Robust Controller for Underwater Vehicle-Manipulator Systems Including Thruster Dynamics

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Abstract: This paper deals with a control scheme for underwater vehicle-manipulator systems with the dynamics of thrusters in the presence of uncertainties in system parameters. We have developed an adaptive controller that overcomes thruster nonlinearities, which cause an uncontrollable system. However, the structure of the adaptive controller is very complex due to the regressors of dynamic system models and parameter estimators. In this paper we develop a robust controller whose structure is much simpler than that of the adaptive controller.

Keywords: Underwater vehicle-manipulator system, thruster dynamics, robust control.

I. INTRODUCTION

An autonomous underwater vehicle with manipulators, referred to as underwater vehicle-manipulator system (UVMS), is expected to play an important role in ocean development [1]. Adaptive or robust control schemes for UVMSs have recently been developed in the presence of not only hydrodynamic forces acting on the UVMS and the dynamic coupling between the vehicle and the manipulator but also uncertainties in system parameters [1–5]. In a general type of UVMS, the vehicle is propelled by marine thrusters, whereas the manipulator is driven by electrical motors. Despite such a different actuator system, the existing control schemes in [1-5] were designed based on the dynamic system models without the thruster dynamics to obtain a simply-structured controller. In each control scheme, furthermore, a high gain control system is constructed in order to achieve good control performance. However, the vehicle propelled by marine thrusters generally has a considerably slower time response than the manipulator driven by electrical motors [4], and hence the high gains may excite the ignored thruster dynamics, which degrades control performance and may even cause instability.

In order to overcome the problem, the authors have developed an adaptive controller for UVMSs with the thruster dynamics [6]. Since the slow thruster dynamics was taken into consideration in the development of controller, control performance can be improved by the adaptive controller with high gains. However, the structure of the adaptive controller is very complex, compared with that of a normal robust controller, due to the regressors of the dynamic system models and the parameter estimators. In this paper we develop a robust controller whose structure is much simpler than that of the adaptive controller proposed in [6].

II. UVMS MODEL

Consider an underwater vehicle equipped with a Dm link manipulator with revolute joints. Without loss of generality, we assume that Dm = Dp + Do, where Dp and Do are the numbers of translational and rotational dimensions, respectively. As in [1,2], the mathematical model without the thruster dynam-

ics is expressed as

$$M(\phi)\ddot{x}(t) + f(\phi, u) = J(\phi)^{-T} \begin{bmatrix} \bar{R}(\phi)\bar{f}_b(t) \\ \tau_m(t) \end{bmatrix}$$
(1)

where the explanation of the main symbols is shown in Table 1. Most of the controllers reported in the literature of UVMS control are designed for the model (1) without the dynamics of $\bar{f}_b(t)$. In our controller design, the following dynamic model for thrusters is used [7]:

$$\left. \begin{array}{l} \bar{f}_b(t) = \bar{K}D(v)v(t) \\ \dot{v}(t) = -\frac{1}{2}AD(v)v(t) + \frac{1}{2}B\tau_b(t) \end{array} \right\}$$
(3)

$$D(v) = \operatorname{diag}\{|v_1|, \dots, |v_{Dm}|\} \in R^{Dm \times Dm}$$

$$\tag{4}$$

Table 1. Symbols in the models (1) to (4)

- Da Number of dimension Dp + 2Do
- Dn Number of dimension 2(Dp + Do) = 2Dm
- x(t) Signal composed of vehicle's and manipulator end-effector's positions and orientations ($\in R^{Dn}$)
- $\phi(t)$ Signal composed of vehicle's orientation and manipulator's joint angles ($\in R^{Da}$)
- $\begin{array}{ll} u(t) & \mbox{Signal composed of vehicle's translational velocity and} \\ \dot{\phi}(t) \; (\in R^{Dn}) \end{array}$
- $\bar{f}_b(t)$ Thrust forces produced by thruster's propellers $(\in R^{Dm})$
- $\tau_m(t)$ Joint torques of manipulator $(\in \mathbb{R}^{Dm})$
- $\begin{array}{ll} J(\phi) & \mbox{Jacobian matrix in the equation } \dot{x}(t) = J(\phi) u(t) \\ & (\in R^{Dn \times Dn}) \end{array}$
- $\overline{R}(\phi)$ Transformation matrix from $\overline{f}_b(t)$ to force and torque concerning inertial coordinate system ($\in R^{Dm \times Dm}$)
- $\bar{M}(\phi)$ Inertia matrix ($\in R^{Dn \times Dn}$)
- $\bar{f}(\cdot)$ Signal composed of centrifugal, Coriolis, gravitational and buoyant forces, fluid drag and bounded disturbances $(\in R^{Dn})$
- v(t) Shaft velocities of thruster's propellers ($\in \mathbb{R}^{Dm}$)
- $\tau_b(t)$ Shaft torques of thruster's propellers ($\in \mathbb{R}^{Dm}$)
- \vec{A}, \vec{B} , Diagonal matrices composed of thruster's system parameters ($\in R^{Dm \times Dm}$)



Fig. 1. Nonlinearity in thruster dynamics

where the explanation of the main symbols is shown in Table 1, and $v_i(t)$ is the *i*th element of v(t). Fig. 1, where $\bar{f}_{bi}(t)$ is the *i*th element of $\bar{f}_b(t)$, shows the schematic representation of the first equation of (3). As shown in Fig. 1, the thruster model (3) has the dead-zone-like nonlinearity that $\bar{f}_{bi}(t) = 0$ and $d\bar{f}_{bi}(t)/dt = 0$ when $v_i(t) = 0$.

In this paper, the backstepping control technology is used to develop a robust controller. To this end, the state v(t) has to be replaced by the new one $z(t) = D(v)v(t) \in \mathbb{R}^{Dm}$. Combining (1) and (3), and rewriting the signal D(v)v(t) as z(t), we obtain the new representation

$$\left. \begin{array}{l} M(\phi)\ddot{x}(t) + f(\phi, u) = J(\phi)^{-T}R(\phi)K\begin{bmatrix} z(t)\\ \tau_m(t) \end{bmatrix} \right\} \quad (5) \\ \dot{z}(t) = -AD(v)z(t) + BD(v)\tau_b(t) \end{array}$$

$$R(\phi) = \begin{bmatrix} \bar{R}(\phi) & 0\\ 0 & I_m \end{bmatrix}, \quad K = \begin{bmatrix} \bar{K} & 0\\ 0 & I_m \end{bmatrix} \in R^{Dn \times Dn}$$
(6)

where $I_m \in \mathbb{R}^{Dm \times Dm}$ is an identity matrix.

The model (5) has the following properties useful for our controller development [1,7]:

P1) The diagonal elements of A, B and \bar{K} are positive constants, and there exists a positive constant c_B such that $c_B \|\bar{y}\|^2 \leq \bar{y}^T B \bar{y}$ for any $\bar{y} \in R^{Dm}$. **P2**) Each of $J(\phi)$ and $R(\phi)$ is composed of the kine-

P2) Each of $J(\phi)$ and $R(\phi)$ is composed of the kinematic parameters (e.g., length) and the functions of $\phi(t)$. In addition, if each of $J(\phi)$ and $R(\phi)$ has a full rank, then there exists a positive constant c_{RKJ} such that $c_{RKJ} \|\bar{x}\|^2 \leq \bar{x}^T J(\phi)^{-T} R(\phi) K R(\phi)^T J(\phi)^{-1} \bar{x}$ for any $\bar{x} \in R^{Dn}$.

P3) If $J(\phi)$ has a full rank, then $M(\phi)$ is symmetric and positive definite, and there exists a positive constant c_{M1} such that $||M(\phi)|| \le c_{M1}$.

III. CONTROLLER DESIGN

The control objective is to develop a controller so that all signals in the closed loop system are bounded and the state x(t) tracks the desired trajectory $x_r(t)$ under the condition that the dynamic and hydrodynamic parameters (e.g., mass and a drag coefficient) are unknown constants.

In order to meet the objective, we make the following assumptions about the model (5) and the reference model (i.e., the desired trajectory $x_r(t)$):

A1) The signals $\phi(t)$, x(t), u(t) and v(t) are available. **A2**) The kinematic parameters in (5) are known constants.

A3) Each of the matrices $J(\phi)$ and $R(\phi)$ in (5) has a full rank.

A4) The desired trajectory $x_r(t)$ and the derivatives $\dot{x}_r(t)$ and $\ddot{x}_r(t)$ exist and are bounded.



Fig. 2. Controller design procedure

It follows from the property P2 and the assumptions A1 and A2 that $J(\phi)$ and $R(\phi)$ are known matrices, and hence $\dot{x}(t)$ is available by using the equation $\dot{x}(t) = J(\phi)u(t)$.

In the following subsections we develop a controller that achieves the control objective by using a two-step backstepping procedure, as shown in Fig. 2. The first step is the design of a robust controller with the inputs z(t) and $\tau_m(t)$, called robust controller I in this paper. The second step is the design of a robust controller with the input $\tau_b(t)$, called robust controller II in this paper. In this step we first replace z(t) determined in the first step by the desired trajectory $z_r(t)$, and then design the control input $\tau_b(t)$ for the second equation of (5) so that z(t) tracks $z_r(t)$.

1. Robust Controller I

According to the design procedure shown in Fig. 2, we make the following assumption in the design of robust controller I:

A5) The control inputs are z(t) for vehicle control and $\tau_m(t)$ for manipulator control.

In order to achieve the aforementioned control objective, we use the tracking errors

$$\tilde{s}(t) = \dot{\tilde{x}}(t) + \alpha \tilde{x}(t), \quad \tilde{x}(t) = x(t) - x_r(t) \tag{7}$$

where $\alpha > 0$ is a design parameter. Using the first equations of (5) and (7), we have the error models

$$\begin{aligned}
M(\phi)\dot{\tilde{s}}(t) &= J(\phi)^{-T}R(\phi)K\begin{bmatrix}z(t)\\\tau_m(t)\end{bmatrix} - \frac{1}{2}\dot{M}(\cdot)\tilde{s}(t)\\ &+ f_x(t) - \tilde{x}(t)\\ \dot{\tilde{x}}(t) &= -\alpha\tilde{x}(t) + \tilde{s}(t)\\ f_x(t) &= -f(\cdot) + M(\phi)[\alpha\dot{\tilde{x}}(t) - \ddot{x}_r(t)]\\ &+ \frac{1}{2}\dot{M}(\cdot)\tilde{s}(t) + \tilde{x}(t) \in R^{Dn} \end{aligned} \tag{8}$$

and $f_x(t)$ has the following property useful for our controller development:

P4) There exists a positive constant c_x such that

$$\|f_x(t)\| \le c_x \,\omega_x(t) \tag{10}$$
$$\omega_x(t) = 1 + \alpha + \alpha^2$$

$$+(1+\alpha^2)\|\tilde{x}(t)\|^2 + \|u(t)\|^2 \in R.$$
 (11)

The robust control law for the error models (8) subject to the assumptions A1 to A5 is given by

$$\begin{bmatrix} z(t) \\ \tau_m(t) \end{bmatrix} = -\mu_x(t)R(\phi)^T J(\phi)^{-1}\tilde{s}(t)$$
(12)

$$\mu_x(t) = \alpha + \beta_x \,\epsilon^2 + \beta_x \,\omega_x(t)^2 \in R \tag{13}$$

where β_x , $\epsilon > 0$ are design parameters. It is shown that the robust controller (12) guarantees an ultimate boundedness of the tracking error $\tilde{x}(t)$.

2. Robust Controller II

According to the design procedure shown in Fig. 2, we make the following assumption instead of the assumption A5 in the design of robust controller II: **AC**). The control input of f(x) for excitate

A6) The control inputs are $\tau_b(t)$ for vehicle control and $\tau_m(t)$ for manipulator control.

As shown in Fig. 2, we first replace the input z(t) in (12) by the desired trajectory $z_r(t)$, i.e.,

$$\begin{bmatrix} z_r(t) \\ \tau_m(t) \end{bmatrix} = -\mu_x(t)R(\phi)^T J(\phi)^{-1}\tilde{s}(t)$$
(14)

and then design robust controller II by using the tracking error of z(t). When we choose the error as the normal one $\tilde{z}_n(t) = z(t) - z_r(t)$, then the error model is written as $\dot{\tilde{z}}_n(t) = -AD(v)z(t) - \dot{z}_r(t) + BD(v)\tau_b(t)$. This model has a situation where the system is uncontrollable due to lack of the rank of BD(v) when some of $v_i(t)$ equal zero. This situation is caused by the thruster nonlinearities shown in Fig. 1. In order to avoid the situation, we propose the following error instead of the normal one $\tilde{z}_n(t)$:

$$\tilde{z}(t) = z(t) - z_r(t) + 2 \epsilon l(v)$$

$$l(v) = \{I_m - E(v)\} \bar{v}(v) \in R^{Dm} \\
E(v) = \text{diag} \{e^{-|v_1|}, \dots, e^{-|v_{Dm}|}\} \in R^{Dm \times Dm} \\
\bar{v}(v) = [\text{sgn}(v_1), \dots, \text{sgn}(v_{Dm})]^T \in R^{Dm}.$$
(15)
(16)

It should be noted that the signal l(v) is bounded for all v(t). As a result of adding the term l(v), the error model of $\tilde{z}(t)$ is expressed as

$$\dot{\tilde{z}}(t) = BL(v)\tau_b(t) - \bar{I}^T K R(\phi)^T J(\phi)^{-1} \tilde{s}(t) + f_z(t)$$
(17)

$$L(v) = D(v) + \epsilon E(v) \in R^{Dm \times Dm}$$

$$\bar{I} = \begin{bmatrix} I_m \\ 0 \end{bmatrix} \in R^{Dn \times Dm}$$

$$f_z(t) = -AL(v)z(t) + \bar{I}^T K R(\phi)^T J(\phi)^{-1} \tilde{s}(t)$$

$$-\dot{z}_r(t) \in R^{Dm}$$

$$(18)$$

and $f_z(t)$ has the following property useful for our controller development:

P5) There exists a positive constant c_z such that

$$\|f_{z}(t)\| \leq c_{z} \,\omega_{z}(t)$$

$$\omega_{z}(t) = w_{z2}(t) + w_{z3}(t) \|\tilde{s}(t)\| \in R$$

$$w_{z1}(t) = 1 + \|u(t)\|^{2} + \|z(t)\| + \|\tau_{m}(t)\|$$

$$w_{z2}(t) = \left[w_{z1}(t) + \alpha \|\dot{\tilde{x}}(t)\|\right] \mu_{x}(t) + \|L(v)z(t)\|$$

$$w_{z3}(t) = 1 + (1 + \alpha^{2})\beta_{x} \,\omega_{x}(t) \|\tilde{x}(t)\| \|\dot{\tilde{x}}(t)\|$$

$$+ \left[\mu_{x}(t) + \beta_{x} \,\omega_{x}(t) w_{z1}(t)\right] \|u(t)\|.$$

$$(19)$$

It is noteworthy that the coefficient matrix BL(v) of the input $\tau_b(t)$ in the error model (17) has a full rank for all v(t), and hence the error model is controllable in spite of the thruster nonlinearities.

The robust control law for the error model (17) subject to the assumptions A1 to A4 and A6 is given by

$$\tau_b(t) = -\mu_z(t)L(v)^{-1}\tilde{z}(t) \tag{21}$$

$$\mu_z(t) = \alpha + \beta_z \omega_z(t)^2 \in R \tag{22}$$

where $\beta_z > 0$ is a design parameter.

In addition to the error $\tilde{z}(t)$, we use the errors $\tilde{s}(t)$ and $\tilde{x}(t)$, introduced for the design of robust controller I, to guarantee the stability of the overall closed loop system. The error models (8) concerning $\tilde{s}(t)$ and $\tilde{x}(t)$ need to be modified because z(t) in the input (12) for robust controller I is replaced by $z_r(t)$. Using (15), we rewrite (8) as

$$M(\phi)\dot{\tilde{s}}(t) = J(\phi)^{-T}R(\phi)K \begin{bmatrix} z_r(t) \\ \tau_m(t) \end{bmatrix} - \frac{1}{2}\dot{M}(\cdot)\tilde{s}(t) \\ +J(\phi)^{-T}R(\phi)K\bar{I}\tilde{z}(t) + f_x(t) - \tilde{x}(t) \\ -2\epsilon J(\phi)^{-T}R(\phi)K\bar{I}l(v) \\ \dot{\tilde{x}}(t) = -\alpha\tilde{x}(t) + \tilde{s}(t) \end{cases}$$

$$(23)$$

For robust controller I and II, the following theorem holds:

Theorem 1 Consider the robust controller (14) and (21) for the error models (17) and (23) subject to the assumptions A1 to A4 and A6. This controller guarantees that the signals x(t), $\dot{x}(t)$, u(t), v(t), z(t), $z_r(t)$, $\tau_m(t)$ and $\tau_b(t)$ in the closed loop system are bounded, and that the tracking error $\tilde{x}(t)$ satisfies the inequality

$$\|\tilde{x}(t)\|^2 \le \rho_1 \, e^{-\gamma \alpha t} + \frac{\rho_2}{\alpha \, \beta} \tag{24}$$

where ρ_1 and ρ_2 are positive constants, $\beta = \min\{\beta_x, \beta_z\}$, and $\gamma = \min\{2c_{RKJ}/c_{M1}, 2, 2c_B\}$. The inequality (24) in Theorem 1 means that an ul-

The inequality (24) in Theorem 1 means that an ultimate bound of $\tilde{x}(t)$ can be arbitrarily reduced by increasing the design parameters α , β_x and β_z . It should be noted that a high gain controller can be constructed for the model (5), since the slow thruster dynamics is taken into consideration in the design of controller. *Proof:* We first choose the positive definite function

$$V(t) = \frac{1}{2} \left[\tilde{s}(t)^T M(\phi) \tilde{s}(t) + \tilde{x}(t)^T \tilde{x}(t) + \tilde{z}(t)^T \tilde{z}(t) \right] (25)$$

and then the time derivative of V(t) along the solutions of (17) and (23) is given by

$$\dot{V}(t) \le -\gamma \alpha V(t) + \frac{\gamma \rho_2}{2\beta}.$$
(26)

In the derivation of (26), we use the control law (14) and (21), the inequalities in the properties P1 to P3 and the inequalities

$$-\beta_x \,\omega_x(t)^2 \tilde{s}(t)^T H(\phi) \tilde{s}(t) + \tilde{s}(t)^T f_x(t) \le \frac{\rho_3}{2\beta_x} \\ -\beta_x \,\epsilon^2 \tilde{s}(t)^T H(\phi) \tilde{s}(t) \\ -2 \,\epsilon \,\tilde{s}(t)^T J(\phi)^{-T} R(\phi) K \bar{I} l(v) \le \frac{\rho_4}{2\beta_x} \\ -\beta_z \omega_z(t)^2 \tilde{z}(t)^T B \tilde{z}(t) + \tilde{z}(t)^T f_z(t) \le \frac{\rho_5}{2\beta_z} \\ H(\phi) = J(\phi)^{-T} R(\phi) K R(\phi)^T J(\phi)^{-1} \in R^{Dn \times Dn}$$
(28)



Fig. 3. UVMS for numerical simulation

where ρ_3 , ρ_4 and ρ_5 are positive constants. From Lemma 3.2.4 in [8], (26) can be rewritten as

$$V(t) \le e^{-\gamma \alpha t} V(0) + \frac{\rho_2}{2 \alpha \beta}.$$
(29)

It follows from (29) that $\tilde{s}(t)$, $\tilde{x}(t)$, $\tilde{z}(t) \in L^{\infty}$. Using (7), (11), (13) to (16), (20) to (22), the assumption A4 and the inequalities in APPENDIX, we can easily prove that x(t), $\dot{x}(t)$, u(t), v(t), z(t), $z_r(t)$, $\tau_m(t)$ and $\tau_b(t) \in L^{\infty}$. Moreover, we can directly derive the inequality (24) from (29), and the proof is complete.

IV. SIMULATION EXAMPLE

In order to confirm the usefulness of our robust controller (14) and (21), we performed numerical simulation. Typical simulation results are presented in this paper. The UVMS simulated here was an underwater vehicle with a two-link manipulator, as shown in Fig. 3. The values of these system parameters were the same as those used in the reference [6]. In this figure, only the values of the main parameters are shown. Except for α , the controller design parameters were chosen as $\beta_x = 0.003, \beta_z = 0.001, \epsilon = 1$. Each of the desired trajectories of the vehicle's position and the manipulator end-effector's position is set up along a straight path. Each of the velocities is given by a filtered trapezoidal function. On the other hand, the desired trajectory of the vehicle's orientation is selected to remain at the initial value.

Fig. 4 shows the simulation result for $\alpha = 10$. It can be seen from this figure that x(t) (the vehicle's position and orientation and the manipulator end-effector's position) tracks the desired trajectory $x_r(t)$ in spite of the nonlinearities of thruster dynamics and the uncertainties of system parameters.

The simulation where α is selected as various values was carried out. In this paper, the tracking errors for $\alpha = 8, 9, 10$ are shown in Fig. 5. As shown in Fig. 5, the control performance is improved by increasing the design parameter α .

V. CONCLUSION

In this paper we developed a robust controller for underwater vehicle-manipulator systems with thruster dynamics. In the controller development we presented a new tracking error model that overcomes uncontrollability caused by the thruster dynamics. It is, furthermore, shown that all signals in the closed loop system are bounded, and that an ultimate bound of the tracking error can be reduced by increasing controller's design parameters.



Fig. 4. Robot motion for $\alpha = 10$





APPENDIX

Inequalities: The following inequalities, where c_* is a positive constant, are used for the design of controller in this paper:

- (i) $||J(\phi)|| \le c_{J1}, ||\dot{J}(\cdot)|| \le c_{J2}||u(t)||, ||R(\phi)K|| \le c_{RK}, ||R(\phi)|| \le c_{R1}, ||\dot{R}(\cdot)|| \le c_{R2}||u(t)||, ||\dot{u}(t)|| \le c_u w_{z1}(t)$
- (ii) $\|\dot{M}(\cdot)\| \le c_{M2} \|u(t)\|, \|M(\phi)^{-1}\| \le c_{M3}, \|f(\cdot)\| \le c_f (1+\|u(t)\|^2), \|J(\phi)^{-1}\| \le c_{J3} \text{ (if } J(\phi) \text{ has a full rank)}$

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