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Observer based control of a manipulator system with structured uncertainty

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Abstract : In this paper, we show an uncertain model of a two link RR manipulator with uncertainties in the two rotation angles of each joint, and show the extended system with uncertainty also in an output matrix. For this system, we apply a guaranteed cost control method based on a linear upper bound. Parameter tuning of γ_i in the linear upper bound is effective to design a feedback gain which have appropriate characteristics. In the numerical simulation, we show an advantage that the state observer is effective to reduce the influence of signal noise in state vector.

Keywords : Uncertain system, Guaranteed cost control, Observer based control

1 Introduction

The guaranteed cost control (GCC) is one of an effective approach to design a robust control system. This method is an extended version of the linear quadratic regulator (LQR) that is one of the efficient method for designing control system, and it is stated as an essential concept in the modern control theorem. However, LQR method is formulated as nominal form, thus, it is weak for the effect of the disturbance which is caused by the uncertainty of the system model, secular distortion, signal noise, ans so on. The system performance is degraded by these disturbance effects.

For such a problem, under the assumption that an uncertain parameter variation is in an admissible closed set, the GCC method guarantees the upper bound of the performance index variation [1]. Takahashi et al. extended the GCC method to the case with uncertainty in an output matrix [2], and they proposed the modeling method for the uncertain inverted pendulum car system which include uncertainty in a pendulum angle and apply the guaranteed cost control method [3]. In this paper, we will propose a method of the GCC to the system with structured uncertainties in input, state and also output matrix. And show the effectiveness of the parameter tuning in the upper bound. At last, we will apply the state observer to consider the influence of disturbance.

2 Formulation of the GCC problem

In this section, we will show the GCC problem with parameter variation in state, input and also output matrix. Let us consider the following uncertain system.

$$\left\{ \begin{array}{rcl} \dot{\boldsymbol{x}}(t) & = & A(\xi)\boldsymbol{x}(t) + B(\zeta)\boldsymbol{u}(t) \\ \boldsymbol{y}(t) & = & C(\psi)\boldsymbol{x}(t) \end{array} \right. \quad (1)$$

where, state, output and input vector are $\boldsymbol{x} \in \Re^n, \boldsymbol{y} \in \Re^m$ and $\boldsymbol{u} \in \Re^l$, respectively. Uncertain state matrix $A(\xi) \in \Re^{n\times n}$, input matrix $B(\zeta) \in \Re^{n\times l}$, and output matrix $C(\psi) \in \Re^{m\times n}$ are defined as

$$A(\xi) = A_0 + \sum_{i=1}^{p} \xi_i A_i, \ |\xi_i| \le 1 \quad (2)$$

$$B(\zeta) = B_0 + \sum_{j=1}^{q} \zeta_j B_j, \ |\zeta_j| \le 1 \quad (3)$$

$$C(\psi) = C_0 + \sum_{k=1}^{r} \psi_k C_k, \ |\psi_k| \le 1 \quad (4)$$

where, A_0, B_0 and C_0 represent the system structure that could be included in the linear system model. We call these matrices as nominal element. A_i, B_j and C_k represent the uncertain system structure that could not be included in the linear system model. We call these matrices as uncertainties. Where p, q and r are numbers of the corresponding uncertainties, ξ_i, ζ_j and ψ_k are indeterminate scalar parameters which represent the scale of uncertainties and included a bounded closed set. These parameters are used to normalize the structures of uncertainties.

Here we consider the GCC problem for the system of eq. (1) which has uncertainty in an output matrix. The performance index function consists of the quadratic forms of the input vector $\boldsymbol{u}(t)$ and the output vector $\boldsymbol{y}(t)$.

$$\begin{aligned} &J(\boldsymbol{y}, \boldsymbol{u}, \xi, \zeta, \psi) \\ &= \int_0^\infty \left\{ \boldsymbol{y}^{\mathrm{T}}(t) Q \boldsymbol{y}(t) + \boldsymbol{u}^{\mathrm{T}}(t) R \boldsymbol{u}(t) \right\} dt \end{aligned} \quad (5)$$

where $Q \in \Re^{m\times m} \geq 0$ and $R \in \Re^{l\times l} > 0$ are weighting matrices of input and output vector, respectively. In virtue of the uncertain structure (2), (3) and (4), the linear upper bound becomes:

$$\begin{aligned} &U_L(A(\xi), B(\zeta), C(\psi), P, Q, R) \\ &= \sum_{i=1}^{p} \left(\gamma_i^{-1} P + \gamma_i A_i^{\mathrm{T}} P A_i \right) + P R_s P + Q_s \end{aligned} \quad (6)$$

where

$$R_s = \sum_{j=1}^{q} (B_j R^{-1} B_0^{\mathrm{T}} + B_0 R^{-1} B_j^{\mathrm{T}}) \quad (7)$$

$$Q_s = \sum_{k=1}^{r} (C_0^{\mathrm{T}} Q C_k + C_k^{\mathrm{T}} Q C_0 + C_k^{\mathrm{T}} Q C_k) \quad (8)$$

By applying this upper bound, we have stochastic algebraic Riccati equation (SARE) based on the linear upper bound:

$$\begin{aligned} &(A_0 + \gamma I)^{\mathrm{T}} P + P(A_0 + \gamma I) + C_0^{\mathrm{T}} Q C_0 + Q_s \\ &\quad - P(R_n - R_s) P + \sum_{i=1}^{p} \gamma_i A_i^{\mathrm{T}} P A_i = O \end{aligned} \quad (9)$$

where $\gamma = 1/2 \sum_{i=1}^{p} \gamma_i^{-1}, R_n = B_0 R^{-1} B_0^{\mathrm{T}}$. Feedback gain is obtained as:

$$F = -R^{-1} B_0^{\mathrm{T}} P$$

3 Skeletal form of the uncertain model

From the past research [4], we have a dynamics of uncertain LTI system of a two link RR manipulator. The link 1 is connected to the base with a rotation joint 1 and the link 2 is connected to another end point of the link 1 with a rotation joint 2. Each joints and arms have physical parameters illustrated in table 1.

Table 1 : Parameters of the Manipulator

Parameters	**Meaning** [unit]
θ_i	Angle of the joint [rad]
m_i	Mass of the arm [kg]
I_i	Inertia moment of the arm [kg · m^2]
l_i	Length of the arm [m]
l_{Gi}	Distance from the joint to the center of gravity of the arm [m]
g	Gravity [m/s^2]
τ_i	Input torque to the joint [N· m]

Let us define a state vector $\boldsymbol{x}(t)$ and an input vector $\boldsymbol{u}(t)$ are

$$\boldsymbol{x}(t) = \begin{bmatrix} \theta_1(t) \\ \dot{\theta}_1(t) \\ \theta_2(t) \\ \dot{\theta}_2(t) \end{bmatrix}, \boldsymbol{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

An input matrix $A(\xi)$ and an output matrix $B(\zeta)$ are obtained as following:

$$A(\xi) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \bar{h}_{11}/h_D & 0 & \bar{h}_{12}/h_D & 0 \\ 0 & 0 & 0 & 1 \\ \bar{h}_{21}/h_D & 0 & \bar{h}_{22}/h_D & 0 \end{bmatrix}$$

$$B(\zeta) = \begin{bmatrix} 0 & 0 \\ h_{22}/h_D & -h_{12}/h_D \\ 0 & 0 \\ -h_{12}/h_D & h_{22}/h_D \end{bmatrix}$$

where

$$h_D = h_{11} h_{22} - h_{12}^2$$

$$\begin{aligned} \bar{h}_{11} &= \Delta c_1 g(h_{22}(m_1 l_{G1} + m_2 l_1) \\ &\quad + \Delta c_2 m_2 l_{G2}(h_{22} - h_{12})) \\ \bar{h}_{12} &= \Delta c_1 \Delta c_2 m_2 g l_{G2}(h_{22} - h_{12}) \end{aligned}$$

$$\begin{aligned} \bar{h}_{21} &= \Delta c_1(-h_{12} g(m_1 l_{G1} + m_2 l_1) \\ &\quad + \Delta c_2 m_2 g l_{G2}(h_{11} - h_{12})) \\ \bar{h}_{22} &= \Delta c_1 \Delta c_2 m_2 g l_{G2}(h_{11} - h_{12}) \end{aligned}$$

h_{11}, h_{12} and h_{22} are given as follows:

$$\begin{aligned} h_{11} &= I_1 + m_1 l_{G1}^2 \\ &\quad + I_2 + m_2(l_1^2 + l_{G2}^2 + 2\Delta c_2 l_1 l_{G2}) \\ h_{12} &= I_2 + m_2(l_{G2}^2 + \Delta c_2 l_1 l_{G2}) \\ h_{22} &= I_2 + m_2 l_{G2}^2 \end{aligned}$$

From $A(\xi)$ and $B(\zeta)$, we can obtain deterministic elements A_0 and B_0 as $(\Delta\theta_1, \Delta\theta_2) = (0, 0)$. A_1 and B_1 are obtained in the condition of $(\Delta\theta_1, \Delta\theta_2) = (max_1, 0)$ and A_2 and B_2 are obtained in the condition of $(\Delta\theta_1, \Delta\theta_2) = (0, max_2)$. Where max_i is a maximum uncertainty of the rotational angle in the joint i.

4 State observer

In this section, we consider the application of an identical state observer to the uncertain system (1). $\hat{\boldsymbol{x}}(t)$ is the state of the observer (estimation of the plant state), $\boldsymbol{y}(t)$ is output from the plant, and $\boldsymbol{u}(t)$ is the input from the controller to the plant, respectively.

$$\dot{\hat{\boldsymbol{x}}}(t) = (A - KC)\hat{\boldsymbol{x}}(t) + G\boldsymbol{y}(t) + H\boldsymbol{u}(t)$$

where, K is observer gain.

5 Numerical example

5.1 Uncertain system

In this section, we will show the numerical example. Here we consider that the joint 2 is passive, thus the input matrix $B(\zeta)$ becomes

$$B(\zeta) = \begin{bmatrix} 0 \\ h_{22}/h_D \\ 0 \\ -h_{12}/h_D \end{bmatrix}$$

The values of the physical parameters are illustrated as in table 2.

Table 2: Parameters

Parameter	Value	Parameter	Value
m_1	1	m_2	1
I_1	0.03	I_2	0.03
l_1	0.3	l_2	0.3
l_{G1}	0.15	l_{G2}	0.15
g	9.8		

For the above parameters, we have uncertain system as follows:

$$A_0 = \begin{bmatrix} 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 30.3093 & 0.0000 & -12.1237 & 0.0000 \\ 0.0000 & 0.0000 & 0.000 & 1.0000 \\ -28.2887 & 0.0000 & 50.5155 & 0.000 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 0.0000 \\ 9.6220 \\ 0.0000 \\ -17.8694 \end{bmatrix}$$

For the disturbance $\varDelta\theta_1 = \varDelta\theta_2 = 0.08$, uncertain system becomes:

$$A_1 = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ -0.0969 & 0.0000 & 0.0388 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0905 & 0.0000 & -0.1616 & 0.0000 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0056 & 0.0000 & 0.1059 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ -0.0168 & 0.0000 & -0.3192 & 0.0000 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.0000 \\ -0.0228 \\ 0.0000 \\ 0.0686 \end{bmatrix}$$

5.2 Parameter Tuning γ_i of the linear upper bound

In this paper, we use the linear upper bound for the SARE. In the past research [2], we had shown that the effective of the parameter tuning of γ_i in the linear upper bound to design a system which have an appropriate characteristic of closed loop system, in the case of the system have only one uncertainty. Here we show the result with two uncertainties of A_i where $p = 2$.

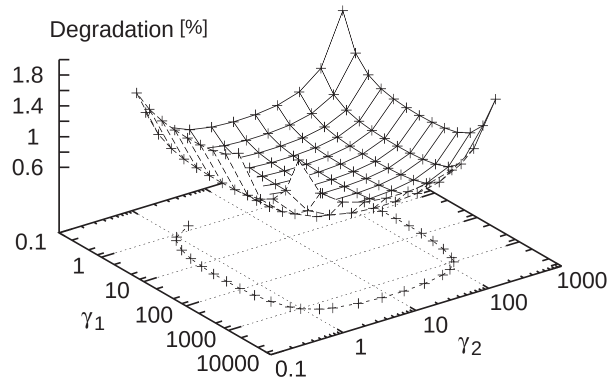

Figure 1 : Degradation of the Performance Index

At first, we solve the SARE on the any point of γ_1 and γ_2 in $0.1 \leq \gamma_i \leq 10000$ $(i = 1, 2)$, and calculate the performance index as $J_{LQR} = \boldsymbol{x}_0^{\mathrm{T}} P_{LQR} \boldsymbol{x}_0$ and $J_{GCC} = \boldsymbol{x}_0^{\mathrm{T}} P_{GCC} \boldsymbol{x}_0$. The weighting matrices are $Q = \mathrm{diag}(1, 1)$ and $R = 1$. From these results, we examine the ratio of the degradation of the performance index. This result is illustrated in the figure 1 by double logarithm 3D-chart, which x- and y-axis are logarithmic and z-axis is plotted with a linear scale. In figure 1, the surface illustrates the degradation ratio of the performance index between GCC and LQR. The contour line on the z-plane denotes a line of J_{LQR} and J_{GCC} are equal. The minimum value takes $J_{GCC}/J_{LQR} = 0.6568$ at a point $(\gamma_1, \gamma_2) = (46.8750, 15.6250)$. This point is surrounded by the square contour line that the corner is round. Inside of this square, the GCC method provides a good result. But in the outside region, a reversed result is provided.

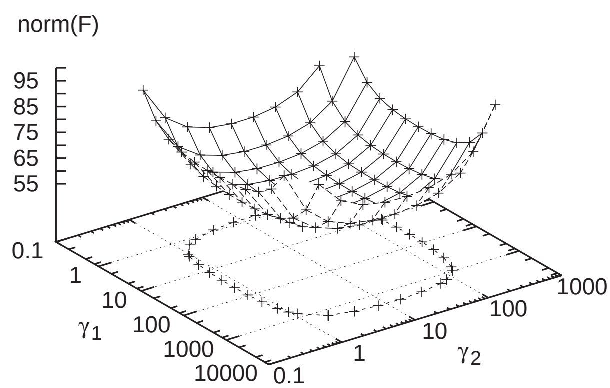

Figure 2 : Norm of F_{GCC}

Next, we will show the comparison of the norm of feedback gain in the same region. In figure 2, the contour line on the z-plane denotes a results of the LQR method that $F_{LQR} = 68.8696$. The minimum value of the norm of feedback gain is norm$(F_{GCC}) = 55.2322$ on the same minimum point of figure 1. These results have a similar tendency.

5.3 Numerical solution of SRAE

Here we show and compare the results of GCC and LQR. The following results is the solution of the SARE (Propsed method) on the minimum point (γ_1, γ_2).

$$P_{GCC} = \begin{bmatrix} 214.9665 & 54.3720 & 163.5782 & 31.4218 \\ 54.3720 & 13.7897 & 41.6448 & 7.9989 \\ 163.5782 & 41.6448 & 128.8797 & 24.5399 \\ 31.4218 & 7.9989 & 24.5399 & 4.6930 \end{bmatrix}$$

Feedback gain is

$$F_{GCC} = [\, -38.3216 \;\; -10.2522 \;\; -37.8073 \;\; -6.8958 \,]$$

Eigenvalues of the closed loop system are:

$$(\, -7.8958 \pm 2.3148i, -4.3929 \pm 0.5460i \,)$$

The LQR result (Ordinary method) is follows.

$$P_{LQR} = \begin{bmatrix} 309.6216 & 80.0140 & 250.8530 & 45.7467 \\ 80.0140 & 20.8360 & 65.4442 & 11.9295 \\ 250.8530 & 65.4442 & 210.2729 & 37.8890 \\ 45.7467 & 11.9295 & 37.8890 & 6.9139 \end{bmatrix}$$

Feedback gain is

$$F_{LQR} = [\, -47.5721 \;\; -12.6897 \;\; -47.3509 \;\; -8.7623 \,]$$

Eigenvalues of the closed loop system are:

$$(\, -23.0041 \;\; -4.3278 \pm 0.7210i \;\; -2.8177 \,)$$

5.4 Comparison of the numerical simulation

Here we will show the numerical simulation of uncertain systems. Now, we compare the performance index function value of eq. (5) with the difference of system composition i) with/without observer, ii) with/without disturbance. The simulation is calculated by Euler's method with step time is 0.01, time interval $(0, 30)$ with an initial state value is $\boldsymbol{x}(0) = [\ 1\ 0\ 1\ 0\ ]^{\mathrm{T}}$ and initial observer state value is $\hat{\boldsymbol{z}}(0) = [\ 0.1\ 0\ 0.1\ 0\ ]^{\mathrm{T}}$. The disturbance is added in the state vector $(x_2, x_4) = (\dot{\theta}_1, \dot{\theta}_2)$ as a signal noise of uniformly random number between $(-0.08, 0.08)$ on every step time.

Table 3: Comparison of the degradation

	No Noise	with Noise	Degradation [%]
LQR1	1108.2367	1214.8047	1.0962
GCC1	1203.4899	1317.1795	1.0945
LQR2	1916.9598	2621.1953	1.3674
GCC2	2866.6822	3257.1295	1.1362

In table1, LQR1 and GCC1 are the result without observer. LQR2 and GCC2 are the result with observer. It express the GCC have high performance value index but have more robustness than the LQR method. In the case with observer, this tendency becomes more remarkable.

6 Conclusion

For the system which haves two uncertainties, we showed the advantage of the state observer for the influence of disturbance and parameter tuning of linear upper bound.

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