Model Matching Adaptive Control of Time Delay Systems with Unknown Relative Degree

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Abstract: This paper considers the adaptive control problem of time delay systems with unknown relative degree based on model matching technique. For single-input single-output (SISO) systems, the only known knowledge of the relative degree is the upper bound of it. An adaptive control scheme is designed so that all signals in the close-loop systems are bounded and the tacking error can converge to zero. A simulation example is included to illustrate the proposed adaptive control scheme.

Keywords: Model matching control; adaptive control; time delay; unknown relative degree

I. INTRODUCTION

Time delay exists in many industrial control systems such as chemical process systems, hydraulically actuated systems and combustion systems. Researches have paid much attention to the control of time delay systems since last century. Stability analysis and controller design for delay systems are more difficult than delay-free systems. Many methods have been proposed to deal with time delay systems. The known smith predictor proposed in [1] could cancel the time delay from the characteristic equation of the closed-loop systems. The finite spectrum assignment method in [6] could assign the eigenvalues of the closed-loop plant at arbitrary prescribed place of the complex plane. However, the two methods are difficult to be applied to adaptive control.

Model matching technique can be easily used to adaptive control scheme design. This technique is to design a controller so that the transfer function of the closed-loop plant coincides exactly with the transfer function of the reference model. Controller design for linear systems based on model matching technique can be found in the book [5]. This method was used to controller design for SISO delay systems in [2]. Then the result was extended to adaptive control in [3]. For multivariable delay systems, a general solution of model matching control of multiple-output-delay systems was given in [4]. However, only unknown parameter uncertainty was considered in [3-4]. In this paper, we will consider the adaptive control of time delay systems with unknown relative degree.

This work was supported by the NSFC (60727002, 60774003, 60921001, 90916024), the MOD (20030006 003), the COSTIND (A2120061303), the National 973 program (2005CB321902).

A strict assumption is that the relative degree is exactly known in the adaptive control literature. This assumption was relaxed in [7] for plants with relative degree n^* satisfying $1 \le n^* \le 3$. A new model reference adaptive control (MRAC) scheme was proposed in [8]. Where the control required only the known upper bound of the relative degree, this result was obtained at the expense of additional complexity in the control and adaptive laws. However, to the best of our knowledge, there is few results about time delay systems with unknown relative degree yet. In this paper, a class of SISO delay systems with unknown relative degree is considered. An adaptive control scheme is designed using the model matching technique.

This paper is organized as follows. Section 2 is problem statement. In section 3, an adaptive control scheme is designed for SISO delay systems and the stability analysis is completed. A simulation example is given in section 4 to illustrate the designed scheme. The last section is a conclusion of this paper.

II. PROBLEM STATEMENT

Consider the SISO linear time-invariant time delay systems

$$y(s) = \frac{gr(s)}{p(s)} e^{-Ls} u(s) \tag{1}$$

Where y(t) and u(t) are the output and the input, respectively. g is the gain, L is the known time delay. $r(s)$ and $p(s)$ are monic polynomials with degree m and n respectively, denote $\partial[r(s)]=m, \partial[p(s)]=n$, where
m and n are unknown constants. The following assumptions are made for the plant (1).

Assumption 1: r(s) is a Hurwitz polynomial.

Assumption 2: The upper bound $\bar{n}$ of the unknown degree n of p(s) is known, i.e. $n \le \bar{n}$.

Assumption 3:The relative degree n^* satisfies $1 \le n_l^* \le n^* \le n_u^*$, where n_l^* and n_u^* are known constants.

The reference model is chosen to be

$$y_d(s) = \frac{g_d r_d(s)}{p_d(s)} e^{-Ls} \upsilon(s) \tag{2}$$

Where $p_d(s)$ is a stable polynomial, its degree is $\bar{n}$, the relative degree of the reference model satisfies $\partial[p_d(s)] - \partial[r_d(s)] \ge n_u^*$. The reference input $\upsilon(t)$ is a uniform bounded piecewise continuous signal. The objective is to design an adaptive control scheme so that all signals in the closed-loop systems are bounded and the plant output tracks the reference model output as close as possible for any given reference input.

III. ADAPTIVE CONTROL DESIGN

First, we design the model matching controller structure for the systems (1). The controller design procedure is different from that the known relative degree case.

Choose monic stable polynomials $r^*(s)$ and $p^*(s)$, $\partial[p^*(s)] = \bar{n}$, $\partial[r^*(s)] = \bar{n} - n_l^*$. Then the reference model can be rewritten as

$$y_d(s) = \frac{r^*(s)}{p^*(s)} e^{-Ls} \frac{g_d r_d(s) p^*(s)}{p_d(s) r^*(s)} \upsilon \tag{3}$$

Denote

$$\bar{\upsilon} = \frac{g_d r_d(s) p^*(s)}{p_d(s) r^*(s)} \upsilon \tag{4}$$

Obviously, $\bar{\upsilon}$ is a realizable dynamical input signal.

When both $r^*(s)$ and p(s) have single or disti nct roots, write

$$\frac{r^*(s)p(s) - gr(s)p^*(s)}{r^*(s)p(s)} = \sum_{i=1}^{\bar{n}-n_l^*+n} \frac{\beta_i}{s - z_i} + 1 - \bar{g} \tag{5}$$

where z_i are roots of $r^*(s)$ for $k = 1, 2, \cdots, \bar{n} - n_l^*$ and roots of p(s) for $k = \bar{n} - n_l^* + 1, \cdots, \bar{n} - n_l^* + n$, respectively. Obviously, we have

$$\bar{g} = \begin{cases} g & n^* = n_l^* \\ 0 & n_l^* < n^* \le n_u^* \end{cases} \tag{6}$$

Define the polynomial $\phi(s)$ satisfying the equation

$$\frac{r^*(s)p(s) - \phi(s)}{r^*(s)p(s)} = \sum_{i=1}^{\bar{n}-n_l^*+n} \frac{\beta_i e^{z_i L}}{s - z_i} + 1 - \bar{g} \tag{7}$$

Remark 1: In order to employ the precompensator In (7)

$$\frac{r^*(s)p(s) - gr(s)p^*(s)}{r^*(s)p(s)} \tag{8}$$

should satisfy

$$\partial[r^*(s)] + \partial[p(s)] \ge \partial[r(s)] + \partial[p^*(s)] \tag{9}$$

i.e.

$$\begin{aligned} \partial[r^*(s)] &\ge \partial[p^*(s)] - \partial[p(s)] + \partial[r(s)] \\ &\ge \bar{n} - n^* \\ &\ge \bar{n} - n^*_l \end{aligned} \tag{10}$$

Therefore, the monic stable polynomial $r^*(s)$ should be chosen as $\partial[r^*(s)] \ge \bar{n} - n_l^*$. In this paper, we choose $\partial[r^*(s)] = \bar{n} - n_l^*$.

Define a polynomial equation by

$$k(s)p(s) + gh(s)r(s) = \bar{g}r^*(s)p(s) - \phi(s) \tag{11}$$

where k(s) and h(s) are unknown polynomials.

Theorem 1. There are solutions k(s) and h(s) for the polynomial equation (11) with degrees $\partial[k(s)] \le \bar{n} - n_l^* - 1$ and $\partial[h(s)] \le \bar{n} - 1$.

Proof. From the equation (7), $\bar{g}r^*(s)p(s) - \phi(s)$ is of degree at most $\bar{n} - n_l^* + n - 1$. It is know that there are unique polynomial solutions k(s) and h(s). k(s) of degree at most $\bar{n} - n_l^* - 1$, polynomial h(s) of degree at most n-1. However, the degree n of $p(s)$ is not known, the only knowledge of it is $n \le \bar{n}$. Therefore, h(s) is of degree at most $\bar{n} - 1$. The proof is completed.

Using the equations (5) and (7), we have the following integral

$$\begin{aligned} &\int_{-L}^{0} \sum_{i=1}^{\bar{n}-n_l^*+n} \beta_i e^{-\sigma z_i} u(s) e^{\sigma s} d\sigma \\ &= \sum_{i=1}^{\bar{n}-n_l^*+n} \frac{\beta_i}{s - z_i} u(s) - \sum_{i=1}^{\bar{n}-n_l^*+n} \frac{\beta_i e^{L z_i}}{s - z_i} u(s) e^{-Ls} \\ &= -\frac{gr(s)p^*(s)}{r^*(s)p(s)} u(s) + \bar{g}u(s) + \frac{\phi(s)}{r^*(s)p(s)} u(s) e^{-Ls} \\ &\quad - \bar{g}u(s) e^{-Ls} \end{aligned} \tag{2}$$

Using the equation (11), the above equation (12) can be rewritten as

$$\int_{-L}^{0} \sum_{i=1}^{\bar{n}-n_l^*+n} \beta_i e^{-\sigma z_i} u(s) e^{\sigma s} d\sigma$$

$$= -\frac{gr(s)p^*(s)}{r^*(s)p(s)}u(s) - \frac{k(s)}{r^*(s)}u(s)e^{-Ls} - \frac{h(s)}{r^*(s)}y(s) + \bar{g}u(s) \quad (13)$$

Another delay compensator is needed to design the controller. Choose any monic stable polynomial $\bar{r}(s)$ with degree $\partial[\bar{r}(s)] = \partial[r^*(s)] = \bar{n} - n_l^*$, then we have

$$-\frac{g\bar{r}(s)}{r^*(s)} = -g + \sum_{i=1}^{\bar{n}-n_l^*} \frac{\bar{\beta}_i}{s - z_i} \quad (14)$$

Define the polynomial $\bar{\phi}(s)$ by

$$-\frac{\bar{\phi}(s)}{r^*(s)} = -g + \sum_{i=1}^{\bar{n}-n_l^*} \frac{\bar{\beta}_i e^{z_i L}}{s - z_i} \quad (15)$$

Using the equations (14) and (15), we can obtain another integral

$$\int_{-L}^{0} \sum_{i=1}^{\bar{n}-n_l^*} \bar{\beta}_i e^{-\sigma z_i} u(s) e^{\sigma s} d\sigma = -\frac{g\bar{r}(s)}{r^*(s)}u(s) + gu(s) + \frac{\bar{\phi}(s)}{r^*(s)}u(s)e^{-Ls} - gu(s)e^{-Ls} \quad (16)$$

Combining the two integrals in the equation (13) and (16) yields

$$\int_{-L}^{0} \sum_{i=1}^{\bar{n}-n_l^*+n} \beta_i e^{-\sigma z_i} u(s) e^{\sigma s} d\sigma + \int_{-L}^{0} \sum_{i=1}^{\bar{n}-n_l^*} \bar{\beta}_i e^{-\sigma z_i} u(s) e^{\sigma s} d\sigma = -\frac{gr(s)p^*(s)}{r^*(s)p(s)}u(s) - \frac{k(s)}{r^*(s)}u(s)e^{-Ls} - \frac{h(s)}{r^*(s)}y(s) - \frac{g\bar{r}(s)}{r^*(s)}u(s) + \frac{\bar{\phi}(s)}{r^*(s)}u(s)e^{-Ls} + \bar{g}u(s) + gu(s) - gu(s)e^{-Ls} \quad (17)$$

Therefore, we can choose the controller u as

$$u(s) = \frac{1}{g}\Big\{\frac{k(s)}{r^*(s)}u(s)e^{-Ls} + \frac{h(s)}{r^*(s)}y(s) + \frac{g\bar{r}(s)}{r^*(s)}u(s) - \frac{\bar{\phi}(s)}{r^*(s)}u(s)e^{-Ls} + \int_{-L}^{0} \sum_{i=1}^{\bar{n}-n_l^*+n} \beta_i e^{-\sigma z_i} u(s) e^{\sigma s} d\sigma + \int_{-L}^{0} \sum_{i=1}^{\bar{n}-n_l^*} \bar{\beta}_i e^{-\sigma z_i} u(s) e^{\sigma s} d\sigma - \bar{g}u(s) + gu(s)e^{-Ls} + \bar{\upsilon}(s)\Big\} \quad (18)$$

Substituting the controller in the equation (18) into the equation (17), we can obtain

$$\frac{gr(s)p^*(s)}{r^*(s)p(s)}u(s) = \bar{\upsilon} \quad (19)$$

It is noting that the polynomials $p^*(s)$ and $r^*(s)$ are chosen to be stable polynomials. Therefore, the above equation (19) can be further rewritten as

$$\frac{gr(s)}{p(s)}e^{-Ls}u(s) = \frac{r^*(s)}{p^*(s)}e^{-Ls}\bar{\upsilon} \quad (20)$$

The system output is equal to the reference model output. Therefore, the controller in (18) is the desired controller. In time domain, the controller can be rewritten as

$$u(t) = \frac{1}{g}\Big\{\frac{k(p)}{r^*(p)}u(t-L) + \frac{h(p)}{r^*(p)}y(t) + \frac{g\bar{r}(p)}{r^*(p)}u(t) - \frac{\bar{\phi}(p)}{r^*(p)}u(t-L) + \int_{-L}^{0} \sum_{i=1}^{\bar{n}-n_l^*+n} \beta_i e^{-\sigma z_i} u(t+\sigma) d\sigma + \int_{-L}^{0} \sum_{i=1}^{\bar{n}-n_l^*} \bar{\beta}_i e^{-\sigma z_i} u(t+\sigma) d\sigma - \bar{g}u(t) + gu(t-L) + \bar{\upsilon}(t)\Big\} \quad (21)$$

where p is the differential operator in time domain. Because $\partial[\bar{\phi}(s)] \le \bar{n} - n_l^*$, $\partial[k(s)] \le \bar{n} - n_l^* - 1$, and $\partial[h(s)] \le \bar{n} - 1$. Thus, they can be written as

$$\bar{\phi}(s) = q_{\bar{n}-n_l^*}s^{\bar{n}-n_l^*} + q_{\bar{n}-n_l^*-1}s^{\bar{n}-n_l^*-1} + \cdots + q_0,$$
$$k(s) = k_{\bar{n}-n_l^*-1}s^{\bar{n}-n_l^*-1} + k_{\bar{n}-n_l^*-2}s^{\bar{n}-n_l^*-2} + \cdots + k_0, \quad (22)$$
$$h(s) = h_{\bar{n}-1}s^{\bar{n}-1} + h_{\bar{n}-2}s^{\bar{n}-2} + \cdots + h_0$$

respectively, where the coefficients of the polynomi als are unknown constants. Define parameter vector

$$\theta = \frac{1}{g}\big[k_{\bar{n}-n_l^*-1}, \cdots, k_0, \cdots, h_{\bar{n}-1}, \cdots, h_0, q_{\bar{n}-n_l^*}, \cdots, q_0, \bar{g}, g, 1\big]^T,$$
$$\lambda = \sum_{i=1}^{\bar{n}-n_l^*+n} \beta_i e^{-\sigma z_i} + \sum_{i=1}^{\bar{n}-n_l^*} \bar{\beta}_i e^{-\sigma z_i} \quad (23)$$

and the signal vector

$$\omega(t) = \Big[\frac{p^{\bar{n}-n_l^*-1}}{r^*(p)}u(t-L), \cdots, \frac{1}{r^*(p)}u(t-L), \frac{p^{\bar{n}-1}}{r^*(p)}y(t), \cdots, \frac{1}{r^*(p)}y(t), -\frac{p^{\bar{n}-n_l^*}}{r^*(p)}u(t-L), \cdots, -\frac{1}{r^*(p)}u(t-L), -u(t), \frac{\bar{r}(p)}{r^*(p)}u(t) + u(t-L), \bar{\upsilon}(t)\Big]^T \quad (24)$$

Then the controller (22) can be represented as

$$u(t) = \hat{\theta}^T(t)\omega(t) + \int_{-L}^{0} \hat{\lambda}(t,\sigma)u(t+\sigma)d\sigma \quad (25)$$

where $\hat{\theta}(t), \hat{\lambda}(t,\sigma)$ are the estimates of the real parameters $\theta, \lambda(\sigma)$, respectively. Define the tracking error by

$$e(t) = y(t) - y_d(t) \quad (26)$$

Theorem 2. The tracking error can be represent ed by the following equation

$$e(t) = g\frac{r^*(p)}{p^*(p)}q^{-L}\Big\{\tilde{\theta}^T(t)\omega(t) + \int_{-L}^{0} \tilde{\lambda}(t,\sigma)u(t+\sigma)d\sigma\Big\} \quad (27)$$

where $\tilde{\theta}(t) = \hat{\theta}(t) - \theta$, $\tilde{\lambda}(t,\sigma) = \hat{\lambda}(t,\sigma) - \lambda(\sigma)$. q^{-L} denotes a time delay operator, $q^{-L}u(t) = u(t-L)$.

In order to design the adaptive law of the controller parameters, define a signal by

$$\eta=\left\{\tilde{\theta}^{T}(t)\bar{\omega}(t)+\int_{-L}^{0}\tilde{\lambda}(t,\sigma)\bar{u}(t+\sigma)d\sigma\right\} \\ -\frac{r^{*}(p)}{p^{*}(p)}q^{-L}\left\{\tilde{\theta}^{T}(t)\omega(t)+\int_{-L}^{0}\tilde{\lambda}(t,\sigma)u(t+\sigma)d\sigma\right\} \qquad (28)$$

where

$$\bar{\omega}(t)=\frac{r^{*}(p)}{p^{*}(p)}\omega(t-L),\ \bar{u}(t)=\frac{r^{*}(p)}{p^{*}(p)}u(t-L) \qquad (29)$$

The augmented error is defined by

$$\varepsilon(t)=e(t)+\hat{g}(t)\eta(t) \\ =g\left\{\tilde{\theta}^{T}(t)\bar{\omega}(t)+\int_{-L}^{0}\tilde{\lambda}(t,\sigma)\bar{u}(t+\sigma)d\sigma\right\} \\ +\tilde{g}(t)\eta(t) \qquad (30)$$

Define a signal by

$$\Omega(t)=[\bar{\omega}(t),\ \sup_{-L\le\sigma\le 0}\bar{u}(t+\sigma),\eta(t)]^{T} \qquad (31)$$

Choose the following adaptive law

$$\dot{\hat{g}}(t)=-\alpha\frac{\eta(t)}{1+\|\Omega(t)\|^{2}}, \\ \dot{\hat{\theta}}(t)=-\beta\frac{\bar{\omega}(t)}{1+\|\Omega(t)\|^{2}}, \qquad (32) \\ \dot{\hat{\lambda}}(t,\sigma)=-\gamma\frac{\bar{u}(t+\sigma)}{1+\|\Omega(t)\|^{2}},\quad -L\le\sigma\le 0,$$

where α,β,γ are positive constant parameters to be chosen.

Theorem 3: The adaptive control scheme consi sts of the controller (25) and the adaptive law (32) designed for the plant (1), it can guarantee that al l signals in the closed-loop plant are bounded and t he tracking error converges to zero.

Proof. Without loss of generality, g is assumed to be positive. Consider a Lyapunov function

$$V(t)=\frac{1}{2\alpha}\tilde{g}^{2}(t)+\frac{g}{2\beta}\tilde{\theta}^{T}(t)\tilde{\theta}(t)+\frac{g}{2\gamma}\int_{-L}^{0}\tilde{\lambda}^{2}(t,\sigma)d\sigma \qquad (33)$$

The proof procedure is similar to the proof of the theorem 7 in [5]. Therefore, it is not detailed here.

IV. SIMULATION EXAMPLE

Consider a SISO time delay system in the form of (1), the known knowledge is $\bar{n}=5$, $n_{l}^{*}=1$, and $n_{u}^{*}=3$. The reference model is chosen to be

$$t_{d}(s)=\frac{1}{s^{5}+9s^{4}+31s^{3}+51s^{2}+40s+12}e^{-3s} \qquad (34)$$

Choose the monic stable polynomials

$$p^{*}(s)=s^{5}+10s^{4}+38s^{3}+68s^{2}+57s+18, \\ r^{*}(s)=s^{4}+10s^{3}+35s^{2}+50s+24, \qquad (35) \\ \bar{r}(s)=s^{4}+3.6s^{3}+3.85s^{2}+1.35s+0.1$$

When the plant model is

$$y(s)=\frac{2(s^{2}+4.5s+2)}{s^{4}+8.5s^{3}+21.5s^{3}+21.5s^{2}+7.5}e^{-3s}u(s) \qquad (36)$$

When the reference input is the unit step signal, use the adaptive control scheme designed in this paper and the parameters $\alpha=\beta=\gamma=2$, the simulation result is given in Figure 1. Figure 1 shows that the tracking error converges to zero. The designed adaptive control scheme achieves the control objective.

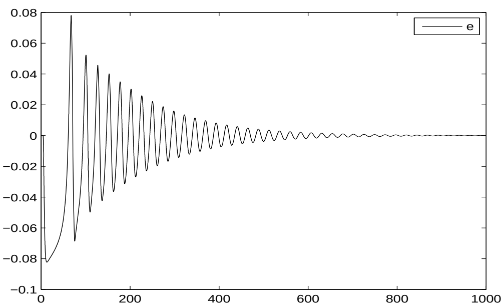

Figure 1. Tracking error of the system.

V. CONCLUSION

This paper design adaptive control schemes for delay systems with unknown relative degree. The schemes are obtained at the expense of updating more parameters than the case of known relative degree.

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