Switching synchronization in a heterogeneous agent network

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Abstract: Motivated by the fact that many real-world networks exhibit a mixture feature of time-invariant and time-varying topologies, we propose a heterogeneous agent network as a simple representation. The presented network consists of two types of nodes: fixed agents and mobile agents, where the connections between fixed agents are constant, while the mobile agents, abstracted as random walkers in plane, interact with the neighboring agents. Under the assumption of fast-switching constraint, we further explore synchronized behavior in the heterogeneous network. The theoretical and numerical results show that the mobile agent density determines synchronization of the considered heterogeneous network. In particular, compared with the network constructed by the fixed agents, synchronizability is enhanced and a global synchronization appears by introducing a proper mobile agent density.

Keywords:Synchronization, switching topology, mobile agent networks.

I. INTRODUCTION

Synchronization in large-scale networks of coupled chaotic oscillators has been intensively investigated in recent years. It has been demonstrated that two or more chaotic oscillators can synchronize by mutual couplings among them, thus a particular interest in this concern is how the network topology influences the propensity of the coupled individuals to synchronize [1-2]. The master stability function (MSF) appraoch relates the stability of the fully synchronized state to the spectral properties of the underlying topological structure, and further provides a framework of analyzing the stability of synchronous state of large populations of identical oscillators [3]. Synchronizability of such a network is then explored based on the notions of MSF and synchronized region.

So far, most investigations have been established on static networks, partially because of a successful performance of the MSF approach in dealing with synchronization problems of static networks. Having examined a variety of network topologies in reality, the classical static network is very restrictive and only reflects a few practical situations. Then the case of connections which do evolve in time is more realistic to depict complex networks, and various synchronization results have been deduced for complex networks with switching topology [4-7]. Of particular interest is synchronization of a set of mobile agents. The mobile agent network, indeed, can be used to explore many problems such as clock synchronization in mobile robots [8], swarming animals or the appearance of synchronized bulk oscillations [9], consensus problem in multi-agent systems [10] and so on.

None of two cases above mentioned, i.e. static network model and mobile agent network model, however, also seems to be an adequate description of many relevant phenomena. For instance, in social networks, lobby groups go about inducing voters whose attitudes are already interacted by a fixed relationship to elect a candidate or to give up an initial view, where the lobbies can be depicted by mobile agents, and static topology seems to be more suitable to characterize interactions between voters [11]; in communication systems, a mobile wireless network attaches the physical network by clock synchronization so that data is transmitted and processed [12]; and in volleyball, the libero as a mobile agent influences the whole team cohesion, while other players share relatively fixed connections. Roughly speaking, there are two types of nodes in all systems above mentioned, and the network corresponding to the system can be decomposed into a (relative) static subnetwork and a switching subnetwork due to the heterogeneity of nodes. Then there appears a question: Is synchronization of the heterogeneous network easier to achieve or not under the existence of mobile agents. There is no doubt, lobbies in social networks, mobile wireless sensors, or volleyball libero player, seem to work as the role of pinned nodes — guiding their neighbor nodes towards the desired objective — in synchronizing a complex network of coupled systems through pinning. This paper is an attempt to explore synchronized behavior based on a heterogeneous agent network model. In this paper, we present a heterogeneous agent network model to characterize a mixture feature of real-world network. Under fast-switching constraints, we investigate the synchronization problem of the heterogeneous agent network. Particularly, we focus on the effect of mobile agents to synchronization of the static network, which provides an insight into regulatory mechanisms and design of complex systems.

II. A HETEROGENEOUS AGENT NETWORK MODEL

Generally, a complex network consisting of l linearly and diffusively coupled nodes is described by

$$\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i) - \sigma \sum_{j=1}^{l} G^s_{ij} H \mathbf{x}_j, \; i = 1, 2, \cdots, l, \tag{1}$$

where $\mathbf{x}_i = (x_{i1}, x_{i2}, \cdots, x_{in})^{\mathrm{T}} \in R^n$ is the state vector of node i, $\mathbf{f}(\cdot): R^n \to R^n$ is a smooth vector-valued function, governing the dynamics of each isolated node, $\sigma > 0$ is the overall coupling strength, $H \in R^{n\times n}$ is the inner linking matrix, and the coupling matrix $G^s = (G^s_{ij}) \in R^{l\times l}$ is a zero-row sum matrix, describing network topology. If network (1) is connected in the sense of having no isolated clusters and edges signify the bidirectional ability, then G^s is symmetric and all its eigenvalues are ranked as $0 = \mu_1 < \mu_2 \le \cdots \le \mu_l$, where eigenratio $R^s = \mu_l/\mu_2$ is used to measure network synchronizability.

To obtain a heterogeneous agent network, we first assign the l nodes in network (1) to be fixed agents. For simplicity, we denote by $\mathscr{N}_l$ the set of fixed agents, and all agents in $\mathscr{N}_l$ are uniformly distributed in a two-dimensional space of size L with periodic boundary conditions . Moreover, we introduce m mobile agents to the plane, each of which is considered as a random walker whose position and orientation are updated according to

$$\begin{cases} \mathbf{y}_i(t+\Delta t) = \mathbf{y}_i(t) + \mathbf{v}_i(t)\Delta t, \\ \theta_i(t+\Delta t) = \eta_i(t+\Delta t), \end{cases} \tag{2}$$

where $i \in \mathscr{N}_m$, $\mathscr{N}_m$ is the set of mobile agents, $\mathbf{y}_i(t)$ is the position of agent i in the plane at time t, $\eta_i(t)$, $i \in \mathscr{N}_m$ are independent random variables chosen at each time unit with uniform probability from the interval $[-\pi, \pi]$, $\mathbf{v}_i(t)$ is the velocity of agent i, and Δt is the time unit. In the following, assume that the time unit is sufficiently small so that fast-switching synchronization is guaranteed. Similar to Ref.[5], each agent of the heterogeneous network is associated with a chaotic oscillator whose state variable is characterized by $\mathbf{x}_i \in R^n$. Then agent i evolves according to $\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i)$. Without lack of generality, we consider the case of Rössler oscillators, where the state dynamics of each agent is given by $\dot{x}_{i1} = -\left(x_{i2} + x_{i3}\right)$, $\dot{x}_{i2} = x_{i1} + a x_{i2}$, $\dot{x}_{i3} = b + x_{i3}\left(x_{i1} - c\right)$ with $\mathbf{x}_i = (x_{i1}, x_{i2}, x_{i2})^{\mathrm{T}}$, and $a = 0.2$, $b = 0.2$, $c = 7$.

It is obvious that, the heterogeneous agent network of order N can be conveniently described by graph $\mathscr{G} = \{\mathscr{N}, \varepsilon\}$, where $\mathscr{N} = \mathscr{N}_l \cup \mathscr{N}_m$ is the node set (representing N agents) and $\varepsilon \subset \mathscr{N} \times \mathscr{N}$ is the edge set of the graph, which is defined as: Each mobile agent, $i \in \mathscr{N}_m$, interacts at a given time with only those agents located within a neighborhood of an interaction radius according to the rule of moving neighborhood network. In detail, agents i and j are said to be adjacent if and only if

$$|\mathbf{y}_i(t) - \mathbf{y}_j(t)| < r, \; \forall i \in \mathscr{N}_m, j \in \mathscr{N} \tag{3}$$

at time t, where r is a parameter that defines the size of a neighborhood, $|\cdot|$ refers to an induced norm. For any two fixed agents, denoted by $i, j \in \mathscr{N}_l$, the connection between them is a constant, i.e., $G_{ij} = G^s_{ij}$. In other words, the constant matrix G^s describes the topology of network $\mathscr{G}_l = \{\mathscr{N}_l, \varepsilon\}$. Hence, we construct a heterogeneous agent network by combining fixed and mobile agents, chaotic oscillators and their dynamical laws, where the heterogeneous couplings include the time-invariant connections between nodes in $\mathscr{N}_l$ and the switching connections due to the moving of agents.

Based on above assumptions, the heterogeneous network can be mathematically formulated as:

$$\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i) - \sigma \sum_{j=1}^{N} G_{ij}(t) H \mathbf{x}_j, \; i \in \mathscr{N}, \tag{4}$$

where the entries of the coupling matrix $G(t) = (G_{ij}(t)) \in R^{N\times N}$ are as follows: for non-diagonal entries, $G_{ij}(t) = G^s_{ij}$ if $i, j \in \mathscr{N}_l$, and $G_{ij}(t) = G_{ji}(t) = -1$ if agent $i \in \mathscr{N}_m$ or agent $j \in \mathscr{N}_m$ are adjacent at time t; and the diagonal entries satisfy $G_{ii}(t) = -\sum_{j=1, j\neq i}^{N} G_{ij}(t)$. Thus there exists a completely synchronized state in network (4), i.e., $\mathbf{x}_1(t) = \mathbf{x}_2(t) = \cdots = \mathbf{x}_N(t) = \mathbf{s}(t)$, which is a solution of an isolated node $\dot{\mathbf{s}} = \mathbf{f}(\mathbf{s})$.

III. ANALYSIS OF SWITCHING SYNCHRONIZATION

This section investigates the synchronized behavior of the heterogeneous network under the constraint of fast-switching. As shown in Ref.[4], Stilwell et al. consider a switching network topology of coupled chaotic oscillators and provide a fast-switching synchronization criterion. Following this result, we will show that synchronization of network (4) can be also assessed by a particular static network.

We first give an average of $G(t)$ for network (4). Consider an infinite sequence of contiguous time intervals $[t_k, t_{k+1})$, $k = 0, 1, \cdots$, with $t_0 = 0$ and $t_{k+1} - t_k = \Delta t$. It is easy to see that $G(t)$ is a constant matrix for any $t \in [t_k, t_{k+1})$ and $k = 0, 1, \cdots$. For simplicity, let G^k be the constant coupling matrix at k-th interval $[t_k, t_{k+1})$, then we derive the average of coupling matrix $G(t)$ satisfying

$$\overline{G} = \sum_{i=1}^{o} p_i G^i, \tag{5}$$

where p_i is the probability that topological configuration i occurs, o is the number of possible configurations.

Recalling the evolution of network (4), we learn that the connections between any fixed agents are time-invariant, i.e., $\overline{G}_{ij} = G_{ij}(t) = G^s_{ij}$, $\forall i, j \in \mathscr{N}_l$. And for other cases, $\overline{G}_{ij} = \sum_{i=1}^{m} p_i G^i_{ij}$, $i \in \mathscr{N}_m$ or $j \in \mathscr{N}_m$. Therefore, we write down the non-diagonal entries of $\overline{G}$ for network (4):

$$\overline{G}_{ij} = \begin{cases} G^s_{ij}, & \textit{if } i, j \in \mathscr{N}_l \\ -p, & \textit{otherwise} \end{cases} \tag{6}$$

where $p = \pi r^2/L^2$ is the probability that two agents are neighbors, I_l is an $l \times l$ identity matrix. By elementary transformation, we calculate the N eigenvalues of the average Laplacian $\overline{G}$ as

$$\lambda_i = \left\{0, \mu_j + mp, \underbrace{pN, \cdots, pN}_{m}, j = 2, \cdots, l\right\}. \quad (7)$$

Note that $\overline{G}$ is a symmetric constant matrix, then synchronization of switching network (4) can be investigated by the network reading

$$\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i) - \sigma \sum_{j=1}^{N} \overline{G}_{ij} H \mathbf{x}_j. \quad (8)$$

Let $\mathbf{e}_i$ be the variation on the i-th node and $\mathbf{e} = \left(\mathbf{e}_1^{\mathrm{T}}, \mathbf{e}_2^{\mathrm{T}}, \cdots, \mathbf{e}_N^{\mathrm{T}}\right)^{\mathrm{T}}$ be the collection of variations. Then linearizing network (8) at $\mathbf{x}_i = \mathbf{s}$ yields $\dot{\mathbf{e}} = \left[I_N \otimes \mathbf{J}_f(\mathbf{s}) - \sigma\overline{G} \otimes H\right]\mathbf{e}$, where $\mathbf{J}_f$ is the Jacobian of the function $\mathbf{f}$ evaluated at $\mathbf{s}(t)$, and $\otimes$ stands for the Kronecker product. It is easy to verify that the linear stability of the synchronized state $\mathbf{s}(t)$ for network (8) can be studied by diagonalizing the variational equations of network (8) into N blocks of the form

$$\dot{\xi}_i = (\mathbf{J}_f - \sigma\lambda_i H)\xi_i, \ i = 1, \cdots, N, \quad (9)$$

where $\xi_i = (U \otimes I_n)\mathbf{e}_i \in R^n$, $U \in R^{N\times N}$ is a unitary matrix such that $U^{\mathrm{T}}\overline{G}U = diag(\lambda_1, \cdots, \lambda_N)$. Furthermore, the synchronized state $\mathbf{s}(t)$ is stable if the Lyapunov exponents for the N blocks are negative i.e., $\Gamma(\sigma\mu_i) < 0$ for $i = 2, \cdots, N$. For the coupled Rössler oscillator with $H = \mathrm{diag}(1,0,0)$, there is a single interval (γ_1, γ_2), in which the largest Lyapunov exponent is negative, where γ_1 and γ_2 are constant. Therefore, synchronization of network (4) can be guaranteed by

$$\frac{\gamma_1}{\sigma} < \lambda_2 < \lambda_N < \frac{\gamma_2}{\sigma}, \quad (10)$$

where λ_2 and λ_N are the second smallest and largest eigenvalues of matrix $\overline{G}$, respectively. Based on Eq.(7) and Eq.(10), we thus derive the synchronization condition for the heterogeneous network.

IV. DISCUSSIONS AND NUMERICAL SIMULATIONS

The existence of mobile agents affects the eigenvalues of $\overline{G}$, which further plays an important role in synchronizing network (4). It is noted that Eq.(10) is fulfilled for some values of σ when the eigenratio R satisfies the following inequality $R \equiv \frac{\lambda_N}{\lambda_2} < \frac{\gamma_2}{\gamma_1}$. We then characterize, similarly to the definition in static network, the synchronizability of network (4) with R.

Compared with the static network (1), the heterogeneous network (4) shows a better synchronizability if $R < R^s$. By solving this inequality, we derive $\rho_m > \rho_m^c$, where ρ_m^c is a critical value of ρ_m satisfying

$$\rho_m^c = \frac{R^s}{R^s - 1} \cdot \max\{0, \frac{\mu_2}{\kappa\rho_l} - 1, \frac{1}{R^s} - \frac{\mu_2}{\kappa\rho_l}\}, \quad (11)$$

$\rho_l = l/L^2$ is the fixed agent density and $\rho_m = m/L^2$ is the mobile agent density. Namely, a smaller ρ_m probably means the heterogeneous network (4) is harder to achieve synchronization from the point of view of the interval width in Eq.(10), while synchronization is probably easier to realize by assigning a larger mobile agent density.

Though a larger mobile agent density ρ_m means a better synchronizability of network (4), it is likely to lead to the largest eigenvalue of $\overline{G}$ over the upper bound in Eq.(10). And synchronization is lost with a large mobile agent density for a particular heterogeneous network. According to Eq.(10), an upper bound of ρ_m is given by

$$\rho_m < \rho_m^u = \frac{\gamma_2}{\sigma\kappa} - \max\{\rho_l, \frac{\mu_l}{\kappa}\}. \quad (12)$$

An obvious result is that, no matter what value of ρ_m is, the heterogeneous network (4) is not synchronizable about synchronized state in the condition of $\mu_l > \gamma_2/\sigma$, where the expression $\mu_l > \gamma_2/\sigma$ implies a nonsynchronized motion of static network (1). It is not difficult to see the existence of mobile agents fails to synchronize the considered heterogeneous network. As a result, we favor the introduction of mobile agents for those static networks (1) whose eigencoupling $\sigma\mu_l$ is located in the negative region of the MSF. In the following, we always assume that $\sigma\mu_l < \gamma_2$ holds.

Similarly, we derive a lower bound of mobile agent density from Eq.(10), i.e.,

$$\rho_m > \rho_m^l = \frac{\gamma_1}{\sigma\kappa} - \min\{\rho_l, \frac{\mu_2}{\kappa}\}. \quad (13)$$

To validate our theoretical findings, we consider the static network G^s to be the case of a Barabási-Albert (BA) network [13], where the parameters of BA model are given by $m_0 = m = 3$, and the degree distribution follows a power law. Fig.1 reports a numerical simulation, where synchronization error $\delta x(t) = (\sum_{i=2}^{N} ||\mathbf{x}_i - \mathbf{x}_1||)/N$. As explained in Fig.1, the considered network achieves synchronization again when $\rho_m > 0.2$. Also notice that a synchronized motion disappears as $\rho_m > 0.5$ due to the bounded synchronization region.

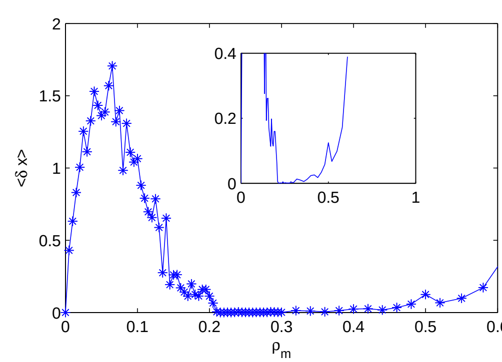

Fig.1. Synchronization index $< \delta x >$ vs mobile agent density ρ_m.

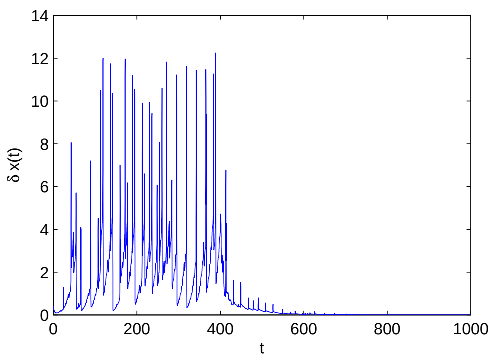

Fig.2. Synchronization index $<\delta x>$ vs mobile agent density ρ_m.

Under the other case $\sigma\mu_2 < \gamma_1$, it has been shown by MSF that static network (1) cannot synchronize. However, synchronization can be easily realized by introducing some, even only one mobile agent to network (1) according to Eq.(13). A numerical example is given in Fig.2 to validate the analytical result, where μ_2 is assigned to be zero. It is obvious that $\mathscr{G}_l$ is an unconnected graph, then a global synchronization of static network (1) cannot be accessed due to isolated clusters in $\mathscr{G}_l$. We observe from Fig.2 that adding several mobile agents (in simulations, five mobile agents are introduced to network at $t = 400s$) can guarantee network synchronization. The role of mobile agents works as a bridge which creates connections among different isolated clusters.

From above discussions, there does exist a bounded region of mobile agent density: synchronization of network (4) is ensured if and only if $\rho_m \in (\rho_m^l, \rho_m^u)$. For a particular static network (1) with given G^s and ρ_m, a too large or a too small mobile agent density will prevent heterogeneous network (4) from achieving synchronization.

V. CONCLUSIONS

In this paper, we propose a heterogenous agent network to capture a mixture feature of time-invariant and time-varying topologies existing in many real-world complex network. The heterogeneous network consists of a certain number of mobile agents and fixed agents, each of which is equipped with a chaotic oscillator in a planar space. In particular, the connections between fixed agents are assigned to be time-invariant, and the mobile agents, abstracted as random walkers, interact with the neighboring agents. Then the heterogenous agent network can be simply regarded as a mixture of a static subnetwork and a switching subnetwork. Under the constraint of fast-switching, we theoretically and numerically show that synchronization of the heterogeneous network depends on the mobile agent density, the fixed agent density and the spectrum of time-invariant subnetwork. For a given heterogeneous network, synchronization motion can be established if mobile agent density of the network lies in an bounded interval, in which its two end-points are determined by the fixed agent density and the static topology. It is worth noting that, compared with the static network, synchronizability can be enhanced when a proper density of mobile agents is introduced to the heterogeneous network. All these results may provide some insights for the future theoretical investigations and practical engineering designs.

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