

Real-Time Stochastic Optimal Control for Traffic Signals of Multiple Intersections

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Abstract: Traffic congestion has become a serious problem with exponential increase of vehicles recently. In urban area, almost all of traffic jams occur at intersections. In such cases, traffic signal control is a reasonable method to reduce the traffic jams. Traffic signal control can be divided into two types: one is off-line (Pre-timed) control and the other is on-line (Adaptive) control. In the pretimed control, empirical formulas were used to calculate the traffic signals off-line using historical traffic data, but it cannot handle the variation of traffic flows. The adaptive control can overcome this limitation by adjusting the traffic signals on-line in the various traffic flows. The adaptive control can also be divided into two types: centralized and distributed systems. However, the centralized system requires the extensive data processing and computational time to calculate optimal traffic signals. On the other hand, the distributed system can achieve a real time control. In this paper, a real time stochastic optimal control method of traffic signal is proposed. A modified Cellular Automaton (CA) traffic model and Bayesian Network (BN) model are used to predict the traffic jams. In addition, H-GA-PSO algorithm is used to search optimal traffic signals based on the stochastic model. The H-GA-PSO algorithm is a modified Hierarchical Particle Swarm Optimization (H-PSO) method based on Genetic Algorithm (GA). Finally, the effectiveness of the proposed method is shown through simulations at multiple intersections using a micro traffic simulator.

Keywords: Traffic signal, Stochastic Optimal Control, Cellular Automaton Traffic model, H-GA-PSO algorithm.

1 INTRODUCTION

Traffic jam has become a serious problem with exponential increase of vehicles recently. In urban area, almost all traffic congestion occurs at intersections. One of the ways to solve this problem is road-expansion, but it is difficult to realize in urban areas. In such cases, traffic signal control is a reasonable method to reduce traffic-jam. Traffic signal control can be divided into two types. One is off-line (Pre-timed) control and the other is on-line (Adaptive) control. In the pre-timed control, Webster's formula is used to calculate the traffic signals off-line using historical traffic data, but it cannot handle the variation of traffic flows. On the other hand, the adaptive control can overcome this limitation by adjusting the traffic signals on-line in the various traffic flows. SCOOT and SCATS have been implemented on urban traffic networks using the centralized systems. However, the centralized systems require the extensive data processing and computational time for calculating optimal traffic signals. The distributed system can achieve the real time control.

In this paper, a real time stochastic optimal control method of traffic signal is proposed. A modified Cellular Automaton (CA) traffic model and Bayesian Network (BN) model are used to predict the probabilistic distributions of standing vehicles of the intersection roads. In addition, a hierarchical H-GA-PSO is proposed to search the optimal

traffic signals to minimize traffic-jam probabilities using an optimal probabilistic model. Finally, the effectiveness of the proposed method is shown through simulations at multiple intersections using a micro traffic simulator.

2 STOCHASTIC MODEL FOR TRAFFIC JAM

2.1 Prediction of probabilistic distribution for standing vehicles at intersection

In order to predict the traffic jam probability, a Bayesian Network (BN) model is built to predict probabilistic distributions of standing vehicles at crossroad according to the relationship between traffic flows and standing vehicles.

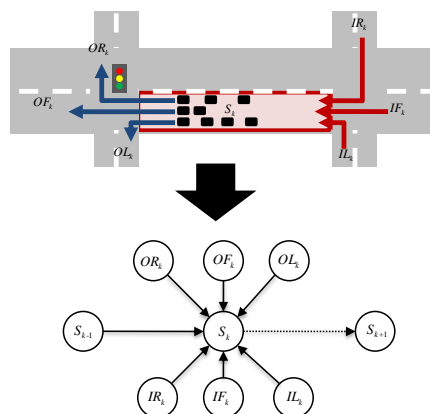


Fig. 1. Crossroad and BN model

Here, we consider a crossroads as shown in Fig.1. The random variables of the inflows and the outflows of the crossroad and the standing vehicles between the two intersections are represented as nodes. The Bayesian network model of the standing vehicles is shown in Fig.1. The number of the standing vehicles of k-th cycle can be calculated as equation (1). The probabilistic distribution of the standing vehicles is obtained by summing over all joint probability distribution of the other variables.

$$S_k = S_{k-1} + IR_k + IF_k + IL_k - OR_k - OF_k - OL_k \quad (1)$$

$$P(S_k) = \sum_{S_{k-1}} \sum_{IF_k} \sum_{IL_k} \sum_{IR_k} \sum_{OF_k} \sum_{OL_k} \sum_{OR_k} P(S_k, S_{k-1}, IF_k, IL_k, IR_k, OF_k, OL_k, OR_k) \quad (2)$$

$$P(S_k, S_{k-1}, IF_k, IL_k, IR_k, OF_k, OL_k, OR_k) = P(S_k | S_{k-1}, IF_k, IL_k, IR_k, OF_k, OL_k, OR_k) \times P(S_{k-1} | IF_k, IL_k, IR_k, OF_k, OL_k, OR_k) \times P(IF_k | IL_k, IR_k, OF_k, OL_k, OR_k) \times P(IL_k | IR_k, OF_k, OL_k, OR_k) \times P(IR_k | OF_k, OL_k, OR_k) \times P(OF_k | OL_k, OR_k) \times P(OL_k | OR_k) \times P(OR_k) \quad (3)$$

With the chain rule, the joint probabilistic distribution is represented as the product of conditional probability as equation (3). Then, according to the d-separation and the relationship between traffic flows and standing vehicles the equation (2) can be represented as

$$P(S_k) = \sum_{S_{k-1}} \sum_{IF_k} \sum_{IL_k} \sum_{IR_k} \sum_{OF_k} \sum_{OL_k} \sum_{OR_k} P(S_k) P(S_{k-1}) P(IF_k) \dots, P(IL_k) P(IR_k) P(OF_k) P(OL_k) P(OR_k) \quad (4)$$

In order to calculate the probabilistic distribution of the standing vehicle at intersection road, the prior probabilities of traffic inflow and outflows must be updated. Generally, the priori probability of the traffic flows is variable under the different traffic signal. Hence, we use the CA traffic model to estimate the traffic flow of the different traffic signals. And then, according to the estimated traffic flows the priori probabilistic distributions will be updated. The updating process is as following:

Step1: Using the CA traffic model to estimate the traffic inflow I_k^e and outflow O_k^e of the different traffic signal of the k-th cycle.

Step2: Using the observed traffic data of before the k-th cycle to calculate the probabilistic distribution of inflow $\sum_i P(I_{k-1} = i)$ and outflow $\sum_i P(O_{k-1} = i)$.

Step3: Using the estimated traffic flow data and probabilistic distribution to calculate the prior probabilities of traffic inflow and outflows according to the equations (5) – (8).

$$\hat{P}(I_k^e) = P(\hat{I}_{k-1}^e) - \frac{\gamma}{E-1} \quad (\hat{I}_{k-1}^e \neq \hat{I}_k^e) \quad (5)$$

$$\hat{P}(I_k^e) = P(\hat{I}_{k-1}^e) + \frac{\gamma \cdot n}{E-1} \quad (\hat{I}_{k-1}^e = \hat{I}_k^e) \quad (6)$$

$$\hat{P}(O_k^e) = P(\hat{O}_{k-1}^e) - \frac{\gamma}{E-1} \quad (\hat{O}_{k-1}^e \neq \hat{O}_k^e) \quad (7)$$

$$\hat{P}(O_k^e) = P(\hat{O}_{k-1}^e) + \frac{\gamma \cdot n}{E-1} \quad (\hat{O}_{k-1}^e = \hat{O}_k^e) \quad (8)$$

$$e \in (0,1,2,\dots,E) \quad \gamma = 5\%$$

Here, the CA traffic model is built based on the SchCh model (highway traffic model). The rules of movements for the vehicles on road network are shown in Fig.3 as an example. A vehicle can be accelerated up to maximum speed (Maximum speed = move 2 cell; 1cell=7.5m/1 step [1sec]) when there is no obstacle. According to the conditions of the road, the speed can be changed randomly. In multi lanes road, a vehicle can move to parallel lanes. If the direction of travel is right turn, the vehicle moves to the right turn exclusive lane.

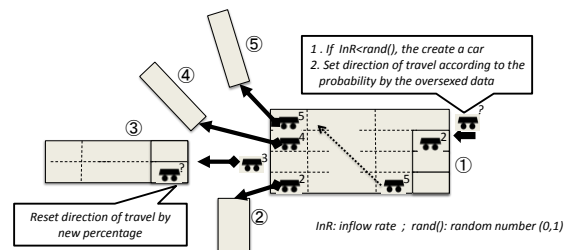


Fig. 2. CA traffic model

2.2 Traffic jam probability optimization problem

The objective of proposed method is to search an optimal traffic signal to minimize the traffic jam probabilities. Here, we formularized the traffic jam probability optimization problem as equation (9) according to the probabilistic distributions of the standing vehicles at crossroad intersection.

$$F(t_k^1, t_k^2, \dots, t_k^i) = a \cdot \sum_{S_i=S_{\min}}^{S_i=+\infty} \hat{P}_{\text{main}}(S_k) + \beta \cdot \sum_{S_i=S_{\min}}^{S_i=+\infty} \hat{P}_{\text{minor}}(S_k) \quad (9)$$

Subject to

$$t_{\min}^n \leq t_k^n \leq t_{\max}^n \quad i = 1, 2, \dots, n$$

Here, t_k^n is the time length of signal phase n , and t_{\min}^n and t_{\max}^n indicates the adjustable range of the signal phase n , $\hat{P}_{\text{main}}(S_k)$ is the probabilistic distributions of the main road $\hat{P}_{\text{minor}}(S_k)$ is the probabilistic distributions of the minor road, a and β are weight value.

3 H-GA-PSO ALGORITHM AND REAL-TIME CONTROL

In order to achieve the real-time control, an H-GA-PSO algorithm is proposed based on the Hierarchical-PSO. In H-GA-PSO a particle is influenced by its own so far best position and by the best position of the particle that is directly above it in the hierarchy. All particles are arranged in a tree that forms the hierarchy so that each node of the tree contains exactly one particle. Fig.3 shows the structure of H-GA-PSO.

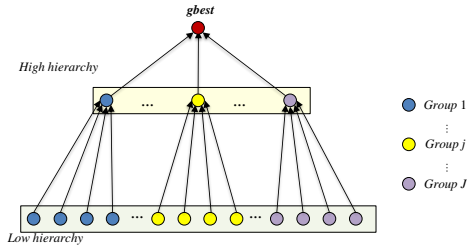


Fig. 3. Structure of H-GA-PSO algorithm

$$v_{ij}^{itr+1} = w_i \cdot v_{ij}^{itr} + c_{11} \cdot rand() \cdot (pbest_{ij}^{itr} - x_{ij}^{itr}) + c_{12} \cdot rand() \cdot (lbest_i^{itr} - x_{ij}^{itr}) + c_{13} \cdot rand() \cdot (gbest^{itr} - x_{ij}^{itr}) \quad (10)$$

$$x_{ij}^{itr+1} = x_{ij}^{itr} + v_{ij}^{itr+1} \quad (11)$$

$$v_{ij}^{itr+1} = w_h \cdot v_{ij}^{itr} + c_{h1} \cdot rand() \cdot (pbest_{ij}^{itr} - x_{ij}^{itr}) + c_{h2} \cdot rand() \cdot (gbest^{itr} - x_{ij}^{itr}) \quad (12)$$

$$x_{ij}^{itr+1} = x_{ij}^{itr} + v_{ij}^{itr+1} \quad (13)$$

In the H-GA-PSO, the particles in low hierarchy are updated using equations (10) and (11), and the particles in high hierarchy are updated using equations (12) and (13). In equation (10), $lbest$ is the position with the best fitness found so far by the corresponding group.

Also, Genetic Algorithm (GA) and a rule of particle's initialization are used to avoid the local minimum problem. The GA is used to update a selected particle's position and basic concepts of initialization for particles are shown in Fig.4. The crossover processing is used to exchange two particles from different group. The mutation processing is used to reset a particle around $gbest$ or $lbest$. The initial particles can keep a certain distance by using the rules.

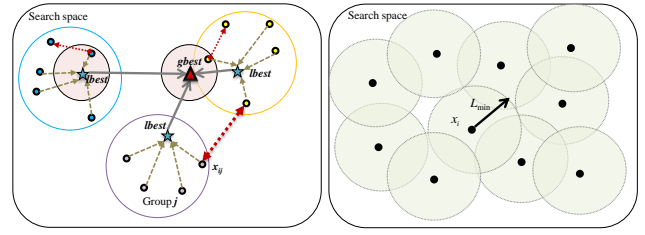


Fig. 4. Structure of H-GA-PSO

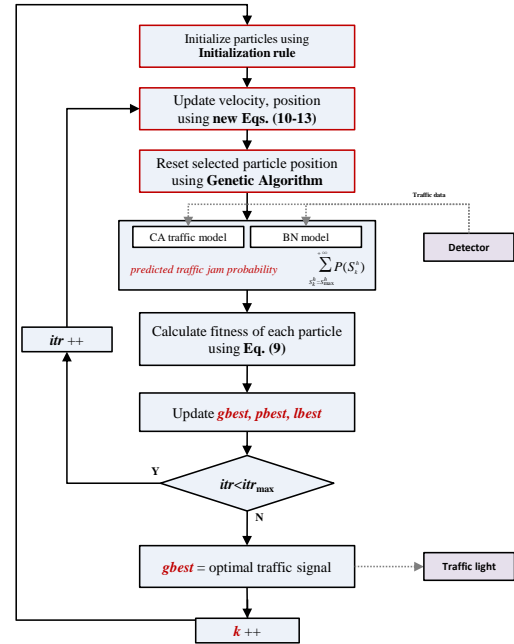


Fig. 5. GA operation and initialization of particles

The H-GA-PSO algorithm is used to search a optimal traffic signal with traffic jam probability optimal model. The procedure is shown in Fig.5.

4 SIMULATION

To prove the effectiveness of the proposed method, a simulation was carried out with a micro-traffic simulator. The micro-traffic simulator is developed based on the CA traffic model. We assume a road network and detector systems.

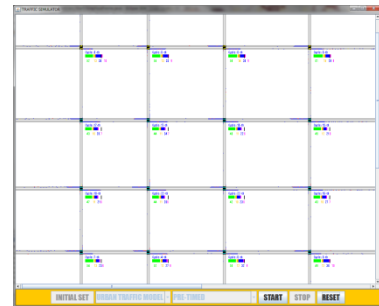


Fig. 6. Frame of micro simulator

The road network of the experiment includes 16 intersections, and all traffic data of the intersections can be observed by the detector systems. The simulator frame is shown in Fig.6. The conditions of the traffic flows and road networks are shown in table 1. Each pre-timed signal of the intersections are calculated according to the Webster's formula.

Table 1. Conditions of traffic flows

Direction	Road length	Lane	direction of ravel			No.	Inflows (pcu/h)
			straight	right	left		
West to East	80 [cell]	3	90%	4%	6%	1	1300
						2	1100
						3	1400
						4	1500
						5	1400
East to West	80 [cell]	3	90%	4%	6%	6	1300
						7	1000
						8	900
						9	600
						10	400
South to North	70 [cell]	2	60%	10%	30%	11	500
						12	500
						13	400
North to South	70 [cell]	2	60%	20%	20%	14	500
						15	500
						16	600

* pcu : passenger car unit ; 1 cell=7.5m

The traffic signals of the intersection 1 suggested by the proposed method are shown in Fig.6. Compared with pre-time signals, the signals by the proposed method are updated according to the changing traffic flows.

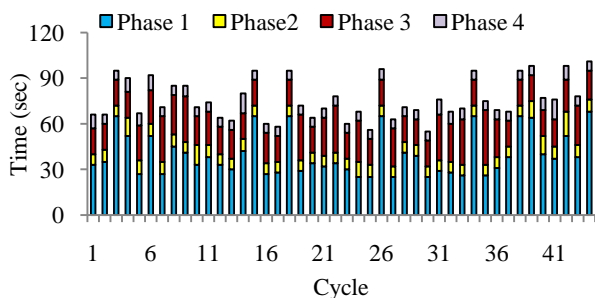


Fig. 7. Optimal Traffic signals of intersection1

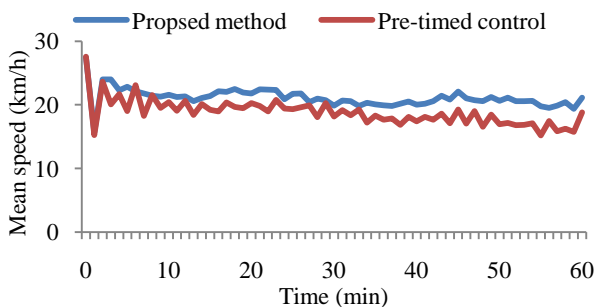


Fig. 8. Comparison of mean speed

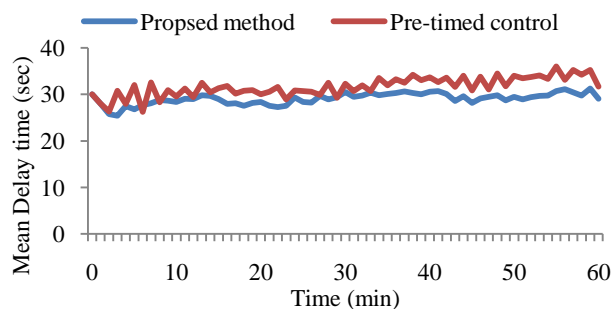


Fig. 9. Comparison of mean delay time

By the proposed the total delay time is decreased about 18.3% compared with pre-timed control, and the result is shown in Fig.9. And, the comparison of mean speed of proposed method and pre-timed control is shown Fig.8. By the proposed method, the mean speed is increased.

5 CONCLUSION

A real-time traffic signals stochastic optimal control was proposed based on the optimization model of traffic-jam probability and the H-GA-PSO algorithm to calculate the optimal signals. Through a simulation with the micro simulator the effectiveness of the proposed method was shown.

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