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Swing-up and LQR stabilization of rotary inverted pendulum

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Abstract: In this paper, we considered swing-up and LQR stabilization of rotary inverted pendulum. A DC motor rotates a rigid arm. At the end of the rigid arm, joint is attached and a pendulum is suspended. Two encoders check the degree of the rigid arm and pendulum every 0.5ms. This paper proposes a modified bang-bang control which swings up a pendulum fast and safe. In order to solve the stabilization problem, this paper used linear quadratic regulator. When the user gives a large disturbance to the pendulum and when the pendulum loses its position, the pendulum quickly recovers to upright position. Experimental results showed that the proposed bang-bang controller and LQR controller can stabilize a rotary inverted pendulum system within 3.0s for any starting point. The system also showed robustness from large disturbance.

Keywords: Inverted pendulum, Swing-up, Linear Quadratic Regulator

I. INTRODUCTION

Inverted pendulum has been a great test bed for decades to deal with the control problems. Inverted pendulum has nonlinear characteristics and are not difficult to analyze systems near the equilibrium point. For these reasons, a lot of researchers used this tool to verify their ideas.

Furuta proposed rotary inverted pendulum which has a direct-drive motor as its actuator source and a pendulum attached to the rotating shaft of the motor [1]. Yoshida proposed an energy-based swing-up control and Åström showed that swinging up a pendulum by energy control is very effective and if acceleration of the pivot is sufficiently large, the pendulum can reach upright position in one swing [2], [3].

In recent years, a lot of control theories have been applied to control inverted pendulum such as fuzzy control [4], adaptive PID control [5], iterative impulsive control [6], neural network control, sliding mode control and other various control methods.

Most papers concentrated on how to swing-up the pendulum and how to stabilize it to upright position. In this paper, we will also deal with situations after the pendulum was stabilized. The user applied large disturbance to a stabilized pendulum which pushed the pendulum to an unstable position. Once the pendulum lost its position, the controller returns it back to stable position quickly. We adjusted the parameters of LQR controller by simulations and experiments.

This paper is organized as follows. In section II, design and kinematics of rotary inverted pendulum by [1] will be introduced. Section III, IV discusses swing-up strategy and LQR controller. Controllers are switched properly by control laws. Experimental results are shown section V. Conclusions are in the last section.

II. ROTARY INVERTED PENDULUM

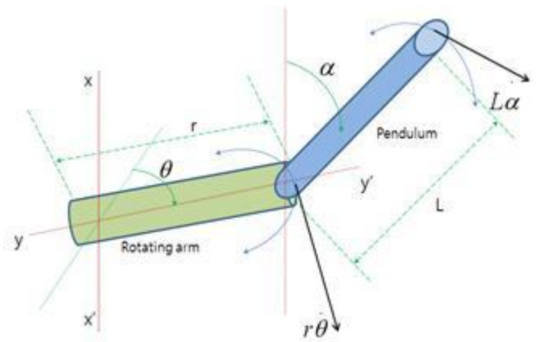

Fig. 1. Sketch of the rotary inverted pendulum

The above figure shows a sketch of the rotary inverted pendulum [1]. To use the Euler-Lagrange dynamic equation,

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}})-(\frac{\partial L}{\partial \theta})=T,\quad \frac{d}{dt}(\frac{\partial L}{\partial \dot{\alpha}})-(\frac{\partial L}{\partial \alpha})=0 \qquad (1)$$

we need to get the kinetic and potential energy. Assuming that mass of the rotating arm is M and the mass of pendulum is m. The kinetic energy of this system is

$$\begin{aligned} K_{net} &= K_{arm}+K_{pen} \\ K_{arm} &= \frac{1}{2}I\dot{\theta}=\frac{1}{6}Mr^2\dot{\theta}^2 \\ K_{pen} &= \frac{m}{2L}\int_0^L\{(r\dot{\theta})^2+(l\dot{\alpha})^2+2r\dot{\theta}l\dot{\alpha}\cos\alpha\}dl \\ &= \frac{1}{2}m(r\dot{\theta})^2+\frac{1}{6}L^2m\dot{\alpha}^2+\frac{1}{2}Lmr\dot{\theta}\dot{\alpha}\cos\alpha \end{aligned} \qquad (2)$$

A rotating arm's potential energy is zero, so the net potential energy of system is

$$P_{net} = P_{pen} = mg\frac{L}{2}\cos\alpha \quad (3)$$

Putting (2) and (3) to Euler-Lagrange equation (1), we can get two equations of motion.

$$L = K_{net} - P_{net}$$
$$T = (\frac{1}{3}Mr^2 + mr^2)\ddot{\theta} + \frac{1}{2}mrL(\ddot{\alpha}\cos\alpha - \dot{\alpha}\sin\alpha) \quad (4)$$
$$0 = \frac{1}{3}mL^2\ddot{\alpha} + \frac{1}{2}mrL\ddot{\theta}\cos\alpha - \frac{1}{2}mgL\sin\alpha$$

The table below shows a specification of SRV02 with ROTPEN.

Table 1. Specification of inverted pendulum

Parameter	Definition	Value	Units
r	Rotating arm length	20	cm
L	Pendulum length	35	cm
m	Pendulum mass	0.128	kg
M	Rotating arm mass	0.278	kg

III. SWING-UP STRATEGY

Swing-up strategy is giving 90% of maximum torque to the opposite direction of pendulum speed when the pendulum heads near the ground. It is a very simple strategy and it works if motor's torque is enough.

However, if a pendulum being pushed by a user lost its position, it could revolve very fast. If a pendulum's speed is high, then swing-up control laws gives acceleration to the pendulum. When acceleration and speed are increased, LQR controller couldn't hold it anymore. This can makes the pendulum rotates all the time and it may cause malfunction of a motor.

To prevent this problem, we need to modify the control laws. We introduce the weighting factor ω, which decides the working range of swing-up controller. Because encoders give α and θ every 0.005sec, we can get pendulum's speed easily. ω decreases when the pendulum's speed is high and vice versa. A motor operates if α located between $-210 < \alpha < -150$ degrees, and ω changes according to the speed of the pendulum. Output torque is 90% of maximum torque multiplied by ω. By trial and error, if $\dot{\alpha}$ is faster than $900(\deg/\sec)$, the pendulum revolves. Therefore we determined ω as follows.

$$\omega = 1 \qquad for\ |\dot{\alpha}| < 600(\deg/\sec)$$
$$\omega = 3 - \frac{|\dot{\alpha}|}{900}\ \ for\ 600 \le |\dot{\alpha}|(\deg/\sec) \quad (5)$$

Modified control law slowly shuts down the motor when the pendulum's speed is too high. It also operates the motor in the opposite direction when $\dot{\alpha}$ is extremely high. Overall, this controller is same as the bang-bang controller when $\dot{\alpha}$ is in the normal range.

If you change the working range of swing-up controller, ω should be reconsidered.

IV. LQR STABILIZATION

When the swing-up controller brings the pendulum to the range between -30 degrees and 30 degrees, the LQR controller will operate. We can linearize (4) near the equilibrium point ($\alpha = 0$). For convenience, we set the auxiliary variables as shown in the equations below.

$$a_1 = \frac{1}{3}Mr^2 + mr^2,\ a_2 = \frac{1}{2}mrL,\ a_3 = \frac{1}{3}mL^2,\ a_4 = \frac{1}{2}mgL$$
$$\ddot{\theta} = \frac{a_2a_4 - a_3T}{a_2^2 - a_1a_3}, \quad \ddot{\alpha} = \frac{a_1a_4 - a_2T}{-a_2^2 + a_1a_3} \quad (6)$$

By (6), we make the state-space representation:

$$\theta = x_1,\ \dot{\theta} = x_2,\ \alpha = x_3,\ \dot{\alpha} = x_4$$
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & \frac{a_2a_4}{a_2^2 - a_1a_3} & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & -\frac{a_1a_4}{a_2^2 - a_1a_3} & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ -\frac{a_3}{a_2^2 - a_1a_3}\\ 0\\ \frac{a_2}{a_2^2 - a_1a_3}\end{bmatrix}T \quad (7)$$

After digitizing (7) with 2 kHz, the system changes to

$$x(k+1) = \Phi x(k) + \Gamma u(k) \quad (8)$$

The system (7) is unstable but controllable. Therefore by using linear quadratic regulator, we can make it stable. LQR minimize the object function

$$J = \frac{1}{2}\sum_{k=0}^{n-1}[x^T(k)Q_1x(k) + u^T(k)Q_2u(k)] + \frac{1}{2}x^T(N)Q_0x(N) \quad (9)$$

and its control law is

$$u(k) = -K_\infty x(k)$$
$$K_\infty = (Q_2 + \Gamma^T S_\infty \Gamma)^{-1} S_\infty \Phi \quad (10)$$

where

$$S_\infty = \Phi^T[S_\infty - S_\infty \Gamma R^{-1}\Gamma^T S_\infty]\Phi + Q_1$$
$$R = Q_2 + \Gamma^T S_\infty \Gamma \quad (11)$$

We want to settle α and θ as fast as possible. Q_2 is just 1 and Q_1 is

$$Q_1 = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (12)$$

which emphasize α and θ. After calculate (10) and (11), we can finally get K_∞.

V. EXPERIMENTAL RESULTS

Controllers discussed in section 3 and 4 are used for these experiments. We used SRV02 with ROTPEN.

1. Swing-up and small disturbance

At the start, the pendulum is located at -180 degree. After the pendulum is stabilized, the user push it to the left and to the right. This changes alpha by almost 20 degrees but not enough to make the pendulum fall down.

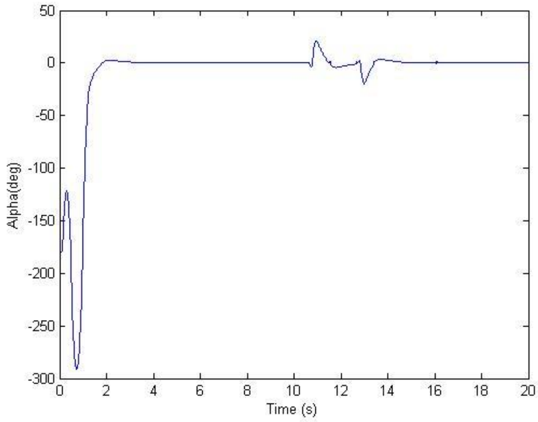

Fig. 2(a). Angle of a pendulum

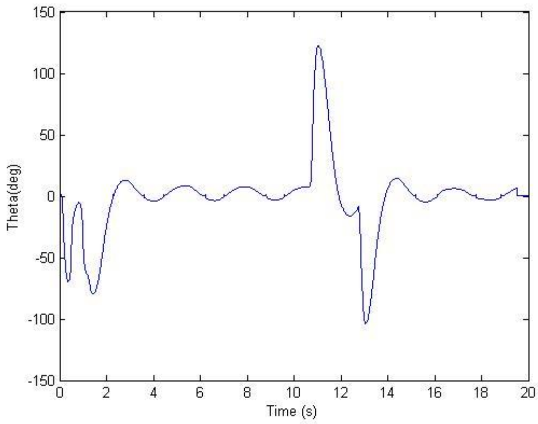

Fig. 2(b). Angle of rotating arm

Fig. 2(a) and Fig. 2(b) show the first experiment result. A pendulum swung to upright position in 2 seconds. Between 10 ~ 14 seconds, the user applied disturbance to the left and right. The pendulum head was moved about 20 degrees and the rotating arm was moved more than 100 degrees but the controller was able to recover their position quickly. This shows that both controllers work correctly.

2. Large disturbance

This time, the user will apply a large disturbance which can make the pendulum fall down to the bottom. We will show that the controller is still able to move it back to upright position.

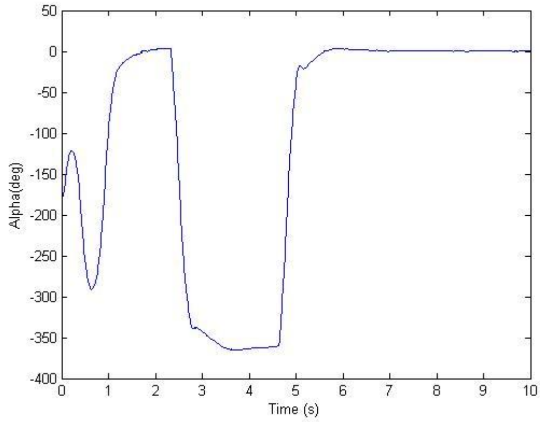

Fig. 3(a). Angle of a pendulum

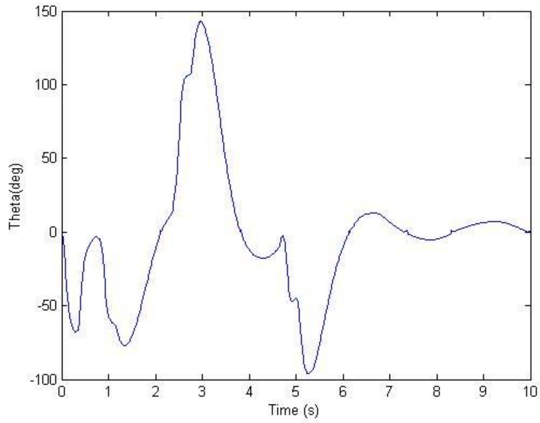

Fig. 3(b). Angle of a rotating arm

Fig. 3(a) and Fig. 3(b) shows the second experiment result. In the figures, 0 and 360 degrees represent same angle. Between 2 and 3 seconds, the pendulum fell down but recovered to the upright position right away. Between 4 and 5 seconds, user applied disturbance to the opposite direction and it still operated.

VI. CONCLUSION

In this paper, the modified bang-bang controller and LQR controller worked successfully. The pendulum showed convergence and achieved robustness from the large disturbance. Swing-up time was less than 3.0s and the whole recovery process worked immediately.

REFERENCES

[1] K. Furuta, M. Yamakita and S. Kobayashi (1991), Swing Up control of Inverted Pendulum. IECON'91, pp. 2193-2198.

[2] K. Yoshida (1999), Swing-up control of an inverted pendulum by energy based methods. Proceedings of the American control conference, pp. 4045-4047.

[3] K. J. Åström, K. Furuta (2000), Swinging up a pendulum by energy control. Automatica, vol. 36, no. 2, pp. 287-295.

[4] J. Krishen and V.M. Becerra (2006), Efficient fuzzy control of a rotary inverted pendulum based on LQR mapping. IEEE International Symposium on Intelligent Control, pp. 2701-2706.

[5] T. C. Kuo, Y. J. Huang and B. W. Hong (2009), Adaptive PID with Sliding Mode Control for the Rotary Inverted Pendulum System. Advanced Intelligent Mechatronics, pp. 1804-1809.

[6] Z. Wang, Y. Chen and N. Fang (2004), Minimum-Time Swing-up of A Rotary Inverted Pendulum by Iterative Impulsive Control. Proceedings of the 2004 American Control Conference, vol. 2, pp. 1335-1340.

[7] N. Muskinja and B. Tovornik (2006), Swinging up and stabilization of a real inverted pendulum. IEEE Transactions on Industrial Electronics, vol. 53, no. 2, pp. 631-639.