The Sixteenth International Symposium on Artificial Life and Robotics 2011 (AROB 16th '11),
B-Con Plaza, Beppu,Oita, Japan, January 27-29, 2011

Emergence of Observable Rules in a Spatial Game System

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Abstract: The present study focuses on player's strategies observed from outside in our original spatial game iterated by players, each of which placed in each lattice site on a two dimensional square lattice. A particularity of the game lies in the point that a player's strategy is not preliminary given, but constructed dynamically in response to a spatial pattern on player's actions. This means the strategy can evolve in time. However simulations bring us unexpected results. Actually all of strategies observed from outside did not evolve and they were fixed in time. This paper enumerates all of the observed strategies in detail and examines their characteristics.

Keywords: Spatial game, Inverse problem, External observer

I. INTRODUCTION

It is very important for many scientific fields ranging from science, medical science to engineering to extrapolate inputs or rules hidden from outside by knowing outputs such as observational results. This kind of problem is called the "inverse problem". Richard et al. [1] try to extract a cellular automaton rule to produce a spatiotemporal pattern of given experimental data by employing the genetic algorithm. Ueda and Ishida [2] deal with one dimensional cellular automata and examine to extrapolate a rule generating given spatiotemporal patterns of cell states. The present study addresses an inverse problem with a spatial game by players placed in a two dimensional square lattice space and discusses whether it is possible to extrapolate player's strategy from spatiotemporal patterns on player's actions. In our model, each player observes actions of its neighborhood players, and builds its own strategy by applying the observed actions for a strategy-building rule and determines its action for the next round game. We suppose an external observer watching a game from outside cannot know what strategy each player applies for determining its action, but can observe actions of all players in each round. Under this setting, the present study discusses a possibility that a player's strategy extrapolated through a global observation by the external observer chimes perfectly with the real player's strategy, and presents a disaccord case: although each player's strategy actually evolves in each game round, the external observer observes any player obeys a unique steady strategy. This result suggests that a model which enables to consistently explain the observation results from outside is not always true one.

II. SPATIAL GAME MODEL

Our study considers a spatial game on a two-dimensional square lattice. Each player is placed on each lattice site. To designate each lattice site, a horizontal axis with a suffix j and a vertical one with a suffix i (i,j=0,1,…,L-1) are prepared. We let a player to select one of two actions: 0, 1. A symbol, $A_{i,j}^{r}$ denotes the player (i,j)'s action at the game round r, and a symbol $S_{i,j}^{r}$ denotes the player (i,j)'s strategy at the game round r. Our model adopts so-called "spatial" strategy. The spatial strategy determines a player's action from the total number of players taking an action "1" in its eight neighborhood players called the *Moore* neighbor.

Figure 1 shows an example of the player (i,j)'s spatial strategy at the round r+1. A symbol "k" is the total number of players taking an action "1" in the *Moore* neighbor of (i,j) site. A symbol "A" is the player (i,j)'s action at the round r+1. In this example, As k amounts to 5, the strategy determines the player (i,j)'s action at the round r+1 becomes 0.

The original of the proposed spatial game [3] lies in that a determination relationship between a strategy and an action is not one-way but mutual way: a strategy determines an action as well as actions determine a strategy. This implies to destroy the original boundary between the strategy and the action. Gunji [4] tries to destroy a boundary between hierarchies on a level of a coarse graining by introducing an internal observer enabling only local observation.

We introduce a strategy building rule to build a strategy based on a spatial configuration of player's actions. The strategy-building rule defines the correspondence between k and player's action A as follows.

When k is 0, let an action A to be $A^r_{i-1,j-1}$.
When k is 1, let an action A to be $A^r_{i-1,j}$.
When k is 2, let an action A to be $A^r_{i-1,j+1}$.
When k is 3, let an action A to be $A^r_{i,j+1}$.
When k is 4, let an action A to be $A^r_{i+1,j+1}$.
When k is 5, let an action A to be $A^r_{i+1,j}$.
When k is 6, let an action A to be $A^r_{i+1,j-1}$.
When k is 7, let an action A to be $A^r_{i,j-1}$.
When k is 8, let an action A to be $A^r_{i,j}$.

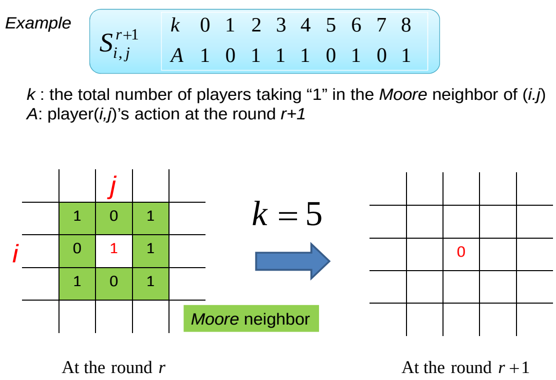

Fig.1. An example of how to build a strategy

Figure 1 shows a process to build the strategy $S_{i,j}{}^{r+1}$ from a spatial configuration of player (i,j)'s *Moore* neighbor through the strategy-building rule.

The spatial game proceeds as stated follows. As an initial condition, all players must preliminarily determine their actions. One round game consists of the two steps:

1. Each player estimates the quantity k from player's actions in its *Moore* neighbor at the previous round, and builds its strategy based on the strategy-building rule.

2. Each player takes the next action corresponding to the quantity k referring to its strategy.

In addition, this study does not interest in relative merits between strategies, thus not consider a scoring process based on a score table representing comparative merits between actions.

III. SIMULATION RESULTS

This study chooses the lattice-size L to be its minimal value: three, hence considers the spatial game on a two dimensional 3x3 square lattice. The number of players is nine. A boundary condition of the square lattice space is periodic.

1. PERIODIC ATTRACTORS

The previous study [3] clarified every observed game becomes periodic. Types of the observed period are period-1 (fixed point), period-3, period-6 and period-9. Some initial bit configurations converge into a periodic game, hence some periodic games are "attractive" in other words, *attractor*. The number of attractors with a certain period: period-1, -3, -6 and -9 is 2, 15, 1 and 2, respectively.

2. EXTERNALLY OBSERVED STRATEGIES

This section shows an odd discrepancy that each player's strategy dynamically evolves in response with change of a bit spatial pattern of its *Moore* neighbor; however a strategy extrapolated by the external observer is irrelevant with the spatial bit pattern thus steady. This section also mentions that interestingly the external observer sees a player have a memory from its behavior.

PERIOD-3 GAME

Strategies observed from outside in the 3-period games are generally described as a "copy" strategy of memorizing a player's action in the previous round and using its action in the next round. Figure 2 exhibits an example of a strategy observed from outside in a period-3 game, each player memorizes an action in the previous round of a certain player placed in the 'right' side for its own site and the player takes the memorized action in the next round. In the other period-3 game, each player memorizes an action in the previous round of the player placed in not the 'right' but the 'bottom left' for its own site and uses the memorized action. It is interesting that in the period-3 games, the external observer must see any player have one bit memory to memorize one bit action.

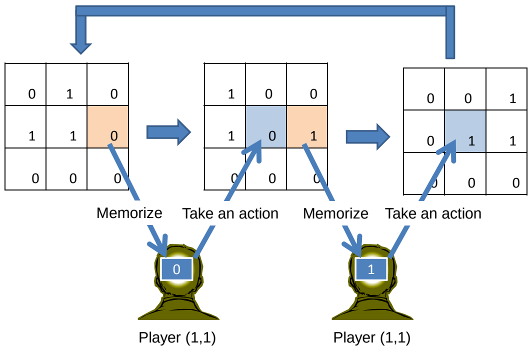

Fig.2. A strategy observed from outside in a period-3 game

PERIOD-6 GAME

A strategy observed from outside in the period-6 game is the same as the one observed in the period-3 game except memorizing a player's action in not the one but two rounds before. Figure 3 exhibits an example of a strategy that each of all players memorizes an action in the two rounds before of the player placed in the right side of each own site, and then takes the memorized action in the next round. In the period-6 game, the external observer must see all players have two bits memory.

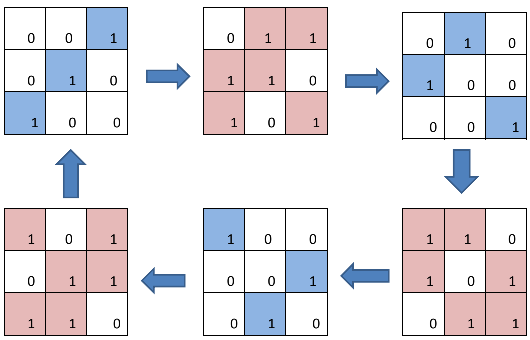

Fig.3. An example of a strategy in the priod-6 game

PERIOD-9 GAME

Strategies observed from outside in the period-9 game is the same as those observed in the period-3 and -6 except memorizing a player's action in not the one/two but three rounds before. Figure 4 exhibits an instance of a strategy in a period-9 game that each player memorizes an action in three rounds before of a certain player placed in the just above for each own site, and the player takes the memorized action in the next round. It is interesting that in the 9-period games, the external observer must see any player have three bits memory.

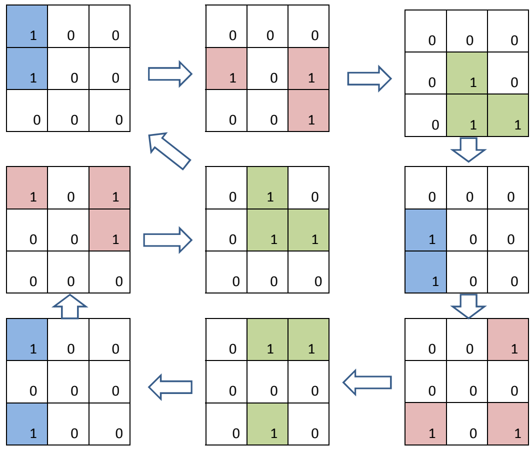

Fig.4. An example of a strategy in a period 9 game

The table 1 sums up all of strategies observed from outside. The strategies are specified by three parameters: *P*, *D* and *T* because all strategies observed in the period "*P*" games are described as a copy strategy of each player memorizes an action in the "*T*" round(s) before of a certain player placed in the "*D*" direction for each own site and then takes the memorized action. The rest of the parameters is *N* which is a number of different games with a certain strategy.

Table1. All types of strategies observed from outside in every periodic game

P	*T*	*D*	*N*
3	1	Just above	1
		Just below	3
		Top right	2
		Right	4
		Bottom right	2
		Top left	0
		Left	1
		Bottom left	2
6	1	Right	1
9	3	Just above	1
		Left	1

The table.1 suggests that in the period-3 games, a strategy to copy an action of player in the top left position is not observed. It also shows the number of games with a strategy to copy an action of a player in the right side is four, whereas in the case of copying an action of player in the opposite left side, number of games is only one. Like this, in response with the difference of the parameter *D*, it features that there is imbalance on the number of the observed games. This asymmetric nature is considered due to the asymmetry of the strategy-building rule.

IV. DISCUSSIONS

It seems to be odd that the strategies observed from outside in the proposed game model do not depend on a bit pattern of actions in the *Moore* neighbor although the actual strategy defined in the model depends on it. Figures 2 and 5 shows the same game from outside and inside respectively. The figure 2 exhibits the player (1,1)'s strategy is just a copy strategy irrelevant with a bit pattern of its *Moore* neighbor. On the other hand, Figure 5 exhibits the player (1,1)'s strategy evolves in response with a bit pattern of its *Moore* neighbor. Here we discuss through this example it is possible the situation that externally observed strategies do not evolve but in fact internally defined strategies evolve. Now it supposes the externally observed strategy in Fig. 2 that all the players memorizes an action in the previous round of the player in the just below for their own site and they take their memorized action in the next round. Then a bit pattern at the round r+1 of the *Moore* neighbor of a player (i,j) becomes equal with a bit pattern at the r round of the *Moore* neighbor of a player (i,j+1). Following the strategy-building rule, if a pair of bit patterns of the *Moore* neighbor of the two players, (i,j) and (i,j+1) are the same, their strategies are the same:

$$S_{i,j}^{r+1} = S_{i,j+1}^{r}. \qquad (1)$$

It would be valid to suppose that at the round r, a bit pattern of the *Moore* neighbor of a player (i,j) does not equal with that of the *Moore* neighbor of a player (i, j+1). Thus,

$$S_{i,j}^{r} \neq S_{i,j+1}^{r} \qquad (2)$$

Eqs. (1) and (2) lead to

$$S_{i,j}^{r+1} \neq S_{i,j}^{r}. \qquad (3)$$

In other words, the strategy of the player (i,j) evolves.

This discussion shows that it is possible that externally observed strategies do not evolve but in fact inner defined strategies evolve.

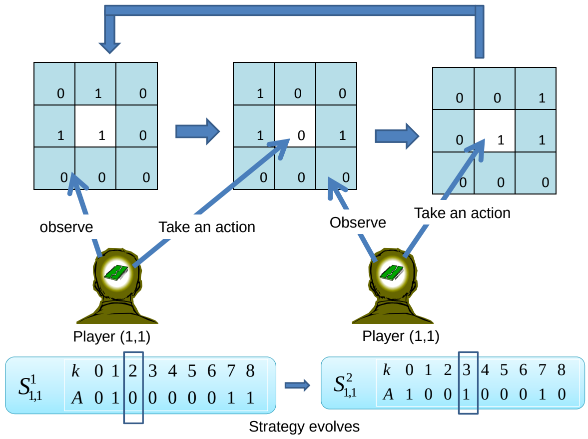

Fig.5 Time evolution of the player (1,1)'s strategy

To solve our inverse problem, what will be needed?

We will need more external observational data on various types of periodic games. However, if we randomly select an initial spatial bit pattern on player's actions and starts the game, it must be hard to observe a certain periodic game with a smaller basin size because a probability of its game being selected becomes lower. For instance, a basin size of the period-6 game in Fig.3 is zero, thus a probability of the game being observed is very low: 6/256. Here the basin size of a periodic game is defined as the number of initial bit patterns on player's actions which finally transit to its game.

It is necessary to focus on identifying not an individual player's strategy but a "meta"-rule to build an individual strategy: the strategy-building rule. However is it possible to discover not individual rules but a meta-rule which is their origin from the observational data? In the future work, we have to think this issue.

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