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Sliding mode control of a quad rotor helicopter using nonlinear sliding surface

Bambang Sumantri[1,2], Naoki Uchiyama[1], Shigenori Sano[1], and Yuma Kawabata[1]

[1]Toyohashi University of Technology, Japan
[2]Electronic Engineering Polytechnic Institute of Surabaya, Indonesia
[1]bambang@eepis-its.edu

Abstract: In this paper, a sliding mode controller (SMC) based on a nonlinear sliding surface (NSS) is designed for controlling a quad rotor helicopter (quadrocopter). It is a well-known that a low overshoot can be achieved with a cost of longer settling time, although a shorter settling time is needed for quick response in a quadrocopter system. In the conventional SMC, the sliding surface is designed as a linear surface that provides a constant damping ratio. The value of damping ratio should be adjusted in order to obtain an optimal performance by making a tradeoff between the two criteria; overshoot and settling time. In this paper, an NSS is designed so that the damping ratio of the control system can be varied from its initial low value to a final high value in a finite time. A low value of damping ratio will cause a quick response, and the later high damping ratio will avoid overshoot, and therefore the control performance can be optimized. First, a dynamics model of a quadrocopter is presented. Next, an SMC with an NSS is designed for tracking control of a quadrocopter. The stability of the proposed control system is proved based on the Lyapunov stability theory. The effectiveness of the proposed design is verified by simulation in which comparative results with the conventional linear sliding surface (LSS) is shown. The NSS is more effective compared to the conventional LSS when the disturbances exist.

Keywords: quad rotor helicopter, sliding mode control, nonlinear sliding surface, tracking control.

1 INTRODUCTION

Quadrocopter is a class of vertical takeoff landing unmanned aerial vehicle that uses two pairs of contra-rotating rotors to provide lift and directional motion. Quadrocopter has some advantages compared to conventional helicopter due to its compactness and simple mechanical structure with high maneuverability. The maneuver is provided by just varying the motors' speed. In order to make the quadrocopter fully autonomous and robustly stable to disturbances are still challenging problems because of its high nonlinearity.

Nonlinear control strategies of a quadrocopter have been concerned by many researchers. Mokhtari *et al.* designed a feedback linearization controller combined with an observer to reduce sensors [1]. They applied feedback linearization to control a partial dynamic system based on rotational motion and an observer to construct translational motion information. Guisser *et al.* proposed an input-output feedback linearization algorithm for stabilizing a quadrocopter and tracking a given trajectory in discrete time [2]. Mian and W. Daobo employed feedback linearization coupled with a PD controller to control translation motion and a backstepping-based PID nonlinear controller for rotational motion [3].

Saturation nonlinearity is also concerned by some researchers. Kendoul *et al.* and Castilo *et al.* proposed a nested saturation strategy for stabilizing a quadrocopter [4][5]. A nested saturation approach was designed based on a priori input bounds of a quadrocopter [6]. This control strategy was designed for stabilizing the attitude of a quadrocopter. Cruz *et al.* used a nested saturation strategy for stabilizing the quadrocopter in a given trajectory [7].

In a real application, a control system of a quadrocopter must be robust to uncertain disturbance such as wind gusting. The SMC is well known as a robust control strategy, and has been applied to a quadrocopter. Yokoyama *et al.* designed a velocity tracking control by using an SMC together with a backstepping method [8]. The robustness of the controller was proved by simulation under a gust of wind. Problems of unmatched disturbance and chattering have been concerned in developing an SMC. Vega *et al.* designed a robust SMC by designing a disturbance estimator [9]. They used a first-order differentiator called a super-twisting algorithm for estimating aerodynamic forces.

As a high speed dynamic system in an uncertain environment, a good control strategy of quadrocopter should be designed to settle its body quickly without any overshoot or oscillation under a disturbance. It is well known that a low overshoot can be achieved with a cost of longer settling time, although a shorter settling time is needed for quick response. Hence, the value of damping ratio should be adjusted in order to obtain an optimal performance by making a tradeoff between the two criteria; overshoot and settling time. In this paper, an NSS is designed so that the damping ratio will be changed from its initial low value to a final high value at a finite time. This NSS is designed based on the algorithm presented in [10]. An SMC with an NSS is employed for stable tracking control of a quadrocopter. The performance and robustness of the proposed controller is proved by simulation. A comparison with a conventional SMC is presented.

2 MODELING OF QUADCOPTER

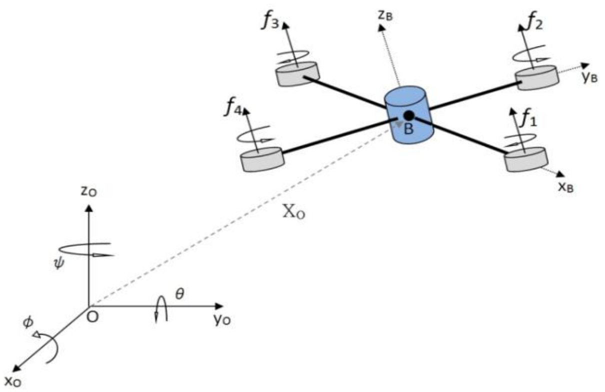

Fig. 1. Coordinate frame of quadrocopter

The position of a quadrocopter, $X_0 = [x, y, z]^T$, is a coordinate position of the centre of gravity of the quadrocopter ("B") with respect to the earth frame ("O") as shown in **Fig. 1**. Its attitude is represented by three angles, $\Theta = [\phi, \theta, \psi]^T$, which are roll, pitch and yaw, respectively. The coordinate position and attitude of the quadrocopter are described in the earth frame, although they are measured in the body frame by using sensors attached to the body of the quadrocopter. Denoting the linear and angular velocities in the earth frame by $\dot{X}_0 \in \mathbb{R}^3$ and $\dot{\Theta} \in \mathbb{R}^3$, respectively, and those in the body frame by $V \in \mathbb{R}^3$ and $\omega \in \mathbb{R}^3$, respectively, we have the following relation:

$$\dot{X}_0 = RV \quad (1)$$
$$T\dot{\Theta} = \omega \quad (2)$$

$$R = \begin{bmatrix} -s\phi s\theta s\psi + c\theta c\psi & -c\phi s\psi & s\phi c\theta s\psi + s\theta c\psi \\ s\phi s\theta c\psi + c\theta s\psi & c\phi c\psi & -s\phi c\theta c\psi + s\theta s\psi \\ -c\phi s\theta & s\phi & c\phi c\theta \end{bmatrix};$$
$$T = \begin{bmatrix} c\theta & 0 & -c\phi s\theta \\ 0 & 1 & s\phi \\ s\theta & 0 & c\phi c\theta \end{bmatrix}$$

where s and c denote sinus and cosines, respectively.

The dynamics of the quadrocopter can be derived by the Newton's second law as follows:

$$\boldsymbol{m}\ddot{X}_0 = \sum F_{ext} \quad (3)$$
$$I\dot{\omega} = -\omega \times I\omega + \sum T_{ext} \quad (4)$$

where $\boldsymbol{m} \in \mathbb{R}^{3x3}$; $I \in \mathbb{R}^{3x3}$; $\sum F_{ext} \in \mathbb{R}^3$ and $\sum T_{ext} \in \mathbb{R}^3$ are the mass matrix, the inertia matrix, the vector of total external forces, and the vector of total external torques, respectively. Eq. (3) can be written as:

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} s\phi c\theta s\psi + s\theta c\psi \\ -s\phi c\theta c\psi + s\theta s\psi \\ c\phi c\theta \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} \quad (5)$$

where m, g and $u_1 = f_1 + f_2 + f_3 + f_4$ are the total mass of quadrocopter, the gravitational acceleration and the resulting thrust of the four rotors, f_1, f_2, f_3 and f_4 , respectively. Eq. (4) is the rotational dynamics of the quadrocopter. Substituting Eq. (2) into Eq. (4), we have

$$I(\dot{T}\dot{\Theta} + T\ddot{\Theta}) = -T\dot{\Theta} \times IT\dot{\Theta} + \sum T_{ext} \quad (6)$$

Eq. (6) leads to

$$J\,[\ddot{\phi}, \ddot{\theta}, \ddot{\psi}]^T = [K_1, K_2, K_3]^T + [u_2, u_3, u_4]^T \quad (7)$$

$$J = \begin{bmatrix} I_x c\theta & 0 & -I_x c\phi s\theta \\ 0 & I_y & I_y s\phi \\ I_z s\theta & 0 & I_z c\phi c\theta \end{bmatrix};$$
$$K_1 = (I_x + I_y - I_z)\dot{\phi}\dot{\theta}s\theta + (-I_x + I_y - I_z)\dot{\phi}\dot{\psi}s\phi s\theta + (I_x + I_y - I_z)\dot{\theta}\dot{\psi}c\phi c\theta + (I_y - I_z)\dot{\psi}^2 s\phi c\phi c\theta;$$
$$K_2 = (-I_y + (I_z - I_x)c2\theta)\dot{\phi}\dot{\psi}c\phi + (I_z - I_x)(\dot{\phi}^2 - \dot{\psi}^2 c^2\phi)s\theta c\theta;$$
$$K_3 = (-I_z + I_x - I_y)\dot{\phi}\dot{\theta}c\theta + (I_z + I_x - I_y)\dot{\phi}\dot{\psi}s\phi c\theta + (I_z - I_x + I_y)\dot{\theta}\dot{\psi}c\phi s\theta - (I_x - I_y)\dot{\psi}^2 s\phi c\phi s\theta;$$

where I_x, I_y and I_z are the moment of inertia of the quadrocopter about x, y and z axes, and $u_2 = (f_4 - f_2)L$, $u_3 = (f_1 - f_3)L$ and $u_4 = (-f_1 + f_2 - f_3 + f_4)D$ are torques resulting from the rotors that cause roll, pitch and yaw motion, respectively. L is the distance of rotor to the centre of gravity, D is the anti-torque coefficient. From Eqs. (5) and (7), the dynamics model is written as

$$\begin{bmatrix} \boldsymbol{m} & 0_{3x3} \\ 0_{3x3} & J \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ m\,g \\ -K_1 \\ -K_2 \\ -K_3 \end{bmatrix} = \begin{bmatrix} (s\phi c\theta s\psi + s\theta c\psi)u_1 \\ (-s\phi c\theta c\psi + s\theta s\psi)u_1 \\ c\phi c\theta u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \quad (8)$$

For notational simplification, Eq. (8) is written as

$$M\ddot{X} + C = U \quad (9)$$

3 NONLINEAR SLIDING MODE CONTROL

To design an NSS, the original dynamics model in Eq. (9) is transformed into a regular form as follows:

$$\dot{z}_1(t) = A_{11}z_1(t) + A_{12}z_2(t)$$
$$\dot{z}_2(t) = A_{21}z_1(t) + A_{22}z_2(t) + B_2u(t) + \rho_d \quad (10)$$

where $Z = [z_1, z_2]^T = T_rX$, $z_1 \in \mathbb{R}^{n-l}$, $z_2 \in \mathbb{R}^l$, T_r is a matrix transforming X into Z, $B_2 \in \mathbb{R}^{lxl}$, l is the number of the inputs, and $\rho_d \in \mathbb{R}^l$ is an estimated disturbance vector. By introducing the disturbance term, Eq. (9) leads to

$$z_1 = [x \quad y \quad z \quad \phi \quad \theta \quad \psi]^T$$
$$z_2 = [\dot{x} \quad \dot{y} \quad \dot{z} \quad \dot{\phi} \quad \dot{\theta} \quad \dot{\psi}]^T \quad (11)$$

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = u(t) + \rho_d \quad (12)$$

where $u(t) = M^{-1}(-C + U)$ as a synthetic input. With regard to the matrices in Eq. (10), $A_{11} = A_{21} = A_{22} \in 0^{6x6}$ are null matrices, and $A_{12} = B_2 = I^{6x6}$ are the identity matrices.

3.1. Nonlinear sliding surface design

The sliding surface is designed as a nonlinear function.

$$s = [F - \Psi P \quad I][e_1 \quad e_2]^T \quad (13)$$

where e_1 and e_2 are tracking error vectors of z_1 and z_2 to desired trajectories z_{1d} and z_{2d}. The matrix $\Psi = \text{diag}(\Psi_i)$, $i = 1,2,..,l$, consists of nonlinear functions as follows:

$$\Psi_i = -\beta_i \frac{\exp(k\|e_i\|)-1}{\exp(\|e_i\|)}, \ e_i \in e_1 \quad (14)$$

where $\beta_i > 0$ and $k > 0$. Next, the control input that considers the system stability will be designed based on the Lyapunov stability theory.

3.2 Stability analysis

Let us define a candidate of the Lyapunov function as

$$V = 0.5s^Ts \quad (15)$$

Differentiating Eq. (15) and considering Eq. (13), we have

$$\dot{V} = s^T\left(-\dot{\Psi}Pe_1 + (F - \Psi P)\dot{e}_1 + \dot{e}_2\right) \quad (16)$$

The tracking errors are

$$e_1 = z_1 - z_{1d}$$
$$e_2 = z_2 - z_{2d} \quad (17)$$

By differentiating Eq. (17) and substituting it into Eq. (16) together with dynamics in Eq. (12), the first derivative of the Lyapunov function can be written as

$$\dot{V} = s^T\left(-\dot{\Psi}Pe_1 + (F - \Psi P)\dot{e}_1 + u(t) + \rho_d - \dot{z}_{2d}\right) \quad (18)$$

Designing the control input $u(t)$ as

$$u(t) = -(F - \Psi P)\dot{e}_1 - Ks - Q\text{sign}(s) + \dot{\Psi}Pe_1 + \dot{z}_{2d} \quad (19)$$

where $Q \in \mathbb{R}^{6x6}$ is a positive definite matrix, and substituting Eq. (19) into Eq. (18), we have

$$\dot{V} = s^T(-K\,s - Q\ \text{sign}(s) + \rho_d) \quad (20)$$

If the minimum eigenvalue of Q is greater than the norm of ρ_d and $K \in \mathbb{R}^{6x6}$ is positive definite, then $\dot{V} < 0$, and hence the control input in Eq. (19) will stabilize the system in Eq. (12).

By substituting the control input in Eq. (19) into Eq. (12) and considering ρ_d as the unknown external disturbance, the closed loop dynamics of the system is

$$\ddot{e}_1 + (F - \Psi P)\dot{e}_1 + Ks + Q\text{sign}(s) - \dot{\Psi}Pe_1 + \rho_d = 0 \quad (21)$$

Because $F \in \mathbb{R}^{6x6}$ and $P \in \mathbb{R}^{6x6}$ are positive definite, the damping ratio of the closed loop dynamics will change according to the magnitude of e_1. If the error is relatively large, then the damping ratio will be reduced, and therefore the system will give faster response toward sliding surface. If the system is close to the sliding surface then the error will be reduced, and the damping ratio will be increased, and therefore the response will be slower and overshoot can be reduced. Next, the effectiveness of the SMC using the NSS in Eq. (19) will be verified by simulation and compared with a conventional linear sliding surface.

4 SIMULATION RESULTS

The parameters used in this simulation as follows are those for the experimental system in our laboratory.

$m = 0.2039$Kg; $I_x = I_y = 5.136 \times 10^{-3}$ Kg.m^2;
$I_z = 1.016 \times 10^{-2}$ Kg.m^2; $L = 0.212$m; $g = 9.807$m.s^{-2}.

The desired trajectories for $[x, y, z, \psi]^T$ are given as $[x_d, y_d, z_d, \psi_d]^T$, and from Eq. (5), desired angles $[\phi_d, \theta_d]^T$ for $[\phi, \theta]^T$ can be calculated as follows:

$$\phi_d = \text{atan}\left(\frac{\ddot{x}_d \sin(\psi_d) - \ddot{y}_d \cos(\psi_d)}{\ddot{z}_d + g}\right)$$

$$\theta_d = \text{atan}\left(\frac{\ddot{x}_d \cos(\psi_d) + \ddot{y}_d \sin(\psi_d)}{\sqrt{\left(\ddot{x}_d \sin(\psi_d) - \ddot{y}_d \cos(\psi_d)\right)^2 + (\ddot{z}_d + g)^2}}\right)$$

The control parameters are as follows:

$F = \text{diag}\{6,6,50,18,18,10\}$; $P = \text{diag}\{10,10,10,10,10,10\}$;

$K = \text{diag}\{1.2, 1.2, 40, 10, 10, 3\}$; $\beta_i = [20, 20, 20, 30, 30, 20]^T$;
$Q = \text{diag}\{0.01, 0.01, 0.01, 0.01, 0.01, 0.01\}$; $k = 4$.

Setting β_i to zero of the control strategy in Eq. (19) leads to a conventional LSS. Both the NSS and the LSS were applied to the tracking control to the same desired trajectory. The result is shown in **Fig. 2**, in which both the NSS and the LSS provides the similar performance.

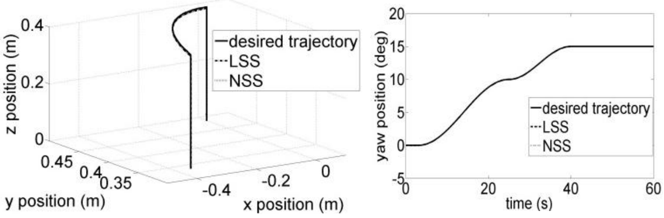

Fig. 2 Helicopter trajectory in NSS and LSS.

The comparison results after applying the step disturbance to the control inputs $[u_1; u_2; u_3; u_4]$ whose values are $[0.5; 0.05; -0.05; 0.005]$, respectively, are shown in **Fig. 3**.

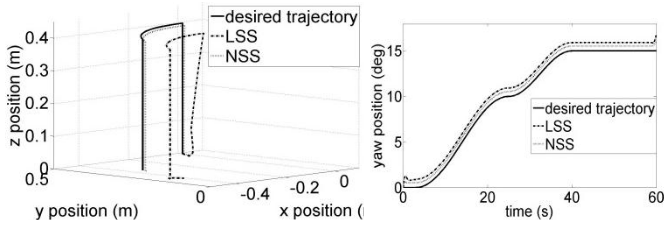

Fig. 3 Helicopter trajectory in NSS and LSS under disturbance

The NSS can track the trajectory with smaller error compared to the LSS. The nonlinear function changes the controller gain according to the error magnitude, and provides the robust performance. The control inputs in **Fig. 3** are shown in **Fig. 4**. The control input magnitudes in the NSS and the LSS show no significant difference.

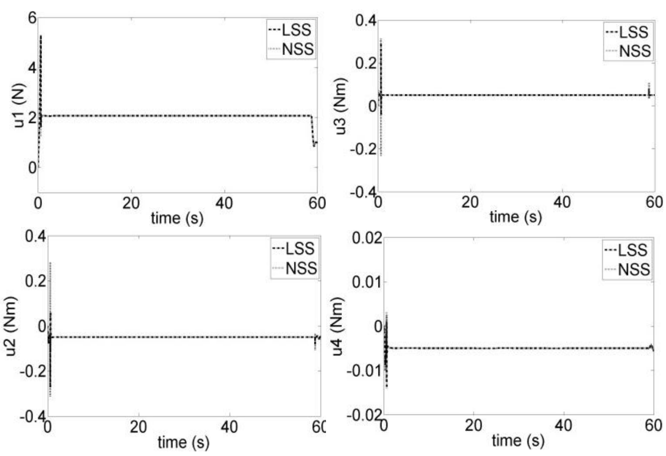

Fig. 4 Control input in NSS and LSS under disturbance.

5 CONCLUSION

A sliding mode controller with a nonlinear sliding surface is designed for robust tracking control of a quadrocopter. The control system stability is proved by the Lyapunov stability theory. The effectiveness of the NSS is proved by simulation. Future work verifies the effectiveness of the NSS by experiment in which the practical disturbance and signal noise exist.

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