

Bottom-Up Pyramid Cellular Acceptors with Four-Dimensional Layers

Yasuo UCHIDA¹, Takao ITO¹, Makoto SAKAMOTO², Takashi IDE², Kazuyuki UCHIDA²,
Ryoju KATAMUNE², Hiroshi FURUTANI², Michio KONO², Satoshi IKEDA², and Tsunehiro YOSHINAGA³

¹ Ube National College of Technology ² University of Miyazaki ³ Tokuyama College of Technology

Abstract

In 1967, M.Blum and C.Hewitt first proposed two-dimensional automata as a computational model of two-dimensional pattern processing. Since then, many researchers in this field have been investigating many properties of two- or three-dimensional automata. In 1997, C.R.Dyer and A.Rosenfeld introduced an acceptor on a two-dimensional pattern (or tape), called the *pyramid cellular acceptor*, and demonstrated that many useful recognition tasks are executed by pyramid cellular acceptors in time proportional to the logarithm of the diameter of the input. They also introduced a *bottom-up pyramid cellular acceptor* which is a restricted version of the pyramid cellular acceptor, and proposed some interesting open problems about bottom-up pyramid cellular acceptors. On the other hand, we think that the study of four-dimensional automata has been meaningful as the computational model of four-dimensional information processing such as computer animation, moving picture processing, and so forth[9]. In this paper, we investigate about bottom-up pyramid cellular acceptors with four-dimensional layers, and show their some accepting powers.

Key Words : cellular automaton, diameter, finite automaton, four-dimension, pattern recognition.

1 Introduction

M.Blum and C.Hewitt first proposed two-dimensional automata as a computational model of two-dimensional pattern processing, and investigated their pattern recognition abilities [1]. Since then, many researchers in this field have been investigating a lot of properties about automata on a two- or three-dimensional tape. In [2], C.R.Dyer and A.Rosenfeld introduced an acceptor on a two-dimensional pattern (or tape), called the *pyramid cellular acceptor*, and demonstrated that many useful recognition tasks are executed by the pyramid cellular acceptors in time proportional to logarithm of the diameter of the input. They also introduced a *bottom-up pyramid cellular acceptor*, which is a restricted version of the pyramid cellular acceptor, and proposed some interesting open problems about it.

On the other hand, the question of whether processing four-dimensional digital patterns is much difficult than two- or three-dimensional ones is of great interest from the theoretical and practical standpoints. Thus, the study of four-dimensional automata as the computational model of four-dimensional pattern processing has been meaningful. From this point of view, we are interested in four-dimensional automata.

In this paper, we study about bottom-up pyramid cellular acceptors with four-dimensional layers, and deal with the following problems (which is one of the open problems) : Does the class of sets accepted by deterministic bottom-up pyramid cellular acceptors with four-dimensional layers include the class of sets accepted by deterministic *four-dimensional finite automata* [3-7]? This paper shows that the class of sets accepted by four-dimensional finite automata is incomparable with the class of sets accepted by deterministic bottom-up pyramid cellular acceptors which operate in time of order lower than the *diameter* of the input.

2 Definition

Let Σ be a finite set of symbols. A *three-dimensional tape* over Σ is a four-dimensional rectangular array of elements of Σ . The set of all the four-dimensional tapes over Σ is denoted by $\Sigma^{(4)}$. Given a tape $x \in \Sigma^{(4)}$, for each j ($1 \leq j \leq 4$), we let $l_j(x)$ be the length of x along the j th axis. The set of all $x \in \Sigma^{(4)}$ with $l_1(x) = n_1, l_2(x) = n_2, l_3(x) = n_3$, and $l_4(x) = n_4$ is denoted by $\Sigma^{(n_1, n_2, n_3, n_4)}$. When $1 \leq i_j \leq l_j(x)$ for each j ($1 \leq j \leq 4$), let $x(i_1, i_2, i_3, i_4)$ denote the symbol in x with coordinates (i_1, i_2, i_3, i_4) . Furthermore, we define $x[(i_1, i_2, i_3, i_4), (i_1', i_2', i_3', i_4')]$, when $i \leq i_j \leq i_j' \leq l_j(x)$ for each integer j ($1 \leq j \leq 4$), as the four-dimensional input tape y satisfying the following (i) and (ii) : (i) for each j ($1 \leq j \leq 4$), $l_j(y) = i_j' - i_j + 1$; (ii) for each r_1, r_2, r_3, r_4 ($1 \leq r_1 \leq l_1(y), 1 \leq r_2 \leq l_2(y), 1 \leq r_3 \leq l_3(y), 1 \leq r_4 \leq l_4(y)$), $y(r_1, r_2, r_3, r_4) = x(r_1 + i_1 - 1, r_2 + i_2 - 1, r_3 + i_3 - 1, r_4 + i_4 - 1)$.

For each $x \in \Sigma^{(n_1, n_2, n_3, n_4)}$ and for each $1 \leq i_1 \leq n_1, 1 \leq i_2 \leq n_2, 1 \leq i_3 \leq n_3, 1 \leq i_4 \leq n_4$, $x[(i_1, 1, 1, i_4), (i_1, n_2, n_3, i_4)], x[(1, i_2, 1, i_4), (n_1, i_2, n_3, i_4)], x[(1, 1, i_3, i_4), (n_1, n_2, i_3, i_4)], x[(i_1, 1, i_3, i_4), (i_1, n_2, i_3, i_4)],$ and $x[(1, i_2, i_3, i_4), (n_1, i_2, i_3, i_4)]$ are called

the i_1 th (2-3) plane of each x , the i_2 th (1-3) plane of each x , the i_3 th (1-2) plane of each x , the i_1 th row on the i_3 th (1-2) plane of each x , and the i_2 th column on the i_3 th (1-2) plane of each x .

We next give some basic concepts about bottom-up pyramid cellular acceptors with four-dimensional layers [7]. A *bottom-up pyramid cellular acceptor with four-dimensional layers* (3-UPCA) is a pyramidal stack of four-dimensional arrays of cells in which the bottom four-dimensional layer has size $2^n \times 2^n \times 2^n \times 2^n$ ($n \geq 0$), the next lowest $2^{n-1} \times 2^{n-1} \times 2^{n-1} \times 2^{n-1}$, and so forth, the $(n+1)$ st four-dimensional layer consisting of a single cell, called the *root*. Each cell is defined as an identical finite-state machine, $M = (Q_N, Q_T, \delta, A)$, where Q_N is a nonempty, finite set of states, $Q_T \subseteq Q_N$ is a finite set of *input states*, $A \subseteq Q_N$ is the set of *accepting states*, and $\delta : Q_N^{17} \rightarrow Q_N$ is the *state transition function*, mapping the current states of M and its sixteen son cells in a $2 \times 2 \times 2 \times 2$ block on the four-dimensional layer below into M 's next state. Let c be some cell on the $(i+1)$ st four-dimensional layer, and let c (UNWP), c (UNWF), c (USWP), c (USWF), c (USEP), c (USEF), c (UNEP), c (UNEF), c (DNWP), c (DNWF), c (DSWP), c (DSWF), c (DSEP), c (DSEF), c (DNEP), and c (DNEF) be sixteen son cells (on the i th four-dimensional layer) of c , where c (UNWF) is c 's upper northwest son in the most future direction, c (DNWP) is c 's lower northwest son in the most past direction, etc. For example, if the coordinates of c on the $(i+1)$ st layer is $(1, 1, 1, 1)$ ($(2^n, 2^n, 2^n, 2^n)$), the coordinates of sixteen son cells of c on the i th layer c (UNWP), c (UNWF), c (USWP), c (USWF), c (USEP), c (USEF), c (UNEP), c (UNEF), c (DNWP), c (DNWF), c (DSWP), c (DSWF), c (DSEP), c (DSEF), c (DNEP), and c (DNEF) are $(1, 1, 1, 1)$, $(1, 1, 1, 2)$, $(1, 2, 1, 1)$, $(1, 2, 1, 2)$, $(1, 2, 2, 1)$, $(1, 2, 2, 2)$, $(1, 1, 2, 1)$, $(1, 1, 2, 2)$, $(2, 1, 1, 1)$, $(2, 1, 1, 2)$, $(2, 2, 1, 1)$, $(2, 2, 1, 2)$, $(2, 2, 2, 1)$, $(2, 2, 2, 2)$, $(2, 1, 2, 1)$, $(2, 1, 2, 2)$, $((2^n - 1, 2^n - 1, 2^n - 1, 2^n - 1)$, $(2^n - 1, 2^n - 1, 2^n - 1, 2^n)$, $(2^n - 1, 2^n, 2^n - 1, 2^n - 1)$, $(2^n - 1, 2^n, 2^n - 1, 2^n)$, $(2^n - 1, 2^n, 2^n, 2^n - 1)$, $(2^n - 1, 2^n, 2^n, 2^n)$, $(2^n, 2^n - 1, 2^n - 1, 2^n - 1)$, $(2^n, 2^n - 1, 2^n - 1, 2^n)$, $(2^n, 2^n, 2^n - 1, 2^n - 1)$, $(2^n, 2^n, 2^n - 1, 2^n)$, $(2^n, 2^n, 2^n, 2^n - 1)$, $(2^n, 2^n, 2^n, 2^n)$, $(2^n, 2^n - 1, 2^n, 2^n - 1)$, $(2^n, 2^n - 1, 2^n, 2^n)$), respectively. Then $q_c(t+1) = \delta(q_c(t), q_{c(\text{UNWP})}(t), q_{c(\text{UNWF})}(t), q_{c(\text{USWP})}(t), q_{c(\text{USWF})}(t), q_{c(\text{USEP})}(t), q_{c(\text{USEF})}(t), q_{c(\text{UNEP})}(t), q_{c(\text{UNEF})}(t), q_{c(\text{DNWP})}(t), q_{c(\text{DNWF})}(t), q_{c(\text{DSWP})}(t), q_{c(\text{DSWF})}(t), q_{c(\text{DSEP})}(t), q_{c(\text{DSEF})}(t), q_{c(\text{DNEP})}(t), q_{c(\text{DNEF})}(t))$, where for example $q_c(t)$ means the state of c at time t . At time $t = 0$, the input tape $x \in Q_T^{(4)}$ [$l_1(x) = l_2(x) = l_3(x) = l_4(x) = 2^n$, $n \geq 0$] is stored as the initial states of the bottom four-dimensional layer, henceforth called the *base*, in such a

way that $x(i_1, i_2, i_3, i_4)$ is stored at the cell of the i_1 th row and the i_2 th column on the i_3 th plane of the i_4 th four-dimensional rectangular array, and the other cells are initialized to a *quiescent state* q_s ($\in Q_N - Q_T - A$). As usual, we let $\delta(q_s, q_s, q_s, q_s, q_s, q_s, q_s, q_s, q_s, q_s, q_s, q_s, q_s, q_s, q_s, q_s, q_s) = q_s$. The input is *accepted* if and only if the root cell ever enters an accepting state. This 4-UPCA is called *deterministic*. A *nondeterministic bottom-up pyramid cellular acceptor* is defined as a 4-UPCA using $\delta : Q_N^{17} \rightarrow 2^{Q_N}$ instead of the state transition function of the deterministic 4-UPCA. Below, we denote a deterministic 4-UPCA by 4-DUPCA, and a nondeterministic 4-UPCA by 4-NUPCA. A 4-DUPCA (or 4-NUPCA) operates in time $T(n)$ if for every four-dimensional tape of size $2^n \times 2^n \times 2^n \times 2^n$ ($n \geq 0$) it accepts the four-dimensional tape, then there is an accepting computation which uses no more than time $T(n)$. By 4-DUPCA($T(n)$) [4-NUPCA($T(n)$)] we denote a $T(n)$ time-bounded 4-DUPCA [4-NUPCA] which operates in time $T(n)$.

We next introduce a four-dimensional finite automaton [8]. A *four-dimensional finite automaton* (4-FA) is a four-dimensional Turing machine with no workspace. A 4-FA M has a read-only four-dimensional tape with boundary symbols $\#$'s, finite control, and an input head. The input head can move in eight direction – east, west, south, north, up, down, future, or past – unless it falls off the input tape. Formally, M is defined by the 5-tuple $M = (K, \Sigma \cup \{\#\}, \delta, q_0, F)$, where K is a finite set of *states*, Σ is a finite set of *input symbols*, $\#$ is the *boundary symbol* (not in Σ), $\delta : K \times (\Sigma \cup \{\#\}) \rightarrow 2^{K \times \{E, W, S, N, U, D, F, P, H\}}$ is the *state transition function*, where E, W, S, N, U, D, F, P , and H represent the move directions of the input head – east, west, south, north, up, down, future, past, and no move, respectively, $q_0 \in K$ is the *initial state*, and $F \subseteq K$ is the set of *accepting states*. The action of M is similar to that of the one-dimensional (or two-dimensional) finite automaton [4], except that the input head of M can move in eight directions. That is, when an input tape $x \in \Sigma^{(4)}$ with boundary symbols is presented to M , M starts in its initial state q_0 with the input head on $x(1, 1, 1, 1)$, and determines the next state of the finite control and the move direction of the input head, depending on the present state of the finite control and the symbol read by the input head. We say that M *accepts* the tape x if it eventually enters an accepting state. We denote a deterministic 4-FA [nondeterministic 4-FA] by 4-DFA [4-NFA].

We let each sidelength of each input tape of three automata, throughout this paper, be equivalent. We denote the set of all four-dimensional tapes accepted by M by $T(M)$. Define \mathcal{L} [4-DUPCA] = $\{T \mid T(M) \text{ is accepted by some 4-DUPCA } M\}$. \mathcal{L} [4-NUPCA], \mathcal{L} [4-DFA], etc. are defined similarly.

Finally, we give definition of *diameter*. Given a

subset S of a tape $x \in \Sigma^{(4)}$, we can define its *extent* in a given direction θ as the length of its projection on a plane in that direction. Here the length of a projection is the distance between its farthest apart nonzero values. Thus the extent of S is the distance between a pair of parallel planes perpendicular to θ that just bracket S . The *diameter* of S is defined as its extent in any direction.

3 Results

In this section, we show that the class of sets accepted by 4-DFA's is incomparable with the class of sets accepted by 4-DUPCA's which operate in time of order lower than the diameter of the input. It has often been noticed that we can easily get several properties of four-dimensional automata by directly applying the results of one- or two-dimensional case, if each sidelength of each four-dimensional input tape of these automata is not equivalent. So we let each sidelength of each input tape, throughout this paper, be equivalent in order to increase the theoretical interest.

Lemma 3.1. *Let $T_1 = \{x \in \{0,1\}^{(4)} \mid \exists n (n \geq 1) [\ell_1(x) = \ell_2(x) = \ell_3(x) = \ell_4(x) = 2^n] \text{ and } x(2^{n-1}, 2^{n-1}, 2^{n-1}, 2^{n-1}) = 1\}$. Then,*
(1) $T_1(x) \notin \mathcal{L}[4\text{-DFA}]$, and
(2) $T_1(x) \in \mathcal{L}[4\text{-DUPCA}(n)]$.

Proof : The Proof of (1) is similar to that of Theorem 3 in [7]. On the other hand, by using the same technique as in the proof of Lemma 1 in [6], we can get Part (2) of the lemma. \square

Lemma 3.2. *Let $T_2 = \{x \in \{0,1\}^{(4)} \mid \exists n (n \geq 1) [\ell_1(x) = \ell_2(x) = \ell_3(x) = \ell_4(x) = 2^n] \text{ and } x[(1,1,1,1), (2^n, 2^n, 2^n, 1)] = x[(1,1,1,2^n), (2^n, 2^n, 2^n, 2^n)]\}$. Let $T(n)$ be a time function such that $\lim_{n \rightarrow \infty} [T(n)/2^{2n}] = 0$. Then,*
(1) $T_2 \in \mathcal{L}[4\text{-DFA}]$, and
(2) $T_2 \notin \mathcal{L}[4\text{-DUPCA}(T(n))]$.

Proof : It is obvious that there is a 4-DFA accepting T_2 , and so (1) of the lemma holds. Below, we prove (2). Suppose that there is a 4-DUPCA B which accepts T_2 and operates in time $T(n)$, and that each cell of B has k states. For each $n \geq 2$, let

$W(n) = \{x \in \{0,1\}^{(4)} \mid \ell_1(x) = \ell_2(x) = \ell_3(x) = \ell_4(x) = 2^n\}$, and

$$\begin{aligned} W'(n) &= \{x \in \{0,1\}^{(4)} \mid \ell_1(x) = \ell_2(x) = \ell_3(x) = \ell_4(x) \\ &= 2^{n-1} \\ &\& x[(1,1,1,1), (2^{n-1}, 2^{n-1}, 2^{n-1}, 1)] \in \{0,1\}^{(4)} \\ &\& x[(1,1,1,2), (2^{n-1}, 2^{n-1}, 2^{n-1}, 2^{n-1})] \in \{0\}^{(4)}\}. \end{aligned}$$

We consider the cases when the tapes in $W(n)$ are presented to B . Let c be the cell which is situated at the first row, the first column, the first plane, and the first three-dimensional rectangular array in the n th layer (i.e., the layer just below the root cell). (Note that there are eight cells in the n th layer.) For each x in $W(n)$ such that $x[(1,1,1,1), (2^{n-1}, 2^{n-1}, 2^{n-1}, 2^{n-1})] \in W'(n)$, and for each $r \geq 1$, let $q_r(x)$ be the state of c at time r when x is presented to B . Then the following proposition must hold.

Proposition 3.1. *Let x, y be two different tapes in $W(n)$ such that both $x[(1,1,1,1), (2^{n-1}, 2^{n-1}, 2^{n-1}, 2^{n-1})]$ and $y[(1,1,1,1), (2^{n-1}, 2^{n-1}, 2^{n-1}, 2^{n-1})]$ are in $W'(n)$ and $x4[(1,1,1,1), (2^{n-1}, 2^{n-1}, 2^{n-1}, 2^{n-1})] \neq y[(1,1,1,1), (2^{n-1}, 2^{n-1}, 2^{n-1}, 2^{n-1})]$. Then, $(q_1(x), q_2(x), \dots, q_{T(n)}(x)) = (q_1(y), q_2(y), \dots, q_{T(n)}(y))$.*

[Proof : For suppose that $(q_1(x), q_2(x), \dots, q_{T(n)}(x)) = (q_1(y), q_2(y), \dots, q_{T(n)}(y))$. We consider two tapes z, z' in $W(n)$ such that

(i) $z[(1,1,1,1), (2^{n-1}, 2^{n-1}, 2^{n-1}, 2^{n-1})] = x[(1,1,1,1), (2^{n-1}, 2^{n-1}, 2^{n-1}, 2^{n-1})]$
and
 $z'[(1,1,1,1), (2^{n-1}, 2^{n-1}, 2^{n-1}, 2^{n-1})] = y[(1,1,1,1), (2^{n-1}, 2^{n-1}, 2^{n-1}, 2^{n-1})]$,

(ii) the part of z except for $z[(1,1,1,1), (2^{n-1}, 2^{n-1}, 2^{n-1}, 2^{n-1})]$ is identical with the part z' except for $z'[(1,1,1,1), (2^{n-1}, 2^{n-1}, 2^{n-1}, 2^{n-1})]$,

and

(iii) $z[(1,1,1,1), (2^n, 2^n, 2^n, 1)] = z[(1,1,1,2^n), (2^n, 2^n, 2^n, 2^n)]$.

By assumption, the root cell of B enters the same states until time $T(n)$, for the tapes z and z' . Since B operate in time $T(n)$ and z is in T_2 , it follows that z' is also accepted by B . This contradicts the fact that z' is not in T_2 . \square

Let $t(n)$ be the number of different sequences of states which c enters until time $T(n)$. Clearly, $t(n) \leq k^{T(n)}$. On the other hand (for any set S , let $|S|$ denote the number of elements of S), $|W'(n)| = 2^{2^{3(n-1)}}$. Since $\lim_{n \rightarrow \infty} T(n)/2^{2n} = 0$ (by assumption of the lemma), it follows that $|W'(n)| > t(n)$ for large n . Therefore, it follows that for large n there must exist two different tapes x, y in $W(n)$ such that

(i) both $x[(1,1,1,1), (2^{n-1}, 2^{n-1}, 2^{n-1}, 2^{n-1})]$ and $y[(1,1,1,1), (2^{n-1}, 2^{n-1}, 2^{n-1}, 2^{n-1})]$ are in $W'(n)$,

(ii) $x[(1,1,1,1), (2^{n-1}, 2^{n-1}, 2^{n-1}, 2^{n-1})] \neq y[(1,1,1,1), (2^{n-1}, 2^{n-1}, 2^{n-1}, 2^{n-1})]$, and

$$\begin{aligned} \text{(iii)} \quad & (q_1(x), q_2(x), \dots, q_{T(n)}(x)) \\ & = (q_1(y), q_2(y), \dots, q_{T(n)}(y)). \end{aligned}$$

This contradicts the above Proposition 3.1, and thus the Part (2) of the lemma holds. \square

From Lemmas 3.1 and 3.2, we can get the following theorem.

Theorem 3.1. *Let $T(n)$ be a time function such that $\lim_{n \rightarrow \infty} [T(n)/2^{2n}] = 0$ and $T(n) \geq n (n \geq 1)$. Then $\mathcal{L}[4\text{-DFA}]$ is incomparable with $\mathcal{L}[4\text{-DUPCA}(T(n))]$.*

Corollary 3.1. *$\mathcal{L}[4\text{-DFA}]$ is incomparable with $\mathcal{L}[4\text{-DUPCA}(n)]$, which is the class of sets accepted by 4-DUPCA's operating in real time.*

Corollary 3.2. *$\mathcal{L}[4\text{-DFA}]$ is incomparable with $\mathcal{L}[4\text{-NUPCA}(n)]$.*

4 Conclusion

In this paper, we investigated the accepting powers of bottom-up pyramid cellular acceptors with four-dimensional layers, and showed that the class of sets accepted by 4-DFA's is incomparable with the class of sets accepted by 4-DUPCA's which operate in time of order lower than the diameter of the input. It is still unknown whether the class of sets accepted by 4-DUPCA's includes the class of sets accepted by 4-DFA's.

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