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Parameter optimization for decoupling controllers of 4WS vehicles

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Abstract: In this paper, the performance analysis and parameter optimization for two typical decoupling controllers of the 4WS vehicles are considered. Firstly, a new relationship between velocity and acceleration in effect on the nonlinear performance is established. Then, for the decoupling controller of a quasi-linearized system, the optimized region of parameters is obtained, in which the damping is bigger than one and eigenvalues are smaller than any given negative number. For the decoupling controller of a linearized system, the necessary and sufficient condition that the overshoot and irritating are avoided is deduced by using a new index. And the region, the measurement-error disturbance is attenuated to under any given positive number, is obtained for any expected eigenvalues. Simulation results show that optimized controllers can improve the safety and comfort obviously.

Keywords: 4WS vehicle, controller optimization, decoupling control, nonlinear performance analysis

1 INTRODUCTION

Decoupling control can improve comfort and safety for the vehicles. Thus its application has received a lot of attention in recent years [1]-[13]. Taking yaw rate and lateral speed as states, the decoupling control with velocity-varying and robust performances of the quasi-linearized system is advocated in [1] and [12]-[13]. Based on linearized systems, the diagonal decoupling is achieved in [3]. Taking yaw rate and lateral velocity as states, the diagonal decoupling in [2] and triangular decoupling in [5]-[7] are obtained.

There are high prices to be payed for using the decoupling control laws, e.g., damping and natural frequency are changed, and the desired disturbance attenuation is hard to be ensured. These prices may reduce the safety and comfort. Hence, the following issues are considered in this paper

1. Are the system performance is affected by the same factors with different states?

2. Are there some special decoupling controllers which can ensure the safety and comfort?

The first issue is answered by employing the linear system theory and joint-point locus approach, and the second is solved by optimizing some controller parameters. Models are described in section 2. System performances are analyzed to different states and the relationship to velocity and acceleration is obtained in section 3. Optimization of two decoupling controllers are discussed in section 4, one is used to control quasi-linear model and the other is linear. Simulations are shown in section 5 and conclusions are given in section 6.

2 VEHICLE MODEL

As mentioned in [7], the essential features of 4WS vehicle steering dynamics can be described by the single-track model as shown in Fig.1. If roll, pitch, and vertical dynamics are

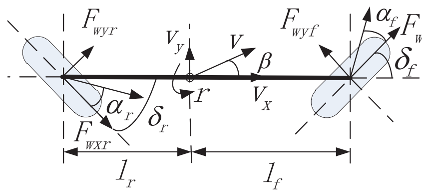

Fig. 1. Single-track model of 4WS vehicles

neglected, then the dynamics can be rewritten as [1]

$$m(\dot{v}_x - r v_y) = \sum_{i=r,l} F_{xi} - f_a \quad (1)$$

$$m(\dot{v}_y + r v_x) = \sum_{i=r,l} F_{yi} + F_y \quad (2)$$

$$J\dot{r} = l_f F_{yf} - l_r F_{yr} + T_z \quad (3)$$

Two typical simplified models can be obtained by linear and quasi-linear methods, which are,

$$\begin{bmatrix} \dot{v}_y \\ \dot{r} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} v_y \\ r \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix} \begin{bmatrix} \delta_f \\ \delta_r \\ F_l \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} \\ \tilde{a}_{21} & \tilde{a}_{22} \end{bmatrix} \begin{bmatrix} v_y \\ r \end{bmatrix} + \begin{bmatrix} \tilde{b}_{11} & \tilde{b}_{12} & \tilde{b}_{13} \\ \tilde{b}_{21} & \tilde{b}_{22} & \tilde{b}_{23} \end{bmatrix} \begin{bmatrix} \delta_f \\ \delta_r \\ F_l \end{bmatrix} \quad (5)$$

where, F_l is the longitudinal acceleration/braking force. Details of $a_{i.j}$ and $\tilde{a}_{i,j}$ please refer to [1].

***Remark* 1.** *Parameters $b_{11}, b_{12}, b_{21}, b_{22}$ are taken as,*

$$b_{11} = \frac{c_f}{m},\ b_{12} = \frac{c_r}{m},\ b_{21} = \frac{c_f l_f}{J},\ b_{22} = -\frac{c_r l_r}{J} \quad (6)$$

while system (4) are linearized, and taken as

$$\begin{bmatrix} \frac{(1-\gamma)F_{l0}+c_f}{m} & \frac{\gamma F_{l0}+c_r}{m} \\ l_f \frac{(1-\gamma)F_{l0}+c_f}{J} & l_r \frac{\gamma F_{l0}+c_r}{J} \end{bmatrix} \quad (7)$$

while simplified by quasi-linearizing.

***Remark* 2.** *b_{i3}, $\tilde{b}_{i3}$ are almost zeros, and $\tilde{b}_{ij}$, b_{ij} satisfy*

$$\tilde{b}_{1j} = v b_{1j},\ \tilde{b}_{2j} = b_{2j},\ j = 1,2 \quad (8)$$

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3 PERFORMANCE ANALYSIS

Linear and nonlinear performances of systems (4) and (5) are analyzed in this section.

3.1 Linear performance analysis

Stability, frequency and damping performances of systems (4) and (5) around equilibria pairs (v_y^*, r^*)(to system (4)) or (β^*, r^*)(to system (5)), are determined by

$$J(v_y^*, r^*) = \begin{bmatrix} -\frac{c_f^*+c_r^*}{mv} & -v-\frac{c_f^* l_f - c_r^* l_r}{mv} \\ -\frac{c_f^* l_f - c_r^* l_r}{Jv} & -\frac{c_f^* l_f^2 + c_r^* l_r^2}{Jv} \end{bmatrix} \quad (9)$$

$$J(\beta^*, r^*) = \begin{bmatrix} -\frac{c_f^*+c_r^*}{mv} & -1-\frac{c_f^* l_f - c_r^* l_r}{mv^2} \\ -\frac{c_f^* l_f - c_r^* l_r}{J} & -\frac{c_f^* l_f^2 + c_r^* l_r^2}{Jv} \end{bmatrix} \quad (10)$$

Both above matrices have characteristic polynomial as

$$p(s) = \omega_0^2 + 2D_0\omega_0 s + s^2 \quad (11)$$

$$\omega_0^2 = \frac{c_r^* c_f^* l^2 + mv^2(c_r l_r - c_f l_f)}{mJv^2} \quad (12)$$

$$2\omega_0 D_0 = \frac{(c_f^* + c_r^*)J + m(c_f^* l_f^2 + c_r^* l_r^2)}{mJv} \quad (13)$$

Thus (4) and (5) have the same linear performances as shown in [6]. Furthermore, if use a_r to replace l_r and a_f replace l_f, while $v > 0$ and the rear tyre isn't working in decaying region, i.e. rear tyres is driving wheels, then the stability factor $K = c_f a_f - c_r a_r$, and the vehicle is understeering while $K > 0$, neutralsteering while $K = 0$ and oversteering while $K < 0$ [9]. And if denote a_f as a_N with $K = 0$, then

$$a_N c_f - (l - a_N)c_r = 0$$

Thus the vehicle is understeering while $a_N > l_f$ and oversteering while $a_N < a_f$. And if the center of mass trends to rear axle, then the vehicle brings about oversteering, which is a dangerous situation with high velocity. From above analysis, we know

***Remark* 3.** *The damping and frequency of systems (4) and (5) mainly depend on v_x. If v_x is so big that $D_0 < 1$, then oscillation occurs and vehicle performance becomes badly.*

***Remark* 4.** *The handling characteristics may be better while a_f is smaller than a_N in a suitable range.*

3.2 Nonlinear performance analysis

By considering the load distributions and friction coefficient factors in the equilibria, we have

$$F_{ywf} + F_{ywr} = mrv_x,\ F_{ywf} l_f - F_{ywr} l_r = 0 \quad (14)$$

$$F_{ywf} = \mu_f N_f f_f(\alpha_f),\ F_{ywr} = \mu_r N_r f_r(\alpha_r) \quad (15)$$

$$N_f = mgl_r(1-\varepsilon_f)/l,\ N_r = mgl_f(1-\varepsilon_r)/l \quad (16)$$

Then we can get

$$\frac{\mu_f(1-\varepsilon_f)}{\mu} f_f(\alpha_f) = \frac{\mu_r(1-\varepsilon_r)}{\mu} f_r(\alpha_r) = \frac{v_x r}{\mu g} \quad (17)$$

If $\lambda_0 \triangleq \mu_f(1-\varepsilon_f)/\mu$, and $\kappa \triangleq (\mu_r(1-\varepsilon_r))/\mu_f(1-\varepsilon_f)$, then λ_0 corresponds to various road conditions which don't effect the nonlinear performance, whereas κ corresponds to load distributions , and (4) , (5) may have both stable and unstable equilibrium points, and the saddle-node bifurcations with $\kappa \leq 1$, while both systems have only globally stable equilibrium points with $\kappa > 1$ [8]. I.e. $\kappa > 1$ can ensure good nonlinear performance. By simply deducing, we know

$$\kappa > 1 \Leftrightarrow \frac{(h_a - h)c_a}{mh} v_x^2 + \dot{v}_x > \frac{l_r l_f(\mu_f - \mu_r)}{l_r\mu_r + l_f\mu_f} g \quad (18)$$

***Remark* 5.** *From formula (18) we know, the velocity and acceleration in the longitudinal direction are very important to handling characteristics. And varying velocity model is more suitable to reflect the actual physical performances of the steering vehicles.*

4 DECOUPLING RESULTS ANALYSIS AND OPTIMIZATION

In this section, decoupling results of quasi-linearization and linearization systems are analyzed and optimized in order to improve vehicle safety and comfort further. As shown in previous section, (4) and (5) have the same location performances, thus we just talk about the system (4).

4.1 Quasi-linearization system

System (4) can be decoupled by decoupling laws [1]

$$u = F^*(x) + G^*(x)\eta,\ \eta = [\eta_1\ \eta_2\ \eta_3]^T \quad (19)$$

Then the quasi-linearization system is decoupled into

$$\begin{bmatrix} \dot{x}_i \\ \ddot{x}_i \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ a_{i1} & a_{i2} \end{bmatrix} \begin{bmatrix} x_i \\ \dot{x}_i \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \eta_i,\ i = 1,2,3 \quad (20)$$

To get good smooth performance and retain the convergence rate, point (a_{1i}, a_{2i}) must be in the following region with any constant $k_{10} > 0$, which is shown in Fig.2,

$$a_{i1} - k_{10}a_{i2} - k_{10}^2 < 0,\ and\ a_{i2}^2 + 4a_{i1} > 0 \quad (21)$$

From Fig.2 we know, the feasible region of (a_{1i}, a_{2i}) is smaller, while k_{10} becomes bigger. And linear $a_{i1} - k_{10}a_{i2} - k_{10}^2 = 0$ and quadratic curve $a_{i2}^2 + 4a_{i1} = 0$ have only one intersection point, which is $(-k_{10}^2, -2k_{10})$, i.e. for any $k_{10} > 0$, points (a_{1i}, a_{2i}) which satisfy inequalities (21), always exist.

4.2 Linearization System

In [3], the decoupling controller is

$$\begin{bmatrix} \delta_c \\ \delta_r \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}^{-1} \begin{bmatrix} -a_{12}r - b_{11}\delta_p \\ -k_p r - x_1 \end{bmatrix} \quad (22)$$

$$\dot{x}_1 = a_{21}(a_{CG_m - rv} - q_1\delta_p + k_1 r + x_2) \quad (23)$$

$$\dot{x}_2 = k_0(r - G(V, \delta_p)\delta_p), x_1(0) = x_2(0) = 0 \quad (24)$$

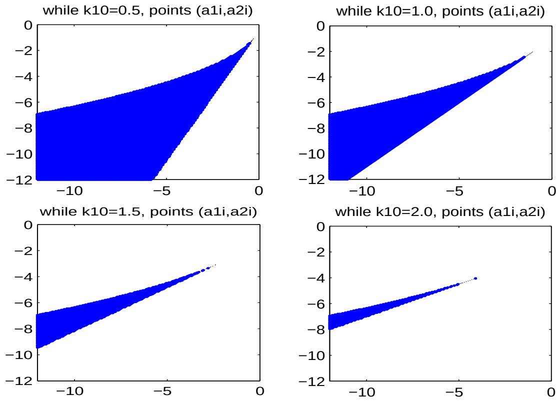

Fig. 2. Regions of (a_{1i}, a_{2i}), with constraints (21)

the controlled linearization system is decoupled into

$$\dot{v}_y = a_{11} v_y \qquad (25)$$

$$\begin{bmatrix} \dot{r} \\ \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} -(k_P - a_{22}) & 1 & 0 \\ -k_1 & 0 & 1 \\ -k_0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r \\ z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} b_{21} \\ q_1 \\ k_0 G(v) \end{bmatrix} \delta_p(t) + \begin{bmatrix} 0 \\ a_{21} \\ 0 \end{bmatrix} d(t) \qquad (26)$$

The transfer function from $\delta_p(s)$ to $r(s)$ is

$$T_{\delta_p}(s) = \frac{b_{21}s^2 + q_1 s + k_0 G(v)}{s^3 + (k_P - a_{22})s^2 + k_1 s + k_0} \qquad (27)$$

The characteristic polynomial is

$$P(s) = s^3 + (k_P - a_{22})s^2 + k_1 s + k_0 \qquad (28)$$

To eigenvalues λ_i, there exist $k_{1,i} \in R, i = 1,2,3$, so that

$$T_{\delta_p}(s) = \frac{k_{1,1}}{s-\lambda_1} + \frac{k_{1,2}}{s-\lambda_2} + \frac{k_{1,3}}{s-\lambda_3} \qquad (29)$$

Hence, the relationship between $r(t)$ and $\delta_p(t)$ is

$$r(t) = k_1 e^{\lambda_1 t}\delta_P(t) + k_2 e^{\lambda_3 t}\delta_P(t) + k_3 e^{\lambda_3 t}\delta_P(t) \qquad (30)$$

Thus, if there were complex λ_i, yaw rate arises overshoot and oscillation. Then the safety and comfort will be worse. So overshoot and oscillation must be avoided.

***Theorem* 1.** *For the cubic equation*

$$ax^3 + bx^2 + cx + d = 0 \qquad (31)$$

Roots x_1, x_2, x_3 *are real and negative, if and only if* $\kappa_0 \leq 0, \frac{b}{a} > 0, \frac{c}{a} > 0,$ *and* $\frac{d}{a} > 0,$ *where* $\kappa_0 = B^2 - 4AC, A = b^2 - 3ac, B = bc - 9ad$ *and* $C = c^2 - 3bd$.

Proof. From Cardano's formula, the equivalent condition that equation (31) only has real roots is $\kappa_0 \leq 0$. The roots are multiple while $\kappa_0 = 0$, and distinct while $\kappa_0 < 0$. And roots are negative while $\frac{b}{a} > 0, \frac{c}{a} > 0, and\ \frac{d}{a} > 0$ [15]. □

The performance of system (29) is decided by κ_0. By employing theorem 1, all eigenvalues of subsystem (26) not only negative but also smaller than $-k_{20}$ equal to

$$k_P - a_{22} > k_{20},\ k_1 > 3k_{20}^2,\ k_0 > k_{20}^3 \qquad (32)$$

$$((k_P - a_{22})k_1 - 9k_0)^2 - 4((k_P - a_{22})^2 - 3k_1)(k_1^2 - 3(k_P - a_{22})k_0) < 0 \qquad (33)$$

And by complexly deducing, we know a sufficient condition for the norm bound $\|T_d(s)\|_\infty \leq \gamma$ is

$$(k_P - a_{22})^2 - 2k_1 > 0,\ k_1^2 - 2k_0(k_P - a_{22}) - \gamma^{-2} > 0 \qquad (34)$$

$$(t_1 t_2 - 9k_0^2)^2 - 4(t_1^2 - 3t_2)(t_2^2 - 3k_0^2 t_1) > 0 \qquad (35)$$

The above desired region of k_P, k_0 and k_1 is shown as Fig.3, where $P_0 = (k_P - a_{22},\ k_0,\ k_1)$. From Fig.3 we know, the region becomes smaller with k_{20} increasing and γ tending to zeros. Specifically, there aren't k_{20} bigger enough or γ near to zeros closely from over zeros direction, so that the region doesn't exist any longer. Such as, k_0/k_1 is taken as smaller than 1 and $(k_P - a_{22})/k_1$ as a constant, then by choosing a large enough $k_P - a_{22}$, inequalities (32)-(35) can be held.

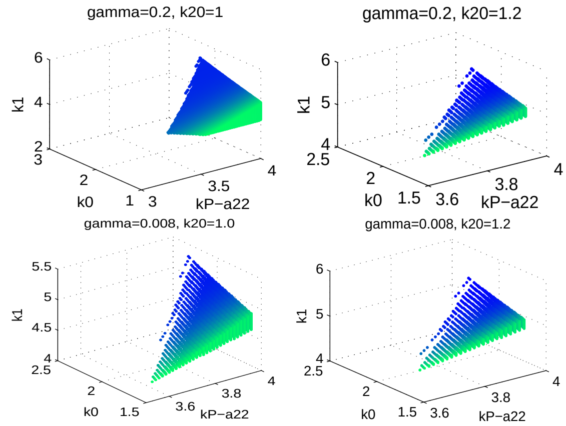

Fig. 3. Regions of P_0 with constraints (32)-(35)

5 SIMULATION

In this section, effects of optimized controllers are examined. The vehicle model BMW 735i is used [1]. The desired track and lateral acceleration are shown in Fig.4. Fig.5 is the simulation of the decoupled system of [1]. Fig.6 and Fig.7 are the decoupled system of [2]-[3], where Fig.6 is the result without disturbance while Fig.7 is the attenuation result of measurement error disturbance of lateral acceleration as $d(t) = 0.1\sin(26\pi t)$. The output and control signals may be both oscillation, however the output signals may be overshoot, while decoupling controllers are without optimizing, especially when the disturbance exists. But the overshoot and oscillation are avoided, and the output and control signals become more smooth, as the controllers are optimized. Furthermore, arbitrary attenuation of measurement error disturbance is arrived by optimizing k_1. So as Shown in simulation results, the safety and comfort are improved further by optimizing the parameters.

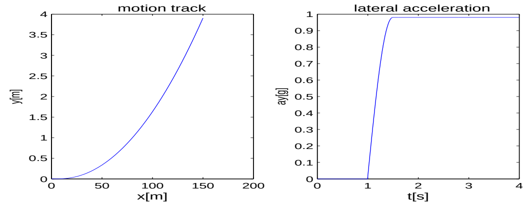

Fig. 4. Desired track and lateral acceleration

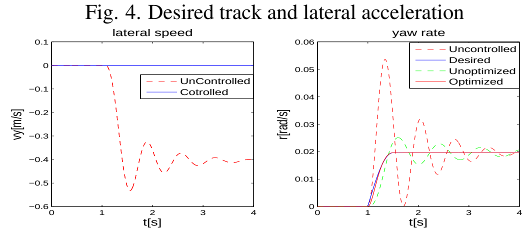

Fig. 5. Steering maneuver of the decoupled system (20)

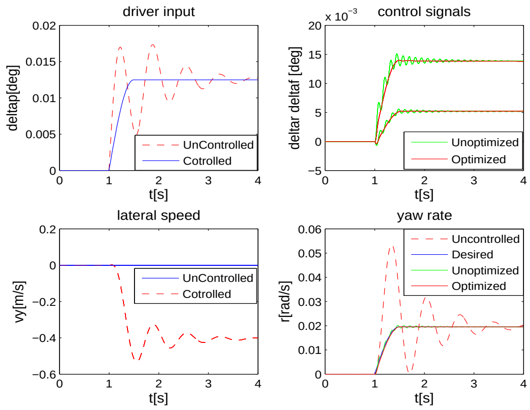

Fig. 6. Steering maneuver of (25)-(26) without disturbing

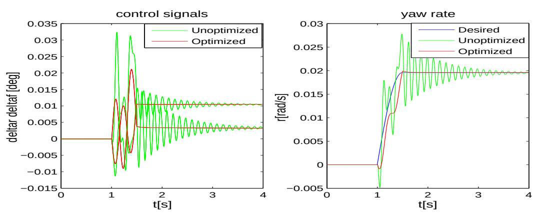

Fig. 7. Steering maneuver of (25)-(26) with disturbing

6 CONCLUSION

The 4WS vehicle performances have been analyzed and a new relationship for nonlinear performance with varying v_x and $\dot{v}_x$ has been given. Specifically, the optimization region of a quasi-linearized system is obtained, in which eigenvalues can be assigned in any desired position and overshoot and oscillation are avoided. Moreover, the necessary and sufficient condition preventing overshoot and irritating of system (25)-(26) is proposed and the optimization region is got, in which any desired H_∞ index γ and specified eigenvalues are achieved with avoiding overshoot and oscillation. And the simulation results show that the comfort and safety for the vehicle driving are improved significantly.

7 ACKNOWLEDGMENTS

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