Distributed Robust Consensus Control of Uncertain Multi-Agent Systems

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Abstract: This paper is devoted to the robust consensus control of multi-agent systems with model parameter uncertainties and external disturbances for networks with switching topology. In particular, a sufficient condition for the consensus performance with a given H_∞ disturbance attenuation level is established for the multi-agent system governed by general linear differential equations, and meanwhile the unknown feedback matrix of the proposed distributed state feedback protocol is determined. The condition is given in terms of linear matrix inequalities (LMIs) and can be easily verified. A numerical example is included to validate the theoretical results.

Keywords: Consensus, Robust H_∞ control, Multi-agent systems, External disturbances, Model uncertainties

I. INTRODUCTION

Recently, some researchers have studied the consensus problem of multi-agent systems with various external disturbances and random communication noises [1]-[6]. However, unavoidable model and parameter uncertainties in the agents' dynamics have not been considered in the existing literature. This motivates us to investigate the consensus problem for multi-agent systems with both model uncertainties and external disturbances.

In this paper, we study the consensus control for networks of multiple agents modeled by general high-dimensional linear differential equations with both model uncertainties and external disturbances, and propose a distributed protocol with an undetermined state feedback matrix. In order to use the existing robust H_∞ theory of linear systems, a controlled output is defined to reformulate the consensus control problem as a robust H_∞ control problem, and a series of model transformations are conducted to convert the original singular closed-loop system to be an equivalent stabilized reduced-order one. Then, sufficient conditions in terms of LMIs are derived to ensure the consensus performance with a given H_∞ index for the disturbed multi-agent system without and with model uncertainties respectively, and the feedback matrix of the proposed protocol is determined accordingly.

II. PROBLEM REFORMULATION AND PROTOCOL DESIGN

A. Problem statement and preliminaries

Consider a multi-agent system consisting of n identical agents with the ith one modeled by

$$\dot{x}_i(t) = Ax_i(t) + B_1\omega_i(t) + B_2u_i(t), \tag{1}$$

where $x_i(t) \in \mathbb{R}^m$ is the state, $u_i(t) \in \mathbb{R}^{m_2}$ is the control input or protocol, and $\omega_i(t) \in \mathbb{R}^{m_1}$ is the external disturbance that belongs to $\mathscr{L}_2[0,\infty)$, the space of square-integrable vector functions over $[0,\infty)$. If system matrices A, B_1, B_2 are uncertain, they are assumed to take the following forms:

$$A = A_0 + \Delta A(t), B_1 = B_{10} + \Delta B_1(t), B_2 = B_{20} + \Delta B_2(t), \tag{2}$$

where A_0, B_{10}, B_{20} are constant matrices, and $\Delta A(t)$, $\Delta B_1(t)$, $\Delta B_2(t)$ are time-varying uncertain matrices satisfying

$$[\Delta A(t)\ \Delta B_1(t)\ \Delta B_2(t)] = E\Sigma(t)[F_1\ F_2\ F_3]. \tag{3}$$

In (3), E and F_i $(i = 1,2,3)$ are constant matrices of appropriate dimensions, and $\Sigma(t)$ is an unknown time-varying matrix that satisfies $\Sigma^{\mathrm{T}}(t)\Sigma(t) \le I$. It is also assumed that (A_0, B_{20}) is stabilized. A protocol $u_i(t)$ is said to asymptotically solve the consensus problem, if and only if the states of agents satisfy

$$\lim_{t\to\infty}[x_i(t) - x_j(t)] = \mathbf{0},\ \ \forall i,j \in \{1,\cdots,n\} \triangleq \mathscr{N}.$$

Undirected graphs are used to model the interaction topologies among agents. Let $\mathscr{G} = (\mathscr{V},\mathscr{E},\mathscr{A})$ be an undirected weighted graph of order n with the set of nodes $\mathscr{V} = \{v_1,\cdots,v_n\}$, the set of undirected edges $\mathscr{E} \subseteq \mathscr{V}\times\mathscr{V}$, and a symmetric adjacency matrix $\mathscr{A} = [a_{ij}]$ with weighting factors $a_{ij} \ge 0$. It is stipulated that the adjacency elements associated with edges are positive, i.e., (v_i,v_j) or $(v_j,v_i) \in \mathscr{E}$ if and only if $a_{ij} = a_{ji} > 0$. In graph $\mathscr{G}$, node v_i represents the ith agent, and edge (v_i,v_j) represents that information is exchanged between agents i and j. Then the set of neighbors of v_i is denoted by $\mathscr{N}_i = \{v_j \in \mathscr{V} : (v_i,v_j) \in \mathscr{E}\}$. The Laplacian of a weighted graph $\mathscr{G}$ is defined as $L = \mathscr{D} - A$, where diagonal matrix $\mathscr{D} = \mathrm{diag}\{d_1,\cdots,d_n\}$ is named the degree matrix of $\mathscr{G}$, whose diagonal elements are $d_i = \sum_{j=1}^{n} a_{ij}$. To describe the variable topologies, a piecewise-constant switching signal function $\sigma(t)$:

$[0,\infty) \mapsto \{1,\cdots,M\} \triangleq \mathscr{M}$ is defined, where $M \in \mathbb{Z}^+$ denotes the total number of all possible undirected interaction graphs. The interaction graph at time instant t is denoted by $\mathscr{G}_{\sigma(t)}$, and the corresponding Laplacian is $L_{\sigma(t)}$. In this paper, the switching graphs $\mathscr{G}_{\sigma(t)}$ are assumed to be always connected for all $\sigma(t) \in \mathscr{M}$.

B. Problem reformulation

Define controlled output functions

$$z_i(t) = x_i(t) - \frac{1}{n}\sum_{j=1}^{n} x_j(t),\ i = 1,\cdots,n \tag{4}$$

to reformulate the consensus control problem of multi-agent system (1) as the following H_∞ control problem:

$$\begin{aligned}\dot{x}(t) &= (I_n \otimes A)x(t) + (I_n \otimes B_1)\omega(t) + (I_n \otimes B_2)u(t)\\ z(t) &= (L_c \otimes I_m)x(t),\end{aligned} \tag{5}$$

where $x(t) = [x_1^{\mathrm{T}}(t)\cdots x_n^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{mn}$, $\omega(t) = [\omega_1^{\mathrm{T}}(t)\cdots \omega_n^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{m_1 n}$, $u(t) = [u_1^{\mathrm{T}}(t)\cdots u_n^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{m_2 n}$, $z(t) = [z_1^{\mathrm{T}}(t)\cdots z_n^{\mathrm{T}}(t)]^{\mathrm{T}} \in \mathbb{R}^{mn}$, and $L_c = [L_{c_{ij}}] \in \mathbb{R}^{n\times n}$ is defined by

$$L_{c_{ij}} = \begin{cases} \frac{n-1}{n}, & i = j\\ -\frac{1}{n}, & i \neq j \end{cases}.$$

Therefore, the objective is to design a distributed protocol $u_i(t)$ ($i \in \mathscr{N}$) such that

$$\|T_{z\omega}(s)\|_\infty = \sup_{\nu\in\mathbb{R}} \bar{\sigma}(T_{z\omega}(\mathrm{j}\nu)) = \sup_{\mathbf{0}\neq\omega(t)\in\mathscr{L}_2[0,\infty)} \frac{\|z(t)\|_2}{\|\omega(t)\|_2} < \gamma$$

holds, where $\gamma > 0$ is a given H_∞ performance index.

C. Protocol design and model transformation

Using the neighbors' local information, the distributed protocol of agent i is designed as

$$u_i(t) = K\sum_{j\in\mathscr{N}_i(t)} a_{ij}(t)[x_i(t) - x_j(t)], \tag{6}$$

where $\mathscr{N}_i(t)$ is the neighbor set of agent i at time instant t, $a_{ij}(t)$ are adjacency elements of the interaction graph $\mathscr{G}_{\sigma(t)}$, and $K \in \mathbb{R}^{m_2\times m}$ is an undetermined feedback matrix. Substituting protocol (6) into the system (5) results in the following closed-loop system

$$\begin{aligned}\dot{x}(t) &= (I_n\otimes A + L_{\sigma(t)}\otimes B_2K)x(t) + (I_n\otimes B_1)\omega(t)\\ z(t) &= (L_c\otimes I_m)x(t),\end{aligned} \tag{7}$$

where $L_{\sigma(t)}$ is the Laplacian matrix of graph $\mathscr{G}_{\sigma(t)}$.

Note that the symmetric matrix $L_{\sigma(t)}$ has a zero eigenvalue. Thus, the state matrix of system (7) can not be robust stable if A_0 is unstable. In order to apply the H_∞ theory, we first convert the system (7) to be an equivalent reduced-order one that is completely stabilized, by a series of model transformations. This will be presented in the following.

By Lemmas 1 and 5 of [4], there exists an orthogonal matrix $U \in \mathbb{R}^{n\times n}$ such that

$$U^{\mathrm{T}}L_cU = \begin{bmatrix} I_{n-1} & 0\\ 0 & 0\end{bmatrix} \triangleq \bar{L}_c,\ U^{\mathrm{T}}L_{\sigma(t)}U = \begin{bmatrix} L_{1\sigma(t)} & 0\\ 0 & 0\end{bmatrix} \triangleq \bar{L}_{\sigma(t)}, \tag{8}$$

and $L_{1\sigma(t)}$ is positive definite since the graph $\mathscr{G}_{\sigma(t)}$ is connected. For the convenience of discussion, denote $U = [U_1\ U_2]$ with $U_2 = \frac{\mathbf{1}}{\sqrt{n}}$ being its last column. Let

$$\begin{aligned}\hat{x}(t) &= (U^{\mathrm{T}}\otimes I_m)\bar{x}(t) = \begin{bmatrix}(U_1^{\mathrm{T}}\otimes I_m)\bar{x}(t)\\ (U_2^{\mathrm{T}}\otimes I_m)\bar{x}(t)\end{bmatrix} \triangleq \begin{bmatrix}\hat{x}^1(t)\\ \hat{x}^2(t)\end{bmatrix}\\ \hat{\omega}(t) &= (U^{\mathrm{T}}\otimes I_{m_1})\omega(t) = \begin{bmatrix}(U_1^{\mathrm{T}}\otimes I_{m_1})\omega(t)\\ (U_2^{\mathrm{T}}\otimes I_{m_1})\omega(t)\end{bmatrix} \triangleq \begin{bmatrix}\hat{\omega}^1(t)\\ \hat{\omega}^2(t)\end{bmatrix}\\ \hat{z}(t) &= (U^{\mathrm{T}}\otimes I_m)z(t) = \begin{bmatrix}(U_1^{\mathrm{T}}\otimes I_m)z(t)\\ (U_2^{\mathrm{T}}\otimes I_m)z(t)\end{bmatrix} \triangleq \begin{bmatrix}\hat{z}^1(t)\\ \hat{z}^2(t)\end{bmatrix},\end{aligned} \tag{9}$$

where

$$\bar{x}(t) = x(t) - \frac{\mathbf{1}}{n}\otimes(\sum_{j=1}^{n} x_j(t)). \tag{10}$$

From (7)-(10), we have

$$\begin{aligned}\dot{\hat{x}}(t) &= (\bar{L}_c\otimes A + \bar{L}_c\bar{L}_{\sigma(t)}\otimes B_2K)\hat{x}(t) + (\bar{L}_c\otimes B_1)\hat{\omega}(t)\\ \hat{z}(t) &= (\bar{L}_c\otimes I_m)\hat{x}(t)\end{aligned} \tag{11}$$

that can be divided into the following two independent subsystems:

$$\begin{aligned}\dot{\hat{x}}^1(t) &= (I_{n-1}\otimes A + L_{1\sigma(t)}\otimes B_2K)\hat{x}^1(t) + (I_{n-1}\otimes B_1)\hat{\omega}^1(t)\\ &\triangleq H_{\sigma(t)}\hat{x}^1(t) + G\hat{\omega}^1(t)\\ \hat{z}^1(t) &= \hat{x}^1(t)\end{aligned} \tag{12}$$

and $\dot{\hat{x}}^2(t) = \mathbf{0}$, $\hat{z}^2(t) = \mathbf{0}$. Then by the definition of H_∞ norm, it can be proved that $\|T_{z\omega}(s)\|_\infty = \|T_{\hat{z}\hat{\omega}}(s)\|_\infty = \|T_{\hat{z}^1\hat{\omega}^1}(s)\|_\infty$. In addition, the state matrix $I_{n-1}\otimes A + L_{1\sigma(t)}\otimes B_2K$ can be robust stable by designing an appropriate matrix K, since (A_0, B_{20}) is assumed to be stabilized and matrix $L_{1\sigma(t)}$ is positive definite. Therefore, we can analyze the H_∞ performance of the stabilized reduced-order system (12) instead of (11). To summarize, the consensus performance of the closed-loop multi-agent system (7) is achieved with H_∞ disturbance attenuation index γ, if the system (12) is asymptotically stable and satisfies the H_∞ level γ.

III. CONDITIONS FOR ROBUST H_∞ CONSENSUS

In this section, the robust H_∞ performance of switched system (12) is analyzed, and sufficient conditions are derived to ensure the desired consensus performance of multi-agent system (1) under the protocol (6). First, we consider the multi-agent system (1) by neglecting matrix uncertainties in (2), i.e., matrices A, B_1, B_2 are known constants. Denote $\lambda_{\sigma(t)i}$ as the ith real positive eigenvalue of matrix $L_{\sigma(t)}$, $i = 1,\cdots,n-1$. Let $\sigma_*(t)i_*$ and $\sigma^*(t)i^*$ be the subscripts associated with the minimum and the maximum nonzero eigenvalues of all Laplacian matrices $L_{\sigma(t)}$, respectively.

The following lemma is derived from Lemma 3.2 of [5] and Schur Complement Formula:

Lemma 1: For a given index $\gamma > 0$, the switched system (12) is asymptotically stable and satisfies $\|T_{\hat{z}^1\hat{\omega}^1}(s)\|_\infty < \gamma$, if there exists a positive definite matrix

$P \in \mathbb{R}^{(n-1)m\times(n-1)m}$ such that

$$H^{\mathrm{T}}_{\sigma(t)}P + PH_{\sigma(t)} + \gamma^{-2}PGG^{\mathrm{T}}P + I < 0 \qquad (13)$$

holds for $\forall\sigma(t) \in \mathscr{M}$.

Theorem 1: Under protocol (6), the multi-agent system (1) achieves consensus with a given H_∞ disturbance attenuation index γ, if there exist $P \in \mathbb{R}^{m\times m} > 0$ and $Q \in \mathbb{R}^{m_2\times m}$ such that the linear matrix inequality (LMI)

$$\begin{bmatrix} PA^{\mathrm{T}}+AP+\lambda_{\sigma(t)i}Q^{\mathrm{T}}B_2^{\mathrm{T}}+\lambda_{\sigma(t)i}B_2Q+\gamma^{-2}B_1B_1^{\mathrm{T}} & P \\ P & -I \end{bmatrix} < 0 \qquad (14)$$

is satisfied for $\sigma(t)i = \sigma_*(t)i_*$ and $\sigma^*(t)i^*$. If the above two LMIs are feasible, then the feedback matrix of the consensus protocol is $K = QP^{-1}$.

Proof: By Lemma 1, the system (12) is asymptotically stable and satisfies $\|T_{\hat{z}^1\hat{\omega}^1}(s)\|_\infty < \gamma$, if there exists a positive definite matrix $P \in \mathbb{R}^{(n-1)m\times(n-1)m}$ satisfying (13). Particularly, take $P = I_{n-1}\otimes X$ with $X \in \mathbb{R}^{m\times m} > 0$.

Since $L_{1\sigma(t)}$ is positive definite, there exists an orthogonal matrix $U_{1\sigma(t)}$ such that $U^{\mathrm{T}}_{1\sigma(t)}L_{1\sigma(t)}U_{1\sigma(t)} = \mathrm{diag}\{\lambda_{\sigma(t)1},\cdots,\lambda_{\sigma(t)(n-1)}\}$. Let $\bar{U}_{1\sigma(t)} = U_{1\sigma(t)}\otimes I_m$. According to the proof of Theorem 3.3 in [5], pre- and post-multiplying the matrix inequality (13) with $\bar{U}^{\mathrm{T}}_{1\sigma(t)}$ and $\bar{U}_{1\sigma(t)}$, respectively, can yield a group of matrix inequalities

$$XA+A^{\mathrm{T}}X+\lambda_{\sigma(t)i}XB_2K+\lambda_{\sigma(t)i}K^{\mathrm{T}}B_2^{\mathrm{T}}X+\gamma^{-2}XB_1B_1^{\mathrm{T}}X+I<0 \qquad (15)$$

$\sigma(t) = 1,\cdots,M,\ i = 1,\cdots,n-1$, which are equivalent to (13). Due to Schur Complement Formula, inequality (15) is also equivalent to

$$\begin{bmatrix} (A+\lambda_{\sigma(t)i}B_2K)^{\mathrm{T}}X+X(A+\lambda_{\sigma(t)i}B_2K)+\gamma^{-2}XB_1B_1^{\mathrm{T}}X & I \\ I & -I \end{bmatrix} < 0. \qquad (16)$$

Then pre- and post-multiplying the inequality (16) with $\mathrm{diag}\{X^{-1},I\}$ yields (14) with $P = X^{-1}$ and $Q = KP$. To summarize, the multi-agent system (1) achieves consensus with a given H_∞ disturbance attenuation index γ, if the LMI (14) holds for $\forall\sigma(t)i$.

Due to the convex property of LMIs, if (14) holds when $\lambda_{\sigma(t)i}$ takes its extreme values $\lambda_{\sigma_*(t)i_*}$ and $\lambda_{\sigma^*(t)i^*}$, then the LMI (14) holds for $\forall\sigma(t)i$, and the desired consensus performance is guaranteed. Further, if the above condition is satisfied, then from $Q = KP$, it is obtained that the feedback matrix of the proposed consensus protocol is $K = QP^{-1}$. □

Based on the previous development, the consensus condition is now given for the multi-agent system (1) with model uncertainties (2). To achieve this, we adopt the following lemma.

Lemma 2 [7]: Given symmetric matrices $X,Y,Z \in \mathbb{R}^{n\times n}$ satisfying $X \geq 0$, $Y < 0$, $Z \geq 0$, if for any nonzero vector $\zeta \in \mathbb{R}^n$, $(\zeta^{\mathrm{T}}Y\zeta)^2 - 4\zeta^{\mathrm{T}}X\zeta\zeta^{\mathrm{T}}Z\zeta > 0$ holds, then there exists a scalar $\lambda > 0$ such that $\lambda^2X + \lambda Y + Z < 0$.

Theorem 2: Under protocol (6), the multi-agent system (1) with model uncertainties (2) can achieve consensus with a given H_∞ disturbance attenuation index γ, if for a scalar $\lambda > 0$, there exist $P \in \mathbb{R}^{m\times m} > 0$ and $Q \in \mathbb{R}^{m_2\times m}$ such that the LMI

$$\begin{bmatrix} \Psi_{\sigma(t)i} & B_{10} & P & \frac{1}{\lambda}(F_1P+\lambda_{\sigma(t)i}F_3Q)^{\mathrm{T}} \\ B_{10}^{\mathrm{T}} & -\gamma^{-2}I & 0 & \frac{1}{\lambda}F_2^{\mathrm{T}} \\ P & 0 & -I & 0 \\ \frac{1}{\lambda}(F_1P+\lambda_{\sigma(t)i}F_3Q) & \frac{1}{\lambda}F_2 & 0 & -I \end{bmatrix} < 0$$

$$\Psi_{\sigma(t)i} = PA_0^{\mathrm{T}}+A_0P+\lambda_{\sigma(t)i}Q^{\mathrm{T}}B_{20}^{\mathrm{T}}+\lambda_{\sigma(t)i}B_{20}Q+\lambda^2EE^{\mathrm{T}} \qquad (17)$$

is satisfied for $\sigma(t)i = \sigma_*(t)i_*$ and $\sigma^*(t)i^*$. If the above two LMIs are feasible, then the feedback matrix of the consensus protocol is $K = QP^{-1}$.

Proof: Due to Schur Complement Formula, the matrix inequality (14) in Theorem 1 is equivalent to

$$\Xi_{\sigma(t)i} = \begin{bmatrix} PA^{\mathrm{T}}+AP+\lambda_{\sigma(t)i}Q^{\mathrm{T}}B_2^{\mathrm{T}}+\lambda_{\sigma(t)i}B_2Q & B_1 & P \\ B_1^{\mathrm{T}} & -\gamma^2 I & 0 \\ P & 0 & -I \end{bmatrix} < 0.$$

By the definition of negative definite matrices, $\Xi_{\sigma(t)i} < 0$ if and only if $\xi^{\mathrm{T}}\Xi_{\sigma(t)i}\xi < 0$ holds for any nonzero vector ξ. Hence, to ensure the desired robust H_∞ consensus performance, we only need to find conditions for $\xi^{\mathrm{T}}\Xi_{\sigma(t)i}\xi < 0$ in the presence of uncertainties (2).

From (2) and (3), it is obtained that $\Xi_{\sigma(t)i} = \Gamma_{\sigma(t)i} + \Delta\Gamma_{\sigma(t)i}$, where

$$\Gamma_{\sigma(t)i} = \begin{bmatrix} PA_0^{\mathrm{T}}+A_0P+\lambda_{\sigma(t)i}Q^{\mathrm{T}}B_{20}^{\mathrm{T}}+\lambda_{\sigma(t)i}B_{20}Q & B_{10} & P \\ B_{10}^{\mathrm{T}} & -\gamma^2 I & 0 \\ P & 0 & -I \end{bmatrix}$$

$$\Delta\Gamma_{\sigma(t)i} = \begin{bmatrix} \Delta\Gamma_{11} & E\Sigma(t)F_2 & 0 \\ F_2^{\mathrm{T}}\Sigma(t)^{\mathrm{T}}E^{\mathrm{T}} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

with $\Delta\Gamma_{11} = PF_1^{\mathrm{T}}\Sigma(t)^{\mathrm{T}}E^{\mathrm{T}}+E\Sigma(t)F_1P+\lambda_{\sigma(t)i}Q^{\mathrm{T}}F_3^{\mathrm{T}}\Sigma(t)^{\mathrm{T}}E^{\mathrm{T}}+\lambda_{\sigma(t)i}E\Sigma(t)F_3Q$. That is, model uncertainties are decoupled from the determined constant system matrices. Let $\xi = [\xi_1^{\mathrm{T}}\ \xi_2^{\mathrm{T}}\ \xi_3^{\mathrm{T}}]^{\mathrm{T}}$ be a nonzero vector. The inequality $\xi^{\mathrm{T}}\Xi_{\sigma(t)i}\xi < 0$ holds if and only if

$$\begin{aligned}&\xi^{\mathrm{T}}\Xi_{\sigma(t)i}\xi \\ &= \xi^{\mathrm{T}}\Gamma_{\sigma(t)i}\xi+2\xi_1^{\mathrm{T}}E\Sigma(t)[(F_1P+\lambda_{\sigma(t)i}F_3Q)\xi_1+F_2\xi_2]<0\end{aligned} \qquad (18)$$

is satisfied for any $\Sigma^{\mathrm{T}}(t)\Sigma(t) \leq I$. Actually, if one takes

$$\Sigma(t) = \frac{(E^{\mathrm{T}}\xi_1)[(F_1P+\lambda_{\sigma(t)i}F_3Q)\xi_1+F_2\xi_2]^{\mathrm{T}}}{||E^{\mathrm{T}}\xi_1||_2||(F_1P+\lambda_{\sigma(t)i}F_3Q)\xi_1+F_2\xi_2||_2}, \qquad (19)$$

then $\xi^{\mathrm{T}}\Xi_{\sigma(t)i}\xi$ reaches its maximum value. Therefore, (18) holds for any $\Sigma(t)$ satisfying $\Sigma^{\mathrm{T}}(t)\Sigma(t) \leq I$ if and only if it holds when $\Sigma(t)$ is taken as (19). Instituting (19) into (18) leads to

$$\xi^{\mathrm{T}}\Gamma_{\sigma(t)i}\xi + 2\sqrt{\xi^{\mathrm{T}}X\xi}\sqrt{\xi^{\mathrm{T}}Z_{\sigma(t)i}\xi} < 0, \qquad (20)$$

where

$$X = \begin{bmatrix} EE^{\mathrm{T}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \geq 0$$

$$Z_{\sigma(t)i}=\begin{bmatrix} Z_{11} & (F_1P+\lambda_{\sigma(t)i}F_3Q)^{\mathrm{T}}F_2 & 0\\ F_2^{\mathrm{T}}(F_1P+\lambda_{\sigma(t)i}F_3Q) & F_2^{\mathrm{T}}F_2 & 0\\ 0 & 0 & 0\end{bmatrix}\geq 0$$
$$Z_{11}=(F_1P+\lambda_{\sigma(t)i}F_3Q)^{\mathrm{T}}(F_1P+\lambda_{\sigma(t)i}F_3Q). \tag{21}$$

Obviously, (20) is satisfied if and only if $\Gamma_{\sigma(t)i}<0$ and

$$(\xi^{\mathrm{T}}\Gamma_{\sigma(t)i}\xi)^2-4\xi^{\mathrm{T}}X\xi\xi^{\mathrm{T}}Z_{\sigma(t)i}\xi>0. \tag{22}$$

Using Lemma 2, we obtain that (22) holds if and only if there exists a scalar $\lambda>0$ satisfying $\Gamma_{\sigma(t)i}+\lambda^2X+\lambda^{-2}Z_{\sigma(t)i}<0$. Inserting (21) into the above inequality results in

$$\Gamma_{\sigma(t)i}+S^{\mathrm{T}}S<0, \tag{23}$$
$$S=\begin{bmatrix} \lambda E^{\mathrm{T}} & 0 & 0\\ \lambda^{-1}(F_1P+\lambda_{\sigma(t)i}F_3Q) & \lambda^{-1}F_2 & 0\end{bmatrix}.$$

According to Schur Complement Formula, the matrix inequality (23) becomes

$$\begin{bmatrix} \Gamma_{\sigma(t)i} & S^{\mathrm{T}}\\ S & -I\end{bmatrix}<0,$$

which is further equivalent to (17). Combining with Theorem 1, we know that if for a scalar $\lambda>0$, there exist $P>0$ and Q such that the LMI (17) holds for $\sigma(t)i=\sigma_*(t)i_*$ and $\sigma^*(t)i^*$, then the multi-agent system (1) with model uncertainties (2) achieves consensus with H_∞ index γ. Further, if the two LMIs are feasible, then $K=QP^{-1}$ is obtained. □

IV. SIMULATION RESULTS

Consider a network of four agents, and the matrices in (2) and (3) are given by

$$A_0=\begin{bmatrix}0&-1\\2&1\end{bmatrix},\ B_{10}=\begin{bmatrix}0\\1\end{bmatrix},\ B_{20}=\begin{bmatrix}1&0\\0&2\end{bmatrix},$$
$$E=\begin{bmatrix}0&0\\0.8&0.8\end{bmatrix},\ \Sigma(t)=\begin{bmatrix}\sin(10t)&0\\0&0\end{bmatrix},$$
$$F_1=\begin{bmatrix}1.2&0\\0&1.2\end{bmatrix},\ F_2=\begin{bmatrix}0.5\\0.5\end{bmatrix},\ F_3=\begin{bmatrix}0.8&0\\0&0.8\end{bmatrix}.$$

The external disturbance is assumed to be band-limited white noise. The H_∞ performance index is chosen as $\gamma=1$. For simplicity, the interaction graphs are constrained to be within the set shown in Fig. 1, whose nonzero weighting factors are all 1.

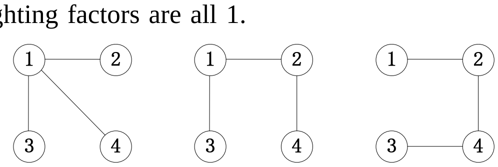

Fig.1. Undirected interaction graphs.

Fig. 2 depicts the state trajectories of four agents $x_i(t)=[x_{i,1}(t)\ x_{i,2}(t)]^{\mathrm{T}}$ under the proposed protocol (6). Fig. 3 gives the energy relationship between the controlled output $z(t)$ and the external disturbance $\omega(t)$. Obviously, the consensus is achieved with H_∞ disturbance attenuation index 1.

V. CONCLUSIONS

This paper has addressed the consensus control problem for switching networks of autonomous agents with both model uncertainties and external disturbances by robust H_∞ theory. Time delays arising in the information exchange among agents are not considered in this paper, and this will be a topic of future research.

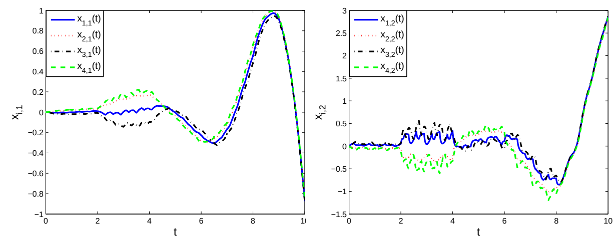

Fig.2. Left: The first element of states. Right: The second element of states.

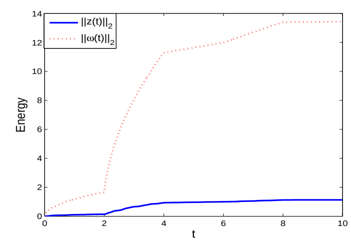

Fig.3. Energy trajectories of $z(t)$ and $\omega(t)$.

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