The Sixteenth International Symposium on Artificial Life and Robotics 2011 (AROB 16th '11),
B-Con Plaza, Beppu,Oita, Japan, January 27-29, 2011

Observer-based Guaranteed Cost Control

Erwin Susanto[1], Mitsuaki Ishitobi[2] and Sadaaki Kunimatsu[3]
[1,2,3]*Department of Mechanical Systems Engineering*
Kumamoto University, Kumamoto 860-8555
Tel: +81-96-342-3866
E-mail: [1]*095d9213@st.kumamoto-u.ac.jp*, [2]*mishi@kumamoto-u.ac.jp*, [3]*kunimatu@mech.kumamoto-u.ac.jp*

Abstract: This paper presents a design scheme of a minimal order observer-based guaranteed cost controller for uncertain linear systems. The perturbations are assumed to be described by structural uncertainties. An iterative linear matrix inequality (ILMI) approach is used to design the observer-based controller since the problems contain inverse relations. We modify the algorithm of Matsunaga et al by optimizing a sufficiently large initial guaranteed cost. This method can be implemented by LMI control toolbox of Matlab. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

Keywords: robust control, guaranteed cost control, minimal order observer

I. INTRODUCTION

Considerable attention to the problem in robust stability analysis and robust stabilization of uncertain systems has been attracting many authors for several last decades. One approach to this problem is the guaranteed cost control method which not only achieves the stability of the uncertain system but also guarantees an adequate level of performance via linear matrix inequality (LMI) techniques (Lien [2], Won and Park [3]).

Although the controller is usually constructed by using state variables, it may not be possible to measure all the states of the system in many cases. The observer-based control is probably well suited and better than the state control feedback in such situations.

Since inverse relations among variables appear, this paper concerns a design method of a minimal order observer-based guaranteed cost controller via an iterative linear matrix inequality (ILMI) technique under an assumption that the statistical properties of the initial state variables are known.

II. PROBLEM STATEMENT

Consider a continuous-time uncertain system

$$\dot{\boldsymbol{x}}(t) = (A + \Delta A(t))\boldsymbol{x}(t) + (B + \Delta B(t))\boldsymbol{u}(t) \quad (1)$$
$$\boldsymbol{y}(t) = C\boldsymbol{x}(t) \quad (2)$$

where $\boldsymbol{x}(t) \in \Re^n$ is the state vector, $\boldsymbol{u}(t) \in \Re^r$ is the control input vector, $\boldsymbol{y}(t) \in \Re^m$ is the measured output vector, A, B, C are known constant real-valued matrices with appropriate dimensions, and C is restricted to the form of $C = [O \;\; I_m]$. Matrices $\Delta A(t)$ and $\Delta B(t)$ denote real-valued matrix functions representing parameter uncertainties. It is assumed that

$$\Delta A(t) = D_A F_A(t) E_A,\ \Delta B(t) = D_B F_B(t) E_B \quad (3)$$

with

$$F_A^T(t)F_A(t) \le I,\ F_B^T(t)F_B(t) \le I$$

where D_A, D_B, E_A, E_B are constant real-valued known matrices with appropriate dimensions, and $F_A(t)$ and $F_B(t)$ are real time-varying unknown continuous and deterministic matrices.

We further assume that the initial state variable $\boldsymbol{x}(0)$ is unknown, but their mean and covariance are known

$$E[\boldsymbol{x}(0)] = \boldsymbol{m}_0 \quad (4)$$
$$E\left[(\boldsymbol{x}(0) - \boldsymbol{m}_0)(\boldsymbol{x}(0) - \boldsymbol{m}_0)^T\right] = \Sigma_0 > O \quad (5)$$

where $E[\cdot]$ denotes the expected value operator.

The problem considered here is to design a minimal order observer

$$\dot{\boldsymbol{z}}(t) = D\boldsymbol{z}(t) + E\boldsymbol{y}(t) + F\boldsymbol{u}(t) \quad (6)$$
$$\hat{\boldsymbol{x}}(t) = P\boldsymbol{z}(t) + W\boldsymbol{y}(t) \quad (7)$$

and a controller

$$\boldsymbol{u}(t) = K\hat{\boldsymbol{x}}(t) \quad (8)$$

with

$$D = A_{11} + LA_{21},\ PT + WC = I_n,$$
$$F = TB,\ TA - DT = EC,\ A = \begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array},$$
$$P = \begin{bmatrix} I_{n-m} & \mathbf{0} \end{bmatrix}^T,\ T = \begin{bmatrix} I_{n-m} & L \end{bmatrix}$$

so as to achieve an upper bound on the following quadratic performance index

$$E[J] = E\int_0^\infty (\boldsymbol{x}^T(t)Q\boldsymbol{x}(t) + \boldsymbol{u}^T(t)R\boldsymbol{u}(t))dt \quad (9)$$

associated with the uncertain system (1) and (2) where Q and R are given symmetric positive-definite matrices.

III. MAIN RESULTS

In this section, a sufficient condition is established for the existence of a minimal order observer-based guaranteed cost controller for the uncertain system (1) and (2). Here, it is assumed that the feedback gain matrix is

$$K = -R^{-1}B^T S_1 \tag{10}$$

where S_1 is a symmetric positive-definite matrix.

The main result of this study is given by Theorem 1.

Theorem 1. If the following matrix inequalities optimization problem; $\min\{\gamma_0+\gamma_1+\gamma_2+\gamma_3+\gamma_4\}$ subject to

$$\begin{bmatrix} \Lambda_0 & XE_A^T & XE_A^T & X^T \\ E_AX & -\zeta I & 0 & 0 \\ E_AX & 0 & -\theta I & 0 \\ X & 0 & 0 & -Q^{-1} \end{bmatrix} < 0 \tag{11}$$

$$\begin{bmatrix} \bar{\Lambda}_0 & G_1^T & G_1^T & G_2^T & G_3^T & G_3^T & G_4^T \\ G_1 & -\delta I & 0 & 0 & 0 & 0 & 0 \\ G_1 & 0 & -\mu I & 0 & 0 & 0 & 0 \\ G_2 & 0 & 0 & -\theta_{inv} I & 0 & 0 & 0 \\ G_3 & 0 & 0 & 0 & -\nu_{inv} I & 0 & 0 \\ G_3 & 0 & 0 & 0 & 0 & -\mu_{inv} I & 0 \\ G_4 & 0 & 0 & 0 & 0 & 0 & -R \end{bmatrix} < 0 \tag{12}$$

$$\sum_{k=1}^{n} \boldsymbol{e}_{nk}^T \Theta_0 \boldsymbol{e}_{nk} < \gamma_0, \quad \sum_{k=1}^{m} \boldsymbol{e}_{mk}^T \Theta_1 \boldsymbol{e}_{mk} < \gamma_1$$
$$\sum_{k=1}^{m} \boldsymbol{e}_{mk}^T \Theta_2 \boldsymbol{e}_{mk} < \gamma_2, \quad \sum_{k=1}^{m} \boldsymbol{e}_{mk}^T \Theta_3 \boldsymbol{e}_{mk} < \gamma_3 \tag{13}$$

$$\begin{bmatrix} -\gamma_4 & \boldsymbol{v}_1^T Y^T & \boldsymbol{v}_2^T Y^T & \cdots & \boldsymbol{v}_m^T Y^T \\ Y\boldsymbol{v}_1 & -S_2 & & & \vdots \\ Y\boldsymbol{v}_2 & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ Y\boldsymbol{v}_m & \cdots & \cdots & \cdots & -S_2 \end{bmatrix} < 0 \tag{14}$$

where

$$\begin{aligned} \Lambda_0 &= AX + XA^T - BR^{-1}B^T + \zeta D_A D_A^T + \epsilon D_B D_B^T \\ &\quad + \epsilon_{inv} BR^{-1}E_B^T E_B R^{-1} B^T + \delta D_B D_B^T \\ &\quad + \nu_{inv} BR^{-1} E_B^T E_B R^{-1} B^T \\ \bar{\Lambda}_0 &= S_2 A_{11} + A_{11}^T S_2 + Y A_{21} + A_{21}^T Y^T \\ Y &= S_2 L, \ Z = [S_2 \ \ Y], \\ G_1 &= E_B R^{-1} B^T S_1 P, \ G_2 = D_A^T Z^T \\ G_3 &= D_B^T Z^T, \ G_4 = B^T S_1 P \\ \Theta_0 &= \frac{1}{2}(S_1(\Sigma_0 + \boldsymbol{m}_0\boldsymbol{m}_0^T) + (\Sigma_0 + \boldsymbol{m}_0\boldsymbol{m}_0^T)^T S_1) \\ \Theta_1 &= \frac{1}{2}(S_2\Sigma_{11} + \Sigma_{11}S_2), \ \Theta_2 = \frac{1}{2}(Y\Sigma_{21} + \Sigma_{21}^T Y^T) \\ \Theta_3 &= \frac{1}{2}(Y^T\Sigma_{12} + \Sigma_{12}^T Y), \ \Sigma_{22}^{1/2} = [\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_m] \\ \Sigma_0 &= \begin{matrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{matrix}, \ \boldsymbol{e}_{ik} = \left[\boldsymbol{0}_{k-1}^T \ 1 \ \boldsymbol{0}_{i-k}^T\right]^T \end{aligned}$$

has a solution $S_1 > 0$, $S_2 > 0$, $X > 0$, Y, Z, $\zeta > 0$, $\delta > 0$, $\epsilon > 0$, $\epsilon_{inv} > 0$, $\theta > 0$, $\theta_{inv} > 0$, $\mu > 0$, $\mu_{inv} > 0$, $\nu_{inv} > 0$, γ_0, γ_1, γ_2, γ_3, γ_4 which satisfy the relation $\epsilon^{-1} = \epsilon_{inv}$, $\theta^{-1} = \theta_{inv}$, $\mu^{-1} = \mu_{inv}$ and $S_1^{-1} = X$, then the minimal order observer-based control law (6)-(8) with (10) is a guaranteed cost controller which gives the minimum expected value of the guaranteed cost

$$E[J^*] = E\left[\boldsymbol{x}^T(0)S_1\boldsymbol{x}(0) + \boldsymbol{\xi}^T(0)S_2\boldsymbol{\xi}(0)\right] \tag{15}$$

where $\boldsymbol{\xi}(t) = \boldsymbol{z}(t) - T\boldsymbol{x}(t)$ is the estimated error of the minimal order observer.

Remark 1: Since (11) and (12) have a constraint of the relationship of the inverse, ILMI approach is introduced to solve (Ghaoui et al [4], Cao et al [5]).

Before giving a proof of Theorem 1, a key lemma is introduced (Mahmoud and Zribi [6]).

Lemma 1. Let D and E be matrices of appropriate dimensions, and F be a matrix function satisfying $F^TF \le I$. Then for any positive scalar α, the following inequality holds

$$DFE + E^TF^TE^T \le \alpha DD^T + \alpha^{-1}E^TE. \tag{16}$$

Proof of Theorem 1.

Equations (1) and (6)-(8) yield the closed-loop system

$$\begin{matrix} \dot{\boldsymbol{x}}(t) \\ \dot{\boldsymbol{\xi}}(t) \end{matrix} = \begin{matrix} \Phi_1 & \Phi_2 \\ \Phi_3 & \Phi_4 \end{matrix} \begin{matrix} \boldsymbol{x}(t) \\ \boldsymbol{\xi}(t) \end{matrix} \tag{17}$$

where

$$\begin{aligned} \Phi_1 &= A + \Delta A(t) + (B + \Delta B(t))K \\ \Phi_2 &= (B + \Delta B(t))KP \\ \Phi_3 &= -T\Delta A(t) - T\Delta B(t)K \\ \Phi_4 &= D - T\Delta B(t)KP \end{aligned}$$

Define a candidate of Lyapunov function as

$$V(t) = \boldsymbol{x}^T(t)S_1\boldsymbol{x}(t) + \boldsymbol{\xi}^T(t)S_2\boldsymbol{\xi}(t) \tag{18}$$

then, the time derivative of (18) along to (17) is calculated as

$$\dot{V}(t) = \boldsymbol{w}^T(t)\Omega\boldsymbol{w}(t) - (\boldsymbol{x}^T(t)Q\boldsymbol{x}(t) + \boldsymbol{u}^T(t)R\boldsymbol{u}(t)) \tag{19}$$

where

$$\boldsymbol{w}(t) = \begin{matrix} \boldsymbol{x}(t) \\ \boldsymbol{\xi}(t) \end{matrix}, \ \Omega = \begin{matrix} \Lambda_1 & \Lambda_2 \\ \Lambda_2^T & \Lambda_3 \end{matrix}$$

$$\begin{aligned} \Lambda_1 &= S_1(A + \Delta A(t)) + (A + \Delta A(t))^T S_1 \\ &\quad - S_1 BR^{-1}B^T S_1 + Q - 2S_1\Delta B(t)R^{-1}B^T S_1 \\ \Lambda_2 &= -S_1\Delta B(t)R^{-1}B^TS_1P - \Delta A^T(t)T^TS_2 \\ &\quad + S_1BR^{-1}\Delta B^T(t)T^TS_2 \\ \Lambda_3 &= S_2D + D^TS_2 + P^TS_1BR^{-1}B^TS_1P \\ &\quad + 2S_2T\Delta B(t)R^{-1}B^TS_1P \end{aligned}$$

Under the condition

$$\Omega < 0 \tag{20}$$

equation (19) leads to

$$\dot{V}(t) < -(\boldsymbol{x}^T(t)Q\boldsymbol{x}(t) + \boldsymbol{u}^T(t)R\boldsymbol{u}(t)) < 0 \tag{21}$$

for any $\boldsymbol{x}(t) \neq \boldsymbol{0}$ and the closed-loop system is asymptotically stable.

Applying lemma 1., pre- and post-multiplying by diag(S_1^{-1}, I) on both sides, denoting $X = S_1^{-1}$, $Y = S_2L$, $\epsilon_{inv} = \epsilon^{-1}$, $\theta_{inv} = \theta^{-1}$, $\mu_{inv} = \mu^{-1}$, $\nu_{inv} = \nu^{-1}$, and using Schur Complement lead to (11) and (12).

Then, integrating (21) from 0 to T and as T tends to the infinity yields

$$\begin{aligned} J &= \int_0^\infty (\boldsymbol{x}^T(t)Q\boldsymbol{x}(t) + \boldsymbol{u}^T(t)R\boldsymbol{u}(t))dt \\ &< \boldsymbol{x}^T(0)S_1\boldsymbol{x}(0) + \boldsymbol{\xi}^T(0)S_2\boldsymbol{\xi}(0) = J^* \end{aligned} \tag{22}$$

where J^* denotes the guaranteed cost. Here, we consider the optimal expected value of the guaranteed cost. It is calculated as

$$E[J^*] = \mathrm{tr}S_1E\left[\boldsymbol{x}(0)\boldsymbol{x}^T(0)\right] + \mathrm{tr}S_2E\left[\boldsymbol{\xi}(0)\boldsymbol{\xi}^T(0)\right] \tag{23}$$

A relation between mean and covarience of $\boldsymbol{x}(0)$ is given by

$$\Sigma_0 = E\left[\boldsymbol{x}(0)\boldsymbol{x}^T(0)\right] - \boldsymbol{m}_0\boldsymbol{m}_0^T \tag{24}$$

Substituting (24) into (23) yields

$$\begin{aligned} E[J^*] &= \mathrm{tr}S_1(\Sigma_0 + \boldsymbol{m}_0\boldsymbol{m}_0^T) \\ &+ \mathrm{tr}S_2E\left[(\boldsymbol{z}(0) - T\boldsymbol{x}(0))(\boldsymbol{z}(0) - T\boldsymbol{x}(0))^T\right] \end{aligned} \tag{25}$$

Here, it is readily seen that

$$\begin{aligned} &E\left[(\boldsymbol{z}(0) - T\boldsymbol{x}(0))(\boldsymbol{z}(0) - T\boldsymbol{x}(0))^T\right] \\ &= T\Sigma_0T^T + (\boldsymbol{z}(0) - T\boldsymbol{m}_0)(\boldsymbol{z}(0) - T\boldsymbol{m}_0)^T \end{aligned} \tag{26}$$

Hence, (25) leads to

$$\begin{aligned} E[J^*] &= \mathrm{tr}S_1(\Sigma_0 + \boldsymbol{m}_0\boldsymbol{m}_0^T) + \mathrm{tr}S_2(T\Sigma_0T^T \\ &+ (\boldsymbol{z}(0) - T\boldsymbol{m}_0)(\boldsymbol{z}(0) - T\boldsymbol{m}_0)^T) \end{aligned} \tag{27}$$

Here, it can be assumed that an initial value of a minimal order observer $\boldsymbol{z}(0)$ satisfies the following equation without loss of generality.

$$\boldsymbol{z}(0) - T\boldsymbol{m}_0 = \boldsymbol{0} \tag{28}$$

Substituting (28) into (27) yields

$$\begin{aligned} E[J^*] &= \mathrm{tr}S_1(\Sigma_0 + \boldsymbol{m}_0\boldsymbol{m}_0^T) \\ &+ \mathrm{tr}S_2(\Sigma_{11} + L\Sigma_{21} + \Sigma_{12}L^T + L\Sigma_{22}L^T) \end{aligned} \tag{29}$$

where

$$\Sigma_0 = \begin{matrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{matrix}$$

Here, we consider positive scalars γ_0, γ_1, γ_2, γ_3, γ_4 satisfying the following inequalities

$$\mathrm{tr}S_1(\Sigma_0 + \boldsymbol{m}_0\boldsymbol{m}_0^T) < \gamma_0 \tag{30}$$

$$\mathrm{tr}S_2\Sigma_{11} < \gamma_1 \tag{31}$$

$$\mathrm{tr}S_2L\Sigma_{21} < \gamma_2 \tag{32}$$

$$\mathrm{tr}S_2\Sigma_{12}L^T < \gamma_3 \tag{33}$$

$$\mathrm{tr}S_2L\Sigma_{22}L^T < \gamma_4 \tag{34}$$

Minimizing $\gamma_0 + \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4$ results in giving min $E[J^*]$. By recalling $\mathrm{tr}(AB) = \mathrm{tr}(BA)$, (30)-(33) lead to (13). Next, by denoting $\Sigma_{22}^{1/2} = [\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_m]$, (34) is calculated as

$$\begin{aligned} &\mathrm{tr}S_2L\Sigma_{22}L^T \\ &= \boldsymbol{v}_1^TY^TS_2^{-1}Y\boldsymbol{v}_1 + \boldsymbol{v}_2^TY^TS_2^{-1}Y\boldsymbol{v}_2 \\ &\quad + \cdots + \boldsymbol{v}_m^TY^TS_2^{-1}Y\boldsymbol{v}_m \\ &= \begin{bmatrix} \boldsymbol{v}_1^TY^T & \boldsymbol{v}_1^TY^T & \cdots & \boldsymbol{v}_1^TY^T \end{bmatrix} S_2^{-1} \begin{bmatrix} Y\boldsymbol{v}_1 \\ Y\boldsymbol{v}_2 \\ \vdots \\ Y\boldsymbol{v}_m \end{bmatrix} < \gamma_4 \end{aligned} \tag{35}$$

Further, Schur complement derives (14) from (35).
Q.E.D.

It is noted that the inequalities (11) and (12) cannot be solved directly by LMI because they contain the scalars ϵ, ϵ_{inv}, θ, θ_{inv}, μ, μ_{inv}, and two matrices S_1, X which satisfy the relation $S_1^{-1} = X$, $\epsilon^{-1} = \epsilon_{inv}$, $\theta^{-1} = \theta_{inv}$, $\mu^{-1} = \mu_{inv}$. There are a number of algorithms available in literature, and we apply the cone complementarity linearization approach (Ghaoui et al [4]) to propose the algorithm as follows.

Step 0: Set k_{max}, γ_{min} and κ.

Step 1: Choose a sufficiently large initial γ such that there exists a feasible solution to LMI conditions

$$\begin{matrix} S_1 & I \\ I & X \end{matrix} > 0,\ \gamma_0 + \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 < \gamma,$$
$$\epsilon\epsilon_{inv} > 1,\ \theta\theta_{inv} > 1,\ \mu\mu_{inv} > 1,$$
inequalities (11)-(14)

Step 2.1 : Set $\bar{\gamma} = \gamma$, $k = 0$, $i = 1$, set $S_1(k) = S_1$, $X(k) = X$, $\epsilon(k) = \epsilon$,$\epsilon_{inv}(k) = \epsilon_{inv}$, $\theta(k) = \theta$, $\theta_{inv}(k) = \theta_{inv}$, $\mu(k) = \mu$, $\mu_{inv}(k) = \mu_{inv}$.

Step 2.2 : Solve the following LMI problem t_k=Minimize(tr$[S_1(k)X + X(k)S_1]$) $+ \epsilon(k)\epsilon_{inv} + \epsilon\epsilon_{inv}(k) + \theta(k)\theta_{inv} + \theta\theta_{inv}(k) +$

$\mu(k)\mu_{inv} + \mu\mu_{inv}(k))$
subject to

$$\begin{matrix} S_1 & I \\ I & X \end{matrix} > 0,\ \gamma_0 + \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 < \gamma,$$

$\epsilon\epsilon_{inv} > 1,\ \theta\theta_{inv} > 1,\ \mu\mu_{inv} > 1,$
inequalities (11)-(14)

Step 3.1 : If $k < k_{max}$ and $t_k > 2n + 6 + \kappa$ then set $k = k + 1$ and go to 2.2.

Step 3.2 : If $k \leq k_{max}$, $t_k \leq 2n + 6 + \kappa$, LMI conditions are satisfied, and $\gamma(0.5)^i > \gamma_{min}$ then $\bar{\gamma}$=$\bar{\gamma}$-$\gamma(0.5)^i$. Else if $\gamma(0.5)^i \leq \gamma_{min}$ then exit and $\bar{\gamma}$ is an optimal value.

Step 3.3 : If $k < k_{max}$, $t_k \leq 2n + 6 + \kappa$, LMI conditions are not satisfied, $i \neq 1$ and $\gamma(0.5)^i > \gamma_{min}$ then $\bar{\gamma}$=$\bar{\gamma}$+$\gamma(0.5)^i$. Else if $\gamma(0.5)^i \leq \gamma_{min}$ then exit and $\bar{\gamma}$ is an optimal value. Else if $i = 1$ then exit and no optimal solution is obtained.

Step 3.4 : If $k = k_{max}$, $t_k > 2n + 6 + \kappa$, $i \neq 1$ and $\gamma(0.5)^i > \gamma_{min}$ then $\bar{\gamma}$=$\bar{\gamma}$+$\gamma(0.5)^i$. Else if $\gamma(0.5)^i \leq \gamma_{min}$ then exit and $\bar{\gamma}$ is an optimal value. Else if $i = 1$ then exit and no optimal solution is obtained.

Step 4 : Set $i = i + 1$ and return to 3.1.

This algorithm allows the optimal value $\bar{\gamma}$ can be reached faster than that of Matsunaga et al [1] because the correction is not fixed but depending on iteration i.

IV. AN ILLUSTRATIVE EXAMPLE

Consider a system with

$$A = \begin{bmatrix} -3 & 0 & -2 & 0 \\ 0 & -2 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 3 \\ 2 \\ -6 \\ 1 \end{bmatrix},$$

$$C = \begin{bmatrix} O_2 \ I_2 \end{bmatrix},\ \boldsymbol{m}_0 = \boldsymbol{0}_4,\ \Sigma_0 = I_4,\ R = 9,$$

$$Q = \mathrm{diag}(7, 15, 1, 3),\ D_A = \begin{matrix} 0.1I_2 & O_2 \\ O_2 & O_2 \end{matrix},$$

$$E_A = \begin{matrix} 0.3I_2 & 0.3I_2 \\ O_2 & O_2 \end{matrix},\ D_B = \mathrm{diag}(0.3, 0.1, 0.3, 0.1),$$

$$E_B = \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix}^T.$$

Applying Theorem 1, with k_{max}=200, γ_{min}=0.0001, κ=0.000001 and initial γ=100, we obtain a solution

$$L = \begin{matrix} -0.1844 & -0.0440 \\ -0.0074 & -0.2493 \end{matrix},$$

$$K = \begin{bmatrix} -0.3837 & -0.4460 & 0.5278 & -0.4720 \end{bmatrix},$$

$$\bar{\gamma} = E\left[J^*\right] = 16.8893.$$

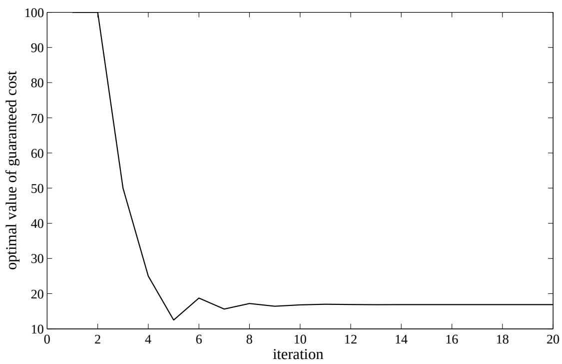

Fig. 1: Trajectory of optimal guaranteed cost γ.

V. CONCLUSION

A guaranteed cost observer-based control problem concerned on a minimal order observer has been discussed. A sufficient condition for the existence of state feedback guaranteed cost controllers is derived on the basis of the ILMI approach to solve inverse relation. A numerical example is given to illustrate the proposed method.

References

[1] D. Matsunaga, M. Ishitobi and S. Kunimatsu (2009), An ILMI approach to guaranteed cost controllers with a minimal order observer. Proc. of ICROS-SICE International Joint Conference 2009:583–588.

[2] C. H. Lien (2004), Robust observer-based control of systems with state perturbations via LMI approach. IEEE Trans. on Automatic Control Vol.49(8):1365–1370.

[3] S. Won and J. H. Park (1999), Design of observer-based controller for perturbed time-delay systems. JSME International Journal C Vol.42(1):129–132.

[4] L. E. Ghaoui, F. Oustry and M. AitRami (1997), A cone complementarity linearization algorithm for static output-feedback and related problems. IEEE Trans. on Automatic Control Vol.42(8):1171–1176.

[5] Y. Cao, J. Lam and Y. Sun (1998), Static output feedback stabilization: an ILMI approach. Automatica Vol.34(12):1641–1645.

[6] M. S. Mahmoud and M. Zribi (2003), Guaranteed cost observer-based control of uncertain time-lag systems. Computers and Electrical Engineering Vol.29(1):193–212.