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Non-uniform Cellular Automata based on Open-ended Rule Evolution

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Abstract: Cellular automata (CAs) are mathematical models of spatially and temporally discrete mathematical systems. Non-uniform CAs are the cellular automata in which each cell may contain a different transition rule and change it with time, while all cells share the same transition rule in regular CAs. Little is still known about the dynamics of open-ended evolution of rules in non-uniform CAs. The purpose of our study is to construct and investigate a model of non-uniform CAs capable of open-ended rule evolution exhibiting a wide variety of behavior across all Wolfram's classes. For this purpose, we construct 1-dimensional 2-state 3-neighborhood non-uniform CAs with evolving transition rules. In the model, we found an interesting dynamics that Class II (periodical behavior) and III (chaotic behavior) patterns emerged alternately, between which Class IV patterns sometimes emerged.

Keywords: Complexity, Elementary cellular automata, Non-uniform cellular automata, Open-ended, Rule evolution

1 INTRODUCTION

Cellular automata (CAs) are mathematical models of spatially and temporally discrete mathematical systems. They consist of a large number of relatively simple units ("cells"). Each cell is a simple finite automaton that repeatedly updates its own state depending on the cell's current state and those of its immediate neighbors. They have been commonly used to describe and explore complex systems. One of the pioneering work is the classification of the CA behavior into four classes (Class I, II, III and IV) by Wolfram [1]. In a typical Class IV CA, initial patterns evolve into structures that interact in complex ways based on a mixture of order and randomness ("edge of chaos"). The connection between such a complexity and the (origin of) life have been frequently discussed in the artificial life community. Additionally, Langton [2] has been successful in assessing Wolfram's each classes quantitatively. He indicated that Class III show the highest entropy and Class IV show the highest mutual information. Moreover, Mori et al. [3] have proposed a rule-changing CA model based on internal dynamics. They indicated that this CA model may show higher mutual information than normal CA.

Non-uniform CAs are the cellular automata in which each cell may contain a different transition rule and change it with time, while all cells share the same transition rule in regular CAs. In the previous studies, non-uniform CAs often have been evolved to perform computational tasks including density classification and synchronization. Before then, Packard [4] and Mitchell [5] used a genetic algorithm to evolve CA rules to perform a specific task. Sipper [6] demonstrated that non-uniform CAs is partially superior to uniform CAs. However, little is still known about the dynamics of open-ended evolution of rules in non-uniform CAs.

The purpose of our study is to construct and investigate a model of non-uniform CAs capable of open-ended rule evolution exhibiting a wide variety of behavior across all Wolfram's classes. We also aim at applying it to visual artwork based on a never-ending autonomous flow of information. For this purpose, we construct 1-dimensional 2-state 3-neighborhood non-uniform CAs with evolving transition rules. Each cell has not only its state but also its transition rules. The transition rules of cells with high fitness that is calculated using entropy tend to propagate to neighboring cells. Specifically, we use the fitness of each transition rule defined as Shannon entropy for the state distribution of the 3-step history of 5-neighborhood cells. In addition, mutation may change the transition rule of each cell into another rule, which brings a novelty into the system.

2 MODEL

2.1 Algorithm

We use 1-dimensional 2-state 3-neighborhood cellular automata (CAs) and their states of the cells. Each cell also has a transition rule, represented as a binary sequence $(x_7x_6x_5x_4x_3x_2x_1x_0)$. This rule maps 3-neigborhood states to a decimal number $x_i \in (0, 1)$: [000] to x_0, [001] to x_1, $\cdots$, and [111] to x_7.

Each cell is updated by the the procdures as follows: (i) N cells are arranged on a 1-dimensional array. Each cell has a state that is set 0 or 1 arbitrarily, and a rule that is chosen arbitrarily from 1-dimensional 2-state 3-neighborhood CAs (elementary CA, ECA) rules. (ii) Each cell determines its next state on the basis of its rule and the states of 3-neighborhood cells including itself. (iii) Each cell calculates its own fitness

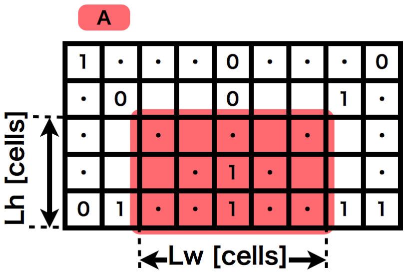

Fig. 1. The state distribution A.

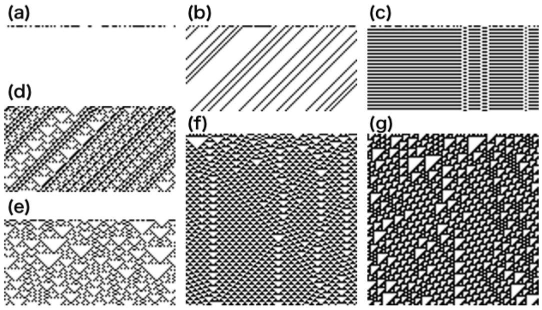

Fig. 2. The typical state patterns of the 7 classes in ECA.

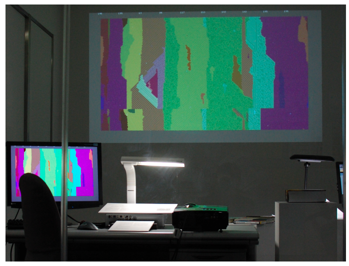

Fig. 3. The artwork.

(Section 2.2). (iv) Each cell replaces its own rule by the rule of the cell with the highest fitness in 3-neighborhood. (v) Each bit x_i in the rule representation $(x_7x_6x_5x_4x_3x_2x_1x_0)$ is flipped with a mutation rate μ as mutation. (vi) Update each cell's state and time step. (vii) Return to (ii) ((ii) - (vi) are repeated G times with (iii) and (iv) being skipped during the first Lh repetitions).

2.2 Fitness

The fitness of each cell is defined as Shannon entropy as follows. Here, K is set to 2 as the number of possible states (0 or 1), and p_i is the appearance ratio of each state in the spatial (Lw-neighborhood cells) and temporal (Lh-step history) domains (A in Fig. 1). Therefore, when the numbers of 0 and 1 in the domain are the same, it will be the highest and, in contrast, when the number of 0 or 1 is 0, it will be the lowest.

$$Fitness = H(A) = -\sum_{i=1}^{K} p_i \log p_i$$

3 EXPERIMENT

We investigate the rule evolution over the long time steps in Section 3.1 and focus on two characteristic patterns of rule evolution in Section 3.2. To do so, we classify the 256 ECA rules into seven groups (Class I, Class II-F, Class II-P, Class III-LC, Class III-C, Class IV-54 and Class IV-110). It covers four classifications by Wolfram [1] and five classifications by Li and Packard [7] as shown in Table 1.

We used the following parameter values: $N = 200$ (number of cells), $G = 2.0 \times 10^6$ (number of steps), $Lw = 5, Lh = 3$ (parameters for the fitness), $\mu = 1.0 \times 10^{-4}$ (mutation rate). All cells began with the state 0 and the rule 0.

3.1 Rule evolution over the long time steps

Fig. 4 shows the distribution of classes to which the existing rules belong (upper), and the number of the different existing rules and the average fitness (lower), across 2 million time steps. We see a general tendency that Class II-F (fixed point) and Class III-C (chaotic) dominate the population alternatively (Phase II and Phase III). There is also a tendency

Table 1. Classifications of ECA rules. Our classification is as follows (The rules inside the parenthesis are equivalent to the representative rule). Class I: 0 (255), 8 (64, 239, 253), 32, (251), 40 (96, 235, 249), 128 (254), 136 (192, 238, 252), 160 (250), 168 (224, 234, 248), Class II-F: The rest of ECA rules, Class II-P: 1 (127), 3, 5 (95), 6 (20, 159,215), 7 (21, 31, 87), 9 (65, 111, 125), 11 (47, 81, 117), 14 (84, 143, 213), 15 (85), 19 (55), 23, 25 (61, 67, 103), 27 (39, 53, 83), 28 (70, 157, 199), 29 (71), 33 (123), 35 (49, 59, 115), 37 (91), 38 (52, 155, 211), 41 (97, 107, 121), 43 (113), 50 (179), 51, 74 (88, 173, 229), 108 (201), 131 (62, 145, 118), 133 (94), 134 (148, 158, 214), 142 (212), 156 (198), 178, Class III-LC: 26 (82, 167, 181), 73 (109), 154 (166, 180, 210), Class III-C: 18 (183), 22 (151), 30 (86, 135, 149), 45 (75, 89, 101), 60 (102, 153, 195), 90 (165), 105, 106 (120, 169, 225), 129 (126), 146 (182), 150, 161 (122), Class IV-54: 54 (147), Class IV-110: 137 (110, 124, 193)

Fig. 2	Wolfram	Li&Packard	This study
(a)	Class I	null	Class I
(b)	Class II	fixed-point	Class II-F
(c)	Class II	periodic	Class II-P
(d)	Class III	locally chaotic	Class III-LC
(e)	Class III	chaotic	Class III-C
(f)	Class IV	chaotic	Class IV-54
(g)	Class IV	chaotic	Class IV-110

that in Phase II, the average fitness is stable and high, and the number of the different rules is high (10-30), and in contrast, in Phase III, the average fitness is rather unstable and lower, and the number of the different rules is lower (-10). Especially, typically in Phase III, the average fitness sometimes has a sudden decrease to around 0.8, that is accompanied by a sudden increase to around 40 in the number of the different rules.

Concerning the Class IV, which is the most interesting class, Class IV-54 sometimes dominates the population mainly in Phase II and Class IV-110 occasionally emerges a little (0-10 cells) almost instantaneously typically in Phase III. This tendency might due to the resemblance of the patterns generated by Class IV-54 to Phase II patterns or the resemblance of the patterns generated by Class IV-110 to Phase III patterns.

Next, we focus on the process which realize the transition from Phase II to Phase III, or vice versa.

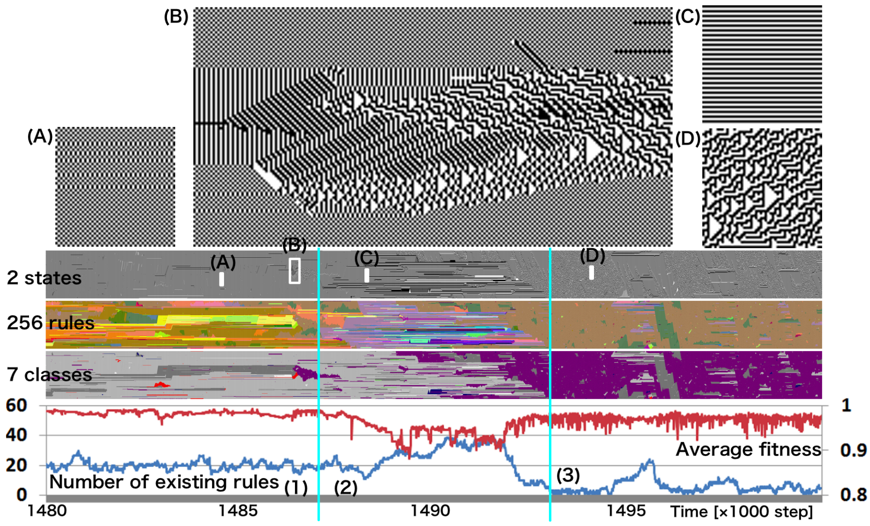

Fig. 5. The number of the existing rules and the average fitness (lower); the distribution of that the 7 classes, the 256 rules and the 2 states (middle); the images of the distinctive patterns (upper) from Phase II to Phase III.

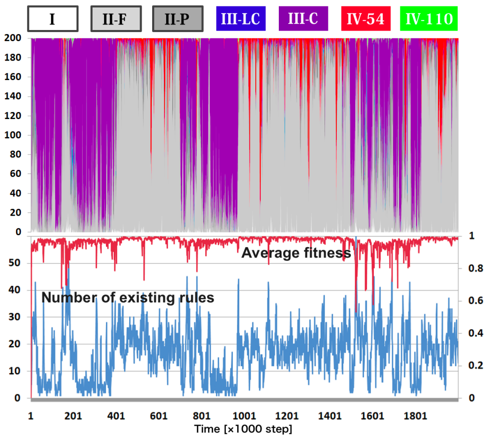

Fig. 4. The number of the existing rules and the average fitness (lower); the distribution of the 7 classes (upper) over 2 million time steps.

3.2 Rule evolution from Phase II (III) to Phase III (II)

Fig. 5 and Fig. 6 show the transitions of the number of the different existing rules, and the average fitness (lower), the state transition between 2 states, rule transition among 256 rules, and class transition among 7 classes (middle), and the enlarged parts of the distinctive patterns in the rule evolution (upper), across 20 thousand time steps, from Phase II to III, and Phase III to II, respectively.

We see from Fig. 5 that a transition from Phase II to III takes place through the three stages: Stage (1) as trigger creation, Stage (2) as rule diversification, and Stage (3) as strategy stabilization. The trigger in Stage (1) shifts the state distribution from Stage (1) to (2) introducing many various transition rules. In Stage (2), the average fitness of the cells is relatively low, and the number of existing transition rules is high. After that, Class III emerges and spreads gradually.

It is also shown from Fig. 6 that a transition from Phase III to II takes place through the three stages similar to the transition from Phase II to III: Stage (1) as trigger creation, Stage (2) as rule diversification, and Stage (3) as strategy stabilization. The trigger in Stage (1) shifts the state distribution from Stage (1) to (2) introducing many various transition rules. Specifically, (D) consists of Class II-F, III-C and IV-54, and (E) consists of Class II-F, II-P and IV-110. In Stage (2), the average fitness is also relatively low, and the number of existing transition rules is high. After that, Class II emerges and spreads gradually.

A closer observation of Fig. 5 reveals that there is a difference in dominant rules and state distribution between Stages (1) and (2), while Class II-F rules dominate the population throughout Stages (1) and (2). We see actually a clear difference between the enlarged patterns (A) and (C) that are the distinct patterns in Stage (1) and (2), respectively. The distinct pattern in Stage (3) is (D), which is a typical pattern of Class III-C behavior.

On the other hand, we see from Fig. 6 that while Class II-F dominates the population throughout Stages (2) and (3), there is also a difference in dominant rules and state distribution between Stages (2) and (3). Similarly, we see from the enlarged patterns that the distinct patterns in Stage (2) and (3) are (C) and (F), respectively, and that in Stage (1) is (A) which is a typical pattern of Class III-C behavior.

It is noticeable fact that the transitions from Phase II to III and from Phase III to II share some common dynamics. Accumulation of mutations suddenly but, in a sense, inevitably induces a state distribution in which several rules work together as a trigger to invoke Stage (2) in both transitions. In

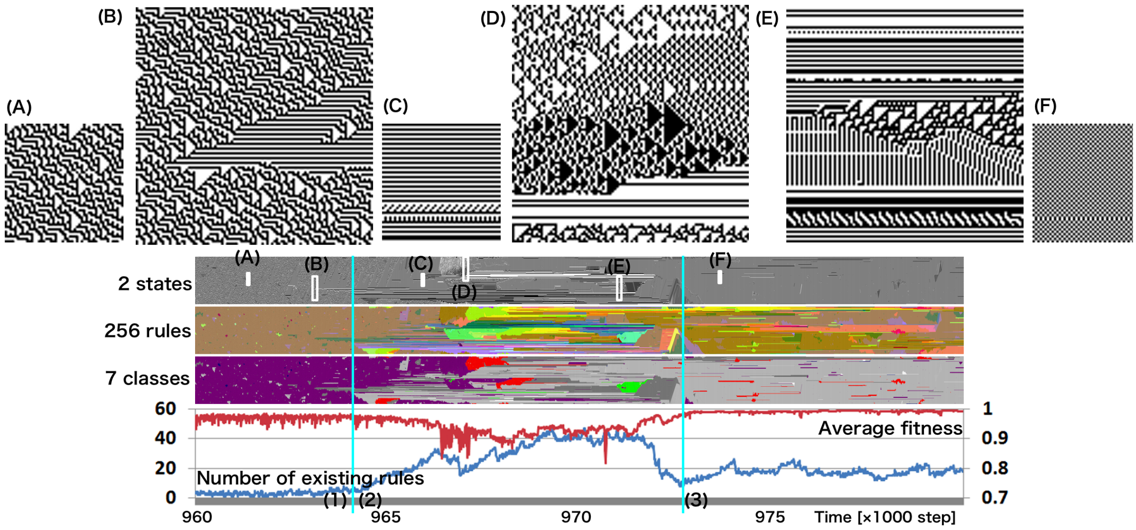

Fig. 6. The number of the existing rules and the average fitness (lower); the distribution of that the 7 classes, the 256 rules and the 2 states (middle); the images of the distinctive patterns (upper) from Phase III to Phase II.

this sense, (B) in both figures can be seen as a part of the triggers.

4 CONCLUSION

We examined the space-time patterns generated by the proposed non-uniform CAs. We found an interesting dynamics that Class II (periodical behavior) and III (chaotic behavior) patterns emerged alternately, between which Class IV patterns sometimes emerged. It was observed continuously over 2 million time steps and therefore, we regard this dynamics as underlying and inherent in the model.

More detailed analysis showed that during the alternation, the average fitness of the cells was relatively low and at the same time, the number of existing transition rules was high. Our interpretation of this is that some decrease in average fitness could start the alternation and this decrease induced various transition rules to emerge. We thus believe that decrease in average fitness and the resulting emergence of various transition rules are the key to the complex dynamics observed in the model.

The art exhibition "ALart_2012" was held at the Nagoya University Project Gallery "clas" in February 2012 featuring the artwork "One-Dimensional Cells". It was based on a never-ending autonomous flow of information created by the proposed model (Fig. 3). The artwork was also used to create the posters of the exhibition [8], which were put up in or around the campus including on the wall in the nearby subway station.

Future work includes investigating analyzing the level of open-ended rule evolution and the complexity of the global state distribution quantitatively.

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