Digital Adaptive Control of a Winged Rocket Applicable to Abort Flight

Tomoaki Shimozawa Shinichi Sagara

Department of Control Engineering, Kyushu Institute of Technology Tobata, Kitakyushu 804-8550, Japan E-mail: sagara@cntl.kyutech.ac.jp

Abstract: Since reusable launch vehicles (RLVs) have wide range flight conditions, the values of parameters of the dynamic equation are not constant. Then some adaptive control methods for the RLVs have been proposed and digital control systems are suited for digital computers. However, the control performance decreases when the nonlinearity strengthens though a linear adaptive control has an excellent performance when the nonlinearity of the controlled system can be disregarded. In this paper, we apply a digital adaptive feedback linearization control method with time-scale separation to a winged rocket in the abort flight. The simulation results show the effectiveness of the control systems.

Keywords: Adaptive Control, Flight Control

Nomenclature

$\delta_a, \delta_e, \delta_r$	=aileron, elevator and rudder
	deflection angles
p, q, r	=rotatonal rates
$lpha,\ eta$	=angle of attack, sideslip angle
$\phi, \; heta$	=bank angle, pitch angle
L_*, M_*, N_*	=aerodynamic rolling, pitching
	and yawing moments
I_{ij}	=moment/product of inertia
V_{TAS}	=true airspeed

1 Introduction

In late years space development is performed lively all over the world, and the research of space transportation systems to enable them is performed. Especially, Reusable Launch Vehicles (RLVs) are expected for the space transportation systems because the RLVs are the low-cost and highly reliable transport system instead of the conventional disposable rockets.

Since the RLVs have wide range flight conditions to the space from the ground, the values of parameters of the dynamic equation of the RLVs are not constant. Therefore, gain scheduling control method [1] that is linearized at a series of design points to be decided by the velocity and the altitude of the RLV and whose parameters of the controller are switched for the change of flight conditions has been applied for the RLVs. However, the gain scheduling control method has drawbacks for the RLVs. First, when the air traffic window expands, the number of required gains to be designed and the schedule becomes very large to guarantee control performance above a certain level in all flight conditions. Second, since the gain scheduling control method can only correspondence to the known change; the control performance of the method becomes worse for the unpredictable change in the flight condition such as the abort flight.

For the change of the flight conditions, adaptive control methods using approximated linear dynamic equation of the RLVs have been researched [2]-[5]. However, the control performance decreases when the nonlinearity strengthens though a linear adaptive control has an excellent performance when the nonlinearity of the controlled system can be disregarded. Then, the feedback linearization method for deleting the nonlinear term by the state feedback for the nonlinear equation of motion of the RLV have been researched [6]. In addition, the method for dividing time-scale by a fast motion and a slow motion is researched for the simplification of the structure of the control system [7]. However, an internal aerodynamic parameter is fixed in their control methods.

In this paper, we apply a digital adaptive feedback linearization control method with time-scale separation to a winged rocket [8] in the abort flight, and the simulations are done to validate the effectiveness of the control systems in wide range flight conditions. The simulation results show that the control system has a good control performance.

2 Model of winged rocket

Fig. 1 shows an outline of a winged rocket [8] and the parameters are shown in Table 1. It has two elevons and two rudders as aerodynamic control surfaces. Since the configuration of the winged rocket



Fig. 1 Winged Rocket

Table 1 Winged rocket parameters

	Parameter
Body length	2.5[m]
Mass	241[kg]
Body outer diameter	0.57[m]
Wing area	$1.05 \ [m^2]$
Wing span	1.8[m]
Mean aerodynamic chord	0.67[m]
Position of center of gravity	65[%]

shown in Fig. 1 is similar for general airplanes, the nonlinear equation of fast states motion is expressed as follows [9]:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}(\boldsymbol{x})\boldsymbol{x}(t) + \boldsymbol{B}(\boldsymbol{x})\boldsymbol{u}(t) + \boldsymbol{C}(x)\boldsymbol{w}(t) \qquad (1)$$

where

$$\begin{aligned} \boldsymbol{x}(t) &= [p \ q \ r]^{T}, \ \boldsymbol{u}(t) = [\delta_{ac} \ \delta_{ec} \ \delta_{rc}]^{T}, \\ \boldsymbol{w}(t) &= [pq \ qr \ rp \ r^{2} - p^{2} \ \alpha \ \beta]^{T}, \\ \boldsymbol{A}(\boldsymbol{x}) &= \boldsymbol{I_d} \begin{bmatrix} L_{p} & 0 & L_{r} \\ 0 & M_{q} & 0 \\ N_{p} & 0 & N_{r} \end{bmatrix}, \\ \boldsymbol{B}(\boldsymbol{x}) &= \boldsymbol{I_d} \begin{bmatrix} L_{\delta_{a}} & 0 & L_{\delta_{r}} \\ 0 & M_{\delta_{e}} & 0 \\ N_{\delta_{a}} & 0 & N_{\delta_{r}} \end{bmatrix}, \\ \boldsymbol{C}(\boldsymbol{x}) &= \boldsymbol{I_d} \\ \vdots \\ \begin{pmatrix} I_{xz} & I_{yy} - I_{zz} & 0 & 0 & 0 \\ I_{xz} - I_{xx} & -I_{xz} & M_{\alpha} & 0 \\ I_{xx} - I_{yy} & -I_{zz} & 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

and I_d is inertia matrix. When \dot{x} is discretizated for forward difference by sampling period T, Eq. (1) is expressed as follows:

$$\begin{aligned} \boldsymbol{x}(k+1) &= (\boldsymbol{I} + T\boldsymbol{A}(\boldsymbol{x}))\boldsymbol{x}(k) \\ &+ T\boldsymbol{B}(\boldsymbol{x})\boldsymbol{u}(k) + T\boldsymbol{C}(\boldsymbol{x})\boldsymbol{w}(k) \\ &= \boldsymbol{A}_T(\boldsymbol{x})\boldsymbol{x}(k) + \boldsymbol{B}_T(\boldsymbol{x})\boldsymbol{u}(k) + \boldsymbol{C}_T(\boldsymbol{x})\boldsymbol{w}(k) \end{aligned}$$
(2)

where, \boldsymbol{I} is identity matrix.

On the other hand, nonlinear equation of slow states motion is expressed as follows [9]:

$$\dot{\boldsymbol{y}}(t) = \boldsymbol{D}(\boldsymbol{y})\boldsymbol{x}(t) + \boldsymbol{E}\boldsymbol{h}(t) + \boldsymbol{g}(t)$$
(3)

where,

$$\begin{split} \boldsymbol{y}(t) &= [\alpha \ \beta \ \phi]^T \\ \boldsymbol{h}(t) &= \left[\frac{\sin \alpha}{V_{TAS} \cos \beta} \ \frac{\cos \alpha}{V_{TAS} \cos \beta} \ \frac{1}{V_{TAS}} \right]^T \\ \boldsymbol{D}(\boldsymbol{y}) &= \left[\begin{matrix} -\cos \alpha \tan \beta & 1 & -\sin \alpha \tan \beta \\ \sin \alpha & 0 & -\cos \alpha \\ 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \end{matrix} \right], \\ \boldsymbol{E} &= \left[\begin{matrix} -\frac{X}{m} & \frac{Z}{m} & 0 \\ 0 & 0 & \frac{Y}{m} \\ 0 & 0 & 0 \end{matrix} \right], \end{split}$$

and g(t) is the vector consisting of the gravitational forces. Similar for fast states motion, \dot{y} is discretizaed for forward difference by sampling period T, Eq. (3) is expressed as follows:

$$\boldsymbol{y}(k+1) = \boldsymbol{y}(k) + \boldsymbol{D}_T(\boldsymbol{y})\boldsymbol{x}(k) + \boldsymbol{E}_T\boldsymbol{h}(k) + T\boldsymbol{g}(k).$$
(4)

3 Control system

The design method in this paper is a digital adaptive feedback linearization control method with timescale separation. To derive the input \boldsymbol{u} to control the output \boldsymbol{y} , this method is separated two time-scale. In the fast time-scale, control surface deflection command $\boldsymbol{u} = [\delta_{ac} \ \delta_{ec} \ \delta_{rc}]^T$ is derived from rotational rate command $\boldsymbol{x}_c = [p_c \ q_c \ r_c]^T$. In the slow time scale, rotational rate command \boldsymbol{x}_c is derived from output command $\boldsymbol{y}_c = [\alpha_c \ \beta_c \ \phi_c]^T$.

3.1 Inner loop for the fast states

Here, control input to Eq. (2) is defined as

$$\boldsymbol{u}(k) = \boldsymbol{B}_T^{-1} \{ \boldsymbol{x}_d(k+1) - \boldsymbol{A}_T(\boldsymbol{x}) \boldsymbol{x}(k) - \boldsymbol{C}_T(\boldsymbol{x}) \boldsymbol{w}(k) - \boldsymbol{\Lambda}(\boldsymbol{x}_d(k) - \boldsymbol{x}(k)) \}$$
(5)

where, $\boldsymbol{\Lambda} = \text{diag}\{\lambda_i\}$ (i = p, q, r) is the gain matrix and \boldsymbol{x}_d is the output of the reference model. From Eqs. (2) and (5), the output error $\boldsymbol{e}_f(k) = \boldsymbol{x}_d(k) - \boldsymbol{x}(k)$ is

$$\boldsymbol{e}_f(k+1) = \boldsymbol{\Lambda} \boldsymbol{e}_f(k). \tag{6}$$

And if λ_i is selected to satisfy $0 < \lambda_i < 1$, the output error $\boldsymbol{e}_f(k)$ tends to zero as k tends to infinity. In a case of unknown parameters, we introduce the matrices $\hat{\boldsymbol{A}}_T(\boldsymbol{x})$, $\hat{\boldsymbol{B}}_T(\boldsymbol{x})$ and $\hat{\boldsymbol{C}}_T(\boldsymbol{x})$ those coefficients are estimated values of the coefficients in $\boldsymbol{A}_T(\boldsymbol{x})$, $\boldsymbol{B}_T(\boldsymbol{x})$ and $\boldsymbol{C}_T(\boldsymbol{x})$, respectively. The coefficients in $\hat{\boldsymbol{A}}_T(\boldsymbol{x})$, $\hat{\boldsymbol{B}}_T(\boldsymbol{x})$ and $\hat{\boldsymbol{C}}_T(\boldsymbol{x})$ are estimated by using adaptive algorithms [11].





3.2 Outer loop for the slow states

From Eq. 4, the rotational rate command is obtained by following equations:

$$\boldsymbol{x}_{c}(k) = \boldsymbol{D}_{T}^{-1} \{ \boldsymbol{y}_{d}(k+1) - \boldsymbol{y}(k) - \boldsymbol{E}_{T}(\boldsymbol{y})\boldsymbol{h}(k) - T\boldsymbol{g}(k) - \boldsymbol{P}(\boldsymbol{y}_{d}(k) - \boldsymbol{y}(k)) \}$$
(7)

where, $\boldsymbol{P} = \text{diag}\{p_i\}$ $(i = \alpha, \beta, \phi)$ is gain matrix and \boldsymbol{y}_d is output of the reference model. From Eq. 3 and 7, output error $\boldsymbol{e}_s(k) = \boldsymbol{y}_d(k) - \boldsymbol{y}(k)$ is

$$\boldsymbol{e}_s(k+1) = \boldsymbol{P}\boldsymbol{e}_s(k). \tag{8}$$

And if p_i is selected to satisfy $0 < p_i < 1$, the output error $e_s(k)$ tends to zero as k tends to infinity.

In this paper, unknown parameters in Eq. 7 uses the fixed values obtained from the wind tunnel examination result.

4 Numerical simulation

To validate the adaptive control system described, computer simulations for a 6-DOF nonlinear winged rocket model [10] considering the atmospheric fluctuation are performed. simulation condition is follows. When the winged rocket is climbing, a trouble is happened at about 4000[m] in altitude. Then

the winged rocket changes the thrust power from 3000[N] into 0[N] for changing the route. That is an abort flight. At the start of the simulation, the altitude of the winged rocket is 3200[m], downrange is 3000[m], crossrange is 0[m], the velocity is 80[m/s]and the pitch angle is 70[deg]. At the target point, downrange and crossrange are 0[m], and altitude is 200[m]. The aerodynamic coefficient of the winged rocket uses the value obtained from the wind tunnel examination result. The actuators of elevons is used that the attenuation coefficient $\zeta = 0.7$ and the natural frequency $\omega_n = 72 [rad/s]$. The sampling period of the control system is T = 0.01[s]. The gain of fast states are $\lambda_p = 0.95$, $\lambda_q = 0.90$ and $\lambda_r = 0.90$. The gain of slow states are $p_{\alpha} = 0.90, p_{\beta} = 0.92$ and $p_{\phi}=0.95.\,$ A constant trace algorithm [11] is utilized for parameter estimation.

Figs. 2 and 3 show the input and the output, respectively. In Fig. 3, the dotted lines show the output of the reference model. From Fig. 2, we can see that the input commands to the actuators is almost smooth. And from Fig. 3, it can be seen that the angle of attack and the bank angle follow to the reference output well, and the side-slip angle is suppressed to 0.5[deg] or less. Next, Fig. 4 is flight trajectory. From Fig. 4, we can see that the winged



rocket arrives at the neighborhood the target point.

From the simulation result, it can be confirmed that the applied control method is effective for the flight control of the winged rocket.

5 Conclusion

In this paper, we apply a digital adaptive feedback linearization control method with time-scale separation to a winged rocket in the abort flight. From the numerical simulation, we showed that the control system of the winged rocket has a good control performance.

References

- T. Tsukamoto, H. Suzuki, T Ninomiya, "Guidance and Control Law Design for High Speed Flight Demonstration Phase II", JAXA Research and Development Report, JAXA-RR-04-006, 2004 (in Japanese).
- [2] Eric N. Johnson, Anthony J. Calise, "Limited Authority Adaptive Flight Control for Reusable Launch Vehicles", *Journal of Guidance, Control, and Dynamics*, vol. 26, no. 6, pp. 906–913, 2003.
- [3] M. Morimoto, K. Uchiyama, Y. Shimada, A. Abe, "Adaptive Attitude Control with Reduced Number of Estimated Parameters for Automatic Landing System", *International Conference on Control, Automation and Systems* 2007, pp. 2865–2870, 2007.
- [4] J. Brinker. and K. Wise, "Reconfigurable Flight

Control for a Tailless Advanced Fighter Aircraft", *Proceedings of AIAA Guidance, Navigation and Control Conference*, pp. 75–87, 1998.

- [5] Eric N. Johnson, Anthony J. Calise, "Pseudo-Control Hedging: A New Method for Adaptive Control", Advances in Navigation Guidance and Control Technology Workshop, November, 1-2, 2000.
- [6] Eric N. Johnson, Anthony J. Calise, "A Six Degree of Freedom Adaptive Flight Control Architecture for Trajectory Following", *Proceedings* of the AIAA Guidance, Navigation, and Control Conference, 2002.
- [7] P. K. Menon, V. R. Iragavarapu, and E. J. Ohlmeyer, "Nonlinear Missile Autopilot Design Using Time-Scale Separation", AIAA paper, 97-3765-CP, pp. 1791-1803, 1997.
- [8] K. Yonemoto, T. Shidooka, K. Okuda, "Development and Flight Test of Winged Rocket", *The* 27th International Symposium on Space Technology and Science, 2009.
- [9] B. Etkin, L. D. Reid, Dynamics of Flight: Stability and Control, New York: John Wiley & Sons, Inc, 1996.
- [10] HIMES Research Group, "Conceptual Design of HIMES (Winged Test Rocket)", Institute of Space and Astronautical Science, Ministry of Education, March 19, 1987 (in Japanese).
- [11] G. C. Goodwin, R. H. Middleton, *Digital Control and Estimation*, Prentice-Hall International, p. 379, 1990.