The Eighteenth International Symposium on Artificial Life and Robotics 2013 (AROB 18th '13),
Daejeon Convention Center, Daejeon, Korea, January 30-February 1, 2013

The Effect of the Internal Parameters on Association Performance of A Chaotic Neural Network

Shun Watanabe[1], Takashi Kuremoto[1], Kunikazu Kobayashi[2], and Masanao Obayashi[1]

[1]Yamaguchi University, Japan
[2]Aichi Prefectural University, Japan
(Tel: +81-(0)836-85-9005, Fax: +81-(0)836-85-9016)
[1]{s005we, wu, m.obayas}@yamaguchi-u.ac.jp
[2]kobayashi@ist.aichi-pu-u.ac.jp

Abstract: The chaotic neural network (CNN) proposed by Aihara et al. is able recollect stored patterns dynamically. But there are difficult cases such as a long time processing of association, and difficult to recall a specific stored pattern during the dynamical associations. We have proposed to find the optimal parameters using meta-heuristics methods to improve association performance, for example, the shorter recalling time and higher recollection rates of stored patterns in our previous works. However, the relationship between the different values of parameters of chaotic neurons and the association performance of CNN was not investigated clearly. In this paper, we analyze how the change of values of internal parameters of chaotic neurons affects the characteristics of chaotic neurons when multiple patterns are stored in a CNN. Q-Q plot, least square approximation (LSM), hierarchical clustering (HC), and Hilbert transform (HT) are used to investigate the similarity of internal states of chaotic neurons, and to classify the neurons. Simulation results showed that the different values of the internal parameter yielded different behaviors of chaotic neurons and it suggests the optimal parameter which generates higher association performance may concern with the stored patterns of the CNN.

Keywords: chaotic neural network, Q-Q plot, least squares approximation, hierarchical clustering, Hilbert transform.

1 INTRODUCTION

The chaotic neural network (CNN) proposed by Aihara et al. is well-known as a recurrent neural network which is able to dynamically recollect stored patterns [1]. And CNN is applied to optimization, parallel distribution processing, robotics, and so on. In our previous works, we have proposed some meta-heuristic methods (e.g. genetic algorithm, particle swarm optimization) to determine the optimal parameters of chaotic neurons to realize higher performance of association memory of CNN [2] [3]. However, the causality between the determined parameters and the association ability was not investigated. In this paper, we intend to analyze how the varied internal states of chaotic neurons are yielded by the different parameters of neuron dynamics. The spatiotemporal change of the internal state of each chaotic neuron are observed, and the comparison of these time series data is given by kinds of methods such as Q-Q plot, least squares approximation (LSA), hierarchical clustering (HC), and Hilbert transform (HT). Q-Q plot, LSA, and HC is used to show the similarity between the dynamical changes of the internal states of chaotic neurons of CNN. HT is used to show the synchronization of neurons during association process. Associative simulation showed the change of the internal parameter affected the characteristics of neurons and the optimal parameter may concern with the stored patterns.

2 CHAOTIC NEURAL NETWORK

Aihara et al. 's chaotic neural network (CNN) is a kind of interconnected recurrent artificial neural network which neurons perform chaotic output. The dynamics of a chaotic neuron i of CNN is defined as follows:

$$\eta_i(t+1) = k_f \eta_i(t) + \sum_{i=0}^{N-1} w_{ij} x_j(t) \tag{1}$$

$$\zeta_i(t+1) = k_r \zeta_i(t) - \alpha x_i(t) + a \tag{2}$$

$$y_i(t+1) = \eta_i(t+1) + \zeta_i(t+1) \tag{3}$$

$$x_i(t+1) = \frac{1}{1+e^{-y_i(t+1)/\varepsilon}} \tag{4}$$

Where N is number of chaotic neurons in network, $\eta_i(t)$ and $\zeta_i(t)$ is internal state for the feedback inputs and refractoriness, k_f and k_r is the decay parameter for the feedback input and the refractoriness, w_{ij} is synaptic weights from jth neuron to ith neuron, α is refractory scaling parameter, a is threshold, $y_i(t)$ is internal state, $x_i(t)$ is chaotic neuron output, ε is steepness parameter. In this paper, the internal states of the chaotic neurons are limited to $y_i(t), i = 0,1,2,\ldots,N-1$ and the internal parameter k_r is investigated in detail.

Hebb learning rule is used to store patterns in CNN, that is, the synaptic weights between two arbitrary neurons is modified as following.

© ISAROB 2013

$$\Delta w_{ij} = \begin{cases} \beta \sum_{m=0}^{M-1} \chi_{mi} \chi_{mj} & if \quad i \neq j \\ 0 & otherwise \end{cases} \tag{5}$$

where $\beta = 1/M$ is learning coefficient, χ_{mi} is ith bipolar value of mth pattern, $m = 0,1,2,\ldots,M-1$.

3 FEATURE EXTRACTION

In this paper, Q-Q plot, least squares approximation (LSA), hierarchical clustering (HC), and Hilbert transform (HT) are used for analyzing the characteristics of the dynamical association in CNN. Firstly, Q-Q plot is used to compare the distribution types of each chaotic neuron internal sates in CNN. Secondly, the curve plotted by Q-Q plot is approximated to a function by LSA. Finally, the approximation errors and approximated parameters are clustered by HC, and the clustered data as the similarity between neurons is shown. Additionally, HT is used to observe the synchronization between internal states of chaotic neuron because the synchronization depends on time, is not able to be represented with the internal state distribution of each chaotic neuron.

3.1. Q-Q plot

Q-Q plot is a graphical method to represent the similarity and it's characteristics between the distributions of two data sets. When the elements of a data set are sorted in ascending order, are plotted on X-dimension in order, and the elements of another data set are plotted on Y-dimension either, then the correlation and the similarity of the two data sets are easily to be observed. If the plotted graph has linearity, it means that their data distributions are similarity. On other hand, if the graph has nonlinearity, it means that their data are not similarity. And the data those are equal to the other, are plotted on a line of the X-Y plane, i.e., a linear function that consists of one gradient coefficient and one bias. In this paper, similarities of all observed internal states of chaotic neurons in CNN are investigated using Q-Q plot.

3.2. Least squares approximation

Least squares approximation (LSA) is a method to find an approximate fitting function using samples of data set. In this paper, linear least square as a LSA method is used to quantitatively measure the plot graph (the sorted data set) in Q-Q plot mentioned in 3.1 section. A polynomial equation of the first degree (linear function) as an approximation function is used to measure the similarity (linearity) between the distributions of two data in Q-Q plot. The advantages of linear function are that squares error is less when the distributions of two data are similar. The similarity of the internal state distributions of neurons is evaluated by LSA here.

3.3. Hierarchical clustering

Hierarchical clustering (HC) is a clustering method that iteratively merges clusters in higher similarity. The method creates the clusters that have a tree-like structure. In this paper, the nearest neighbor method is used in HC and Euclidean distance as the degree of similarity of the internal state is used to classify the chaotic neurons. Shorter Euclidean distance represents higher similarity between two clusters. In this paper, the quantitative values calculated by LSA in Q-Q plot between each chaotic neuron and all are used in HC. So the similarity between the internal state distributions of each neuron are able to be represented in the tree-like structure of HC.

3.4. Hilbert transform

Hilbert transform (HT) is a transform technique to calculate the complex signal from an observed time series signal. In this paper, the complex signal is transformed from the internal state time series signal of each chaotic neuron in CNN. And the synchronization between neurons is observed with the complex signals. The method of HT is defined as follows:

$$y_i(t) \xrightarrow{FT} y_i(\omega) \tag{6}$$

$$y_i(\omega) = \mathrm{Re}[y_i(\omega)] + I\,\mathrm{Im}[y_i(\omega)] \tag{7}$$

$$y'_i(\omega) = \mathrm{Im}[y_i(\omega)] - I\,\mathrm{Re}[y_i(\omega)] \tag{8}$$

$$y'_i(\omega) \xrightarrow{FT^{-1}} y_{iH}(t) \tag{9}$$

where $y_i(t)$ is an observed signal, FT is Fourier transform, I is the imaginary unit, FT^{-1} is inverse Fourier transform, $y_{iH}(t)$ is the signal of HT.

The phase difference $\phi_i(t) - \phi_j(t)$ of two signals ($y_i(t), y_j(t)$) is calculated by the following equation:

$$\phi_i(t) - \phi_j(t) = \tan^{-1} \frac{y_{iH}(t) y_j(t) - y_i(t) y_{jH}(t)}{y_i(t) y_j(t) + y_{iH}(t) y_{jH}(t)} \tag{10}$$

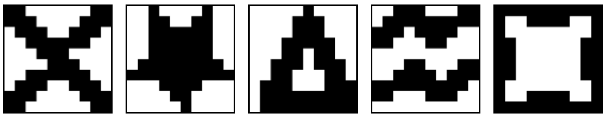
4 COMPUTATER SIMULATIONS

The features of the internal states in CNN are observed when the value of the internal parameter k_r of CNN is different ($k_r = 0.9, k_r = 0.8$). Other parameters of the associative model are set as $N = 100$ (10x10 network), $\zeta_i(0) = 0$, $\eta_i(0) = 0$, $k_f = 0.2$, $\alpha = 10$, $a = 2$, $\varepsilon = 0.015$, $\beta = 0.25$. The stored patterns in CNN are "cross", "star", "triangle", and "wave" as shown in Fig.1. "initial" in Fig.1 means the initial state of CNN in

© ISAROB 2013

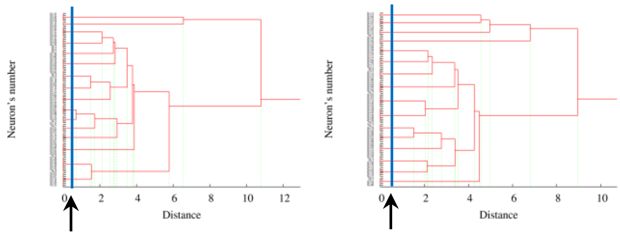
recollection process. The internal states of chaotic neurons in CNN were observed from $t = 2048$ to $t = 4095$ during a recalling.

The HC results of the case $k_r = 0.9$ (which is the value used in the original paper [1]) and the case $k_r = 0.8$ (which is an optional value used in this simulation) are shown in Fig.2 (a) and (b) respectively.



"cross" "star" "triangle" "wave" "initial"

Fig. 1. Stored patterns and initial pattern (width=10, height=10, black: $\chi_{mi} = -1$, white: $\chi_{mi} = +1$).



(a) $k_r = 0.9$ (b) $k_r = 0.8$

Fig. 2. The results of hierarchical clustering (HC).

In Fig. 2, 100 neurons are plotted on the vertical axis which orders are according to their clusters. When the distance between the clusters (horizontal axis) is the point with arrows in Fig.2 (a) and (b), the numbers of clusters are the same16, but neurons in those clusters are different when the parameter $k_r = 0.9$ and $k_r = 0.8$. Neurons in 16 clusters yielded by the different parameters are listed in Table 1 (a) and (b) respectively.

Table 1. The results of clustered neurons which internal state s were similar during recollection using HC.

(a) $k_r = 0.9$

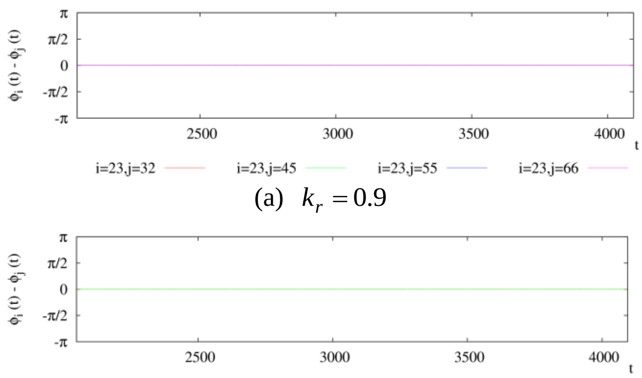
Cluster's No.	Neuron's No.
C0	23, 32, 45, 55, 66
C1	14, 15, 86
C2	33, 34, 37, 43, 44, 56
C3	3, 4, 20, 30, 31, 59, 70
C4	0, 1, 10, 28, 90
C5	16, 24, 25 53, 58, 68, 85
C6	8, 9, 11, 18, 19, 21, 80
C7	12, 17, 22, 27, 64, 65, 76
C8	46, 47, 52, 57, 84, 95
C9	82, 88, 89, 91, 98, 99
C10	2, 13, 69, 74, 75
C11	7, 42, 51, 60, 61
C12	71, 72, 77, 78, 81, 87
C13	5, 79, 83, 92, 93, 94, 96, 97
C14	6, 29, 38, 39, 40, 41, 48, 49, 50
C15	26, 35, 36, 54, 62, 63, 67, 73

(b) $k_r = 0.8$

Cluster's No.	Neuron's No.
C0	14, 15, 86
C1	71, 72, 77, 78, 81, 87
C2	7, 42, 51, 60, 61
C3	23, 32, 45, 55, 66
C4	46, 47, 52, 57, 84, 95
C5	12, 17, 22, 27, 64, 65, 76
C6	8, 9, 11, 18, 19, 21, 80
C7	5, 79, 83, 92, 93, 94, 96, 97
C8	26, 35, 36, 54, 62, 63, 67, 73
C9	6, 29, 38, 39, 40, 41, 48, 49, 50
C10	2, 13, 69, 74, 75
C11	82, 88, 89, 91, 98, 99
C12	16, 24, 25 53, 58, 68, 85
C13	3, 4, 20, 30, 31, 59, 70
C14	33, 34, 37, 43, 44, 56
C15	0, 1, 10, 28, 90

The phases between the neurons in the same cluster showed no any difference ($\phi_i(t) - \phi_j(t) = 0.0$). Fig. 3 shows some examples of the phase comparison using neuron No.23 in C0 via neuron No. 32, 45, 55, and 66 (The first raw in Table 1 (a) where $k_r = 0.9$), and No. 14 via No. 15, and 86 (The first raw in Table 1 (b) where $k_r = 0.8$). So the value of parameter k_r did not affect the synchronization of the neurons in the same cluster.

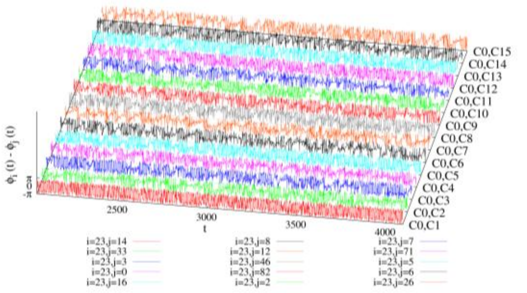
Fig. 4 shows the phase differences between the cluster C0 and other clusters (C1-C15). In Fig.4, the horizontal axis indicates the time of recollection, the vertical axis is the phase difference, and the depth axis corresponds to different clusters (from C1 to C15). So we can find that the phase difference happened between the neurons of different clusters. In other word, it can be conclude that the value of parameter k_r affected the synchronization of the neurons in the different clusters.



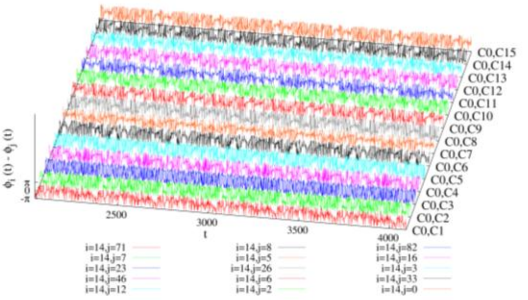
(a) $k_r = 0.9$

(b) $k_r = 0.8$

Fig. 3. The phase differences between the neurons of cluster C0.



(a) $k_r = 0.9$



(b) $k_r = 0.8$

Fig. 4. The phase difference between the neurons of different clusters.

The maximum Lyapunov exponent of CNN with $k_r = 0.9$ was 0.290, and CNN with $k_r = 0.8$ was 0.245. Both values of them were positive, nearby zero. These maximum Lyapunov exponent suggested that either the CNN with $k_r = 0.9$ or the CNN with $k_r = 0.8$ were able to generate chaotic associative process, that is, dynamical attracts were available to appear. Table 3 shows the recalling times of the different stored patterns. The CNN with $k_r = 0.9$ which was used in [1] had a higher recalling rates.

Furthermore, to investigate the relationship between the stored patterns and the neuron spatial positions, we listed the output of neurons in 16 clusters, and it was interested that the number of clusters (i.e. 16) equaled to the number of the combination of stored patterns (see Table 4). For example, neuron No. 23, 32, 45, 55, and 66 in C0 when $k_r = 0.9$ and in C3 when $k_r = 0.8$, output "-1" when stored pattern "cross" was recalled. "+1", "+1", "-1" were output corresponding to the recalling of "star", "triangle" and "wave" respectively.

5 Conclusion

In this paper, we investigated the effect of the internal parameter of CNN on the association process. The spatiotemporal change of the internal states of neurons was used to show the difference yielded by the change of the value of the internal parameter. The chaotic neurons were clustered according to their similarity of internal states, and the characterization seemed affected by the parameter values. So these results support our previous works which suggested the importance of the optimal parameter for the dynamic association model CNNs.

Table 3. Recalling times of stored patterns of each CNN.

CNN	"cross"	"star"	"triangle"	"wave"	Total
k_r =0.9	6	2	8	14	30
k_r =0.8	0	7	12	0	19

Table 4. The causality between elements combinations of sto red patterns and clustered neurons.

Neuron's output co rresponding to the stored patterns (cross, star, triangl e, wave)	Neuron's No.	Cluster (k_r 0.9)	Cluster (k_r 0.8)
-1, -1, -1, -1	26, 35, 36, 54, 62, 63, 67, 73	C15	C8
-1, -1, -1, +1	71, 72, 77, 78, 81, 87	C12	C1
-1, -1, +1, -1	33, 34, 37, 43, 44, 56	C2	C14
-1, +1, -1, -1	12, 17, 22, 27, 64, 65,76	C7	C5
+1, -1, -1, -1	16, 24, 25 53, 58, 68, 85	C5	C12
-1, -1, +1, +1	82, 88, 89, 91, 98, 99	C9	C11
-1, +1, -1, +1	8, 9, 11, 18, 19, 21, 80	C6	C6
-1, +1, +1, -1	23, 32, 45, 55, 66	C0	C3
+1, -1, -1, +1	14, 15, 86	C1	C0
+1, -1, +1, -1	46, 47, 52, 57, 84, 95	C8	C4
+1, +1, -1, -1	2, 13, 69, 74, 75	C10	C10
-1, +1, +1, +1	0, 1, 10, 28, 90	C4	C15
+1, -1, +1, +1	5, 79, 83, 92, 93, 94, 96, 97	C13	C7
+1, +1, -1, +1	3, 4, 20, 30, 31, 59, 70	C3	C13
+1, +1, +1, -1	7, 42, 51, 60, 61	C11	C2
+1, +1, +1, +1	6, 29, 38, 39, 40, 41, 48, 49, 50	C14	C9

REFERENCES

[1] Adachi M., Aihara K. (1997), Associative dynamics in a chaotic neural network, Neural Networks, 10(1):83-98

[2] Kuremoto T., Watanabe S., Kobayashi K., Feng. L.-B., and Obayashi M. (2011), Dynamical recollection of interconnected neural networks using meta-heuristics (in Japanese), IEEJ Transactions on Electronics, Information and Systems, 131(8):1475-1484

[3] Kuremoto T., Watanabe S., Kobayashi K., Feng. L.-B., and Obayashi M. (2012), Dynamical recollection of interconnected neural networks using meta-heuristics, Electronics and Communications in Japan, 95(6):12-23