,2,3)}$, $A_{(t-2,2,1,3)}$, $A_{(t-2,2,2,3)}$ are denoted by C_{UNW}, C_{USW}, C_{USE}, C_{UNE}, C_{DNW}, C_{DSW}, C_{DSE}, C_{DNE}, respectively. For each x in $W(n)$, $x(UNW)$, $x(USW)$, $x(USE)$, $x(UNE)$, $x(DNW)$, $x(USW)$, $x(USE)$, $x(UNE)$ are the states of C_{UNW}, C_{USW}, C_{USE}, C_{UNE}, C_{DNW}, C_{DSW}, C_{DSE}, C_{DNE}, at time t-2, respectively. Let $\sigma\ (x) = (x(UNW)$, $x(USW)$, $x(DNW)$, $x(DSW))$, $\gamma\ (x) = (x(USE)$, $x(UNE)$, $x(DSE)$, $x(DNE))$. and $\rho(x) = (x(UNW)$, $x(USW)$, $x(DNW)$, $x(DSW)$, $x(USE)$, $x(UNE)$, $x(DSE)$, $x(DNE))$. Then, the following two propositions must hold:

Proposition 2.1. (i) *For any two tapes* $x, y \in W(n)$ *whose* $1st(1-3)$ *planes are same,* $\sigma\ (x) = \sigma\ (y)$. (ii) *For any two tapes* $x, y \in W(n)$ *whose* $n-th(1-3)$ *planes are same,* $\gamma\ (x) = \gamma\ (y)$.

[**Proof :** From the mechanism of each cell, it is easily seen that the states of C_{UNW}, C_{USW}, C_{DNW}, C_{DSW} are not influenced by the information of $x(1-3)_t$'s. From this fact, we have (i). The proof of (ii) is the same as that of (i). Q.E.D.]

Propositon 2.2. *For any two tapes* $x, y \in W(t)$ *whose* 1*st* (1-3) *planes are different,* $\sigma(x) \ne \sigma(y)$.

[**Proof :** Suppose to the contrary that $\sigma(x) = \sigma(y)$. We consider two tapes x',$y' \in W(t)$ satisfying the follwing :
(i) $x(1-3)_1$ and $x(1-3)_t$, are equal to $x(1-3)_1$ of x, respectively
(ii) $y'(1-3)_1$ is equal to $y(1-3)_1$, and $y'(1-3)_t$ is equal to $x(1-3)_1$.

As is easily seen, $x' \in V$ and so x' is accepted by M. On the other hand, from Proposition 2.1(ii), $\gamma(x') = \gamma(y')$. From Proposition 2.1(i), $\sigma(x) = \sigma(x')$, $\sigma(y) = \sigma(y')$. It follows that y' must be also accepted by M. This contradicts the fact that y' is not in V. *Q.E.D*]

Proof of Theorem 2.1 (*continued*) **:** Let $p(t)$ be the number of tapes in $W(t)$ whose 1st (1-3) planes are different, and let $Q(t) = \{\sigma(x) | x \in W(t)\}$, where k is the number of states of each cell of M. Then, $p(t) = 2^{t^2}$, and $Q(t) \le k^4$. It follows that $p(n) > Q(t)$ for large t. Therefore, it follows that for large t, there must be two tapes x,y in $W(t)$ such that their 1st (1-3) planes are different and $\sigma(x) = \sigma(y)$. This contradicts Proposition 2.2, so we can conclude that $V \notin \pounds^R[3\text{-}HSPA]$. In the case of n-dimention, we can show that $V \notin \pounds^R[n\text{-}HSPA]$ by using the same technique. This completes the proof of Theorem 2.1. *Q.E.D.*

We next show the recognizability of n-dimensional connected tapes by real-time n-$HSPA$'s by using the name technique of Ref.[3]. Let x in $\{0,1\}^{(n)}$. A maximal subset P of $\boldsymbol{N}^n$ satisfying the following conditons is called a 1-*component* of x.
(i) For any $(i_1,i_2,\ldots,i_n \in P$, we have $1 \le i_1 \le l_1(x)$, $1 \le i_2 \le l_2(x), \ldots, 1 \le i_n \le l_n(x)$, and $x(i_1, i_2, \ldots, i_n) = 1$.
(ii) For any $(i_1,i_2,\ldots,i_n)$, $(i'_1,i'_2,\ldots,i'_n) \in P$, there exists a sequence $(i_{1,0},i_{2,0},\ldots,i_{n,0}),(i_{1,1},i_{2,1},\ldots,i_{n,1}),\ldots,(i_{1,n},i_{2,n},\ldots,i_{n,n})$ of elements in P such that $(i_{1,0},i_{2,0},\ldots,i_{n,0}) = (i_1,i_2,\ldots,i_n)$, $(i_{1,n},i_{2,n},\ldots,i_{n,n}) = (i'_1,i'_2,\ldots,i'_n)$, and $|i_{1,j}\text{-}i_{1,j-1}| + |i_{2,j}\text{-}i_{2,j-1}| + \ldots + |i_{n,j}\text{-}i_{n,j-1}| \le 1 (1 \le j \le n)$. A tape $x \in \{0,1\}^{(n)}$ is called *connected* if there exists exactly one 1-component of x.

Let T_c be the set of all the n-dimensional connected tapes. Then, we have

Theorem 2.2. $T_c \notin \pounds^R[n\text{-}HSPA]$.

3 Conclusion

We investigated a relationship between the accepting powers of homogeneous systolic pyramid automaton with n-dimensional layers(n-$HSPA$) and one-way

n-dimensional cellular automata (1-nCA) in real time, and showed that real-time n-$HSPA$'s are less powerful than real time 1-nCA's.

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