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Extracting probabilistic cellular automata rules from spatio-temporal patterns and analyzing features of these rules

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***Abstract*:** This research deals with one of the inverse problem. It is estimating rules or strategies of generating spatio-temporal patterns which is generated by natural phenomena or social phenomena. We try to consolidate identifying method and evaluation method to clarify generative mechanism. In this research, mainly, we use probabilistic cellular automata (PCA) to describe generative mechanism. And we restrict spatio-temporal patterns to ASEP patterns. In particular, we discuss conservation of mass in ASEP model from spatio-temporal patterns.

***Keywords*:** Probabilistic Cellular Automata (PCA), Reverse problem, Spatio-temporal patterns, ASEP

1 Introduction

Natural phenomena including physical and biological ones present us inexhaustible amount of spatial patterns: patterns of ice crystal, cloud, coastal railroad, forest fire, leaf arrangement, shells, and butterflies, to name only a few. Social phenomena also generate spatial patterns (e.g. traffic jams). They are a few compared to natural one.

Constructing physical or mathematical models are known as the way of research for understanding the generative mechanisms of these spatial patterns. On the other hand, Richards studied extracting cellular automaton rules directly from experimental data [1]. They dealt with spatial patterns of dendrites formed by NH_4Br. They searched the space of rules which is a set of probabilistic CA (PCA) rules as possible models for spatio-temporal patterns, with a learning algorithm. Ichise proposed a general and theoretical method for identifying a generative mechanism of spatio-temporal patterns in CA frameworks [2]. They restricted rules to one dimensional and straightforward ones. Hence Richards's and Ichise's researches are one of the reverse problems for traditional ones.

In this paper we expand the Ichise's method which can identify generative mechanism of spatio-temporal patterns to target more complex mechanisms. The previous method targets on spatio-temporal patterns which is generated by one dimensional and straightforward rules of CA. However, we deal with characteristic patterns (e.g. patterns of traffic jams, diffusion of matters and other physical phenomena) which satisfy conservation of mass. In fact, we succeeded to develop a method to discover the conservation of mass from these patterns.

Section 2 states definitions and notations used in this paper. Section 3 represents the method of the generative mechanism identification in the previous study, and its problem. Section 4 expands the previous method for patterns of conservation of mass, and applies to the ASEP model ones.

2 Definitions and Notations

After von Neumann used cellular automata (CA) in his designing self-reproducing automata [3], not only deterministic cellular automata (DCA) (e.g. [4, 5, 6]) but also probabilistic cellular automata (PCA) (e.g. [7]) have been studied extensively. Cellular automaton consists of cells arranged in a d-dimensional lattice where d is a natural number. Each cell is an automaton which has a certain number of states; whose inputs are the state of neighbor cells; and the output is the state of the cell itself. In this paper, we restrict ourselves to the case of binary state: 0 and 1 and one-dimensional lattice with periodic boundary condition where each cell has two neighbor cells: right and left. s_i^t denotes the state of the cell i at the time step t and its state at the next time step defined in equation (1).

$$s_i^{t+1} = f(n_i^t) \tag{1}$$

n_i^t is states of the neighbor cells of the cell i at time step t and defined in equation (2) with neighborhood radius r.

$$n_i^t = (s_{i-r}^t, \cdots, s_i^t, \cdots, s_{i+r}^t) \tag{2}$$

$f : N \to S$ is a mapping and called "*transition rule*". S is the set of state and $N(= S^{2r+1})$ is the set of the neighbor cells state. Where equation (3) represents the graph f^* of f, $l_f(\in f^*)$ is called "*local rule of f*". Also, $l \in N \times S$ is called "*local rule*".

$$f^* = \{l_f | l_f = (n, s), n \in N, s = f(n)\} \tag{3}$$

When each cell changes state stochastically, the probability is described by equation (4).

$$P(s|n) \quad \text{Where,} \quad \forall n \in N \sum_{s \in S} P(s|n) = 1 \tag{4}$$

The condition of the cells is expressed by $c \in S^m$ where m is the number of cells and c_t indicates the condition at time step t. Then $C_T = (c_0, c_1, \cdots, c_T)$ represents a spatio-temporal pattern with T time steps.

3 Generative Mechanisms Identification

3.1 Deterministic Mechanisms

We consider to identify the transition rule f from a spatio-temporal pattern C_T. Scanning the spatio-temporal pattern C_T gives the local rules $f^* = \{l_f\}$, and we get the transition rule f. For example, the spatio-temporal pattern such as Table 1 is given, we can identify the transition rule of Table 2.

Table 1: An example of spatio-temporal pattern when $m = 5$ and $T = 1$.

c_0	$(0, 0, 1, 0, 0)$
c_1	$(0, 1, 0, 1, 0)$

Table 2: The identified deterministic rule from the spatio-temporal pattern of Table 1.

n_i^t	$(1, 0, 0)$	$(0, 1, 0)$	$(0, 0, 1)$	$(0, 0, 0)$
s_i^{t+1}	0	1	0	1

3.2 Probabilistic Mechanisms

If given C_T is generated by probabilistic rule, we need to estimate probabilistic distribution of local rule from it. Hence we calculate the occurrence ratios of each local rule. For example, Table 3 is a spatio-temporal pattern which is generated by the probabilistic rule and Table 4 is the probabilistic distribution of local rules of it.

Table 3: An example of spatio-temporal pattern when $m = 5$ and $T = 1$.

c_0	$(1, 0, 1, 0, 0)$
c_1	$(1, 0, 0, 1, 0)$

Table 4: The identified probabilistic rule from the spatio-temporal pattern of Table 3.

n_i^t	$(1, 0, 1)$	$(1, 0, 0)$	$(0, 1, 0)$	$(0, 0, 1)$
$P(s_i^{t+1} = 1 \| n_i^t)$	0	1	0.5	0

4 Conservation of Mass Patterns Identification

4.1 ASEP and its Patterns

ASEP (Asymmetical Simple Exclusion Process) [8] is known as the traffic jam model which satisfy the conservation of mass. ASEP is also one of PCA. Each car is arranged on the cell and goes forward at each time step with probability p if there is not any car to front (e.g. Figure 1). Each cell has binary state, 1 indicate existing a car and State 0 is not existing.

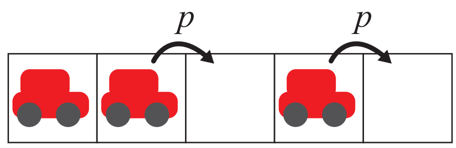

Figure 1: ASEP model. Each cell can be occupied by only single car. Only if there is not a car to front, each car goes forward at each time step with probability p.

Figure 2 shows the spatio-temporal pattern which generated by ASEP. Clumps of black cells indicate that the traffic jams are occurring.

4.2 Problem of Previous Method

The previous method has a problem when the spatio-temporal pattern satisfies the conservation of mass. Because the previous method gives the rule of straightforward CA (such as defined in Section 2). In general, straightforward CA changes states synchronously and cannot generate patterns which satisfy the conservation of mass [9].

Table 5,6 show the rules identified from Figure 2, 3. Figure 2 is different from Figure 3. Figure 2 satisfy the conservation of mass and Figure 3 is not. However, the

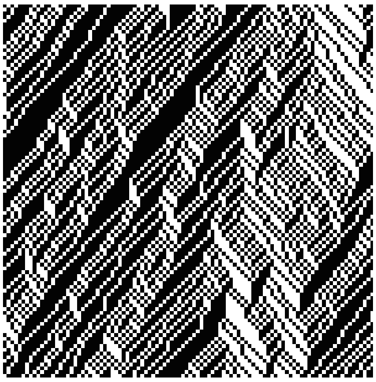

Figure 2: The spatio-temporal pattern which generated by ASEP with $p = 0.8$. The black cells indicate 1 and the white cells are 0. The vertical axis represents time step and the horizontal is space.

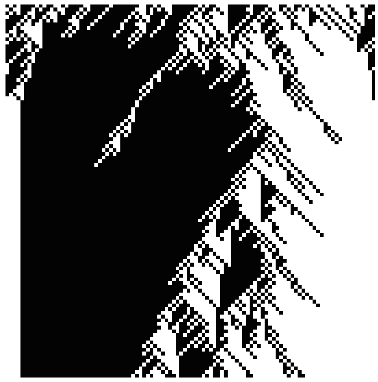

Figure 3: The spatio-temporal pattern which generated by PCA without the conservation of mass. The black cells indicate 1 and the white cells are 0. The vertical axis represents time step and the horizontal is space.

rules are remarkably similar. Because the rule of Table 5 lose the conservation of mass. Hence we need expand the previous method in order to solve the problem.

Table 5: The identified probabilistic rule from Figure 2.

n_i^t	(0,1,1)	(0,1,0)	(0,0,1)	(0,0,0)
$P(s_i^{t+1} = 1 \mid n_i^t)$	1.00	0.18	0.00	0.00
n	(1,1,1)	(1,1,0)	(1,0,1)	(1,0,0)
$P(s_i^{t+1} = 1 \mid n_i^t)$	1.00	0.19	0.81	0.80

Table 6: The identified probabilistic rule from Figure 3.

n_i^t	(0,1,1)	(0,1,0)	(0,0,1)	(0,0,0)
$P(s_i^{t+1} = 1 \mid n_i^t)$	1.00	0.18	0.00	0.00
n_i^t	(1,1,1)	(1,1,0)	(1,0,1)	(1,0,0)
$P(s_i^{t+1} = 1 \mid n_i^t)$	1.00	0.19	0.80	0.80

4.3 Identification

In order not to lose the conservation of mass when identifying the transition rule, investigating whether the spatio-temporal pattern satisfies the conservation of mass is necessary. Equation (5) is the necessary and sufficient condition for satisfying the conservation of mass. Equation (5) shows that the sum of variation of mass of cells is 0 at every time step.

$$\sum_{t=0}^{T-2} \sum_{i=0}^{m-1} (s_i^{t+1} - s_i^t) = 0 \tag{5}$$

In addition, the spatio-temporal pattern generated by ASEP (such as Figure 2) also satisfies equation (6) where $*$ is the wild card: 0 or 1 and $l_i^t = (n_i^t, s_i^{t+1})$. When a car goes forward, equation (6) means that other car does not suddenly appear and the car does not either suddenly disappear.

$$\begin{array}{lcl} l_i^t = ((*, 1, 0), 0) & \rightarrow & l_{i+1}^t = ((1, 0, *), 1) \\ l_i^t = ((*, 1, 0), 1) & \rightarrow & l_{i+1}^t = ((1, 0, *), 0) \end{array} \tag{6}$$

We consider a pattern and assume that the pattern satisfies equation (6). Also we assume that the rule of Table 7 identified from the pattern by the previous method. Of course, the rule does not satisfy the conservation of mass. However we can transform the rule to the hierarchical one. The hierarchical rule satisfies the conservation of mass.

Table 8,9 show the hierarchical rule which transformed from the rule of Table 7. In the hierarchical rule, the state of cell is described as $s_i^t = (a_i^t, b_i^t)$.

In the case of ASEP with probability p, the hierarchical rule is represented by equation (7). Where a_i^t indicates whether there is a car on the cell and b_i^t indicates whether a car goes forward. The hierarchical rule consists of the two phase. The first phase is deciding that the car on the cell goes forward with probability p (such as Table 8). The second phase is actually moving the car on the cell to next cell based on the decision of the first phase (such as 9).

$$
\begin{aligned}
s_i^{t+\frac{1}{2}} &= (a_i^{t+\frac{1}{2}}, b_i^{t+\frac{1}{2}}) \\
b_i^{t+\frac{1}{2}} &= a_i^t(1-a_{i+1}^t)\sigma_p \\
s_i^t &= (a_i^t, b_i^t) = (a_i^{t+1}, b_i^{t+\frac{1}{2}}) \\
a_i^{t+1} &= (1-b_{i-1}^{t+\frac{1}{2}})(1-b_i^{t+\frac{1}{2}})a_i^t \\
&\quad +b_{i-1}^{t+\frac{1}{2}}(1-b_i^{t+\frac{1}{2}})a_{i-1}^t(1-a_i^t)
\end{aligned} \tag{7}
$$

$$
\sigma_p = \begin{cases} 0 & \text{(with probability } 1-p) \\ 1 & \text{(with probability } p) \end{cases} \tag{8}
$$

Table 7: The straightforward probabilistic rule.

n_i^t	(0,1,1)	(0,1,0)	(0,0,1)	(0,0,0)
$P(s_i^{t+1}=1 \mid n_i^t)$	p_3	p_2	p_1	p_0
n_i^t	(1,1,1)	(1,1,0)	(1,0,1)	(1,0,0)
$P(s_i^{t+1}=1 \mid n_i^t)$	p_7	p_6	p_5	p_4

Table 8: The first phase of the transformed hierarchical rule from the rule of Table 7.

n_i^t	(0,1,1)	(0,1,0)	(0,0,1)	(0,0,0)
$P(b_i^{t+\frac{1}{2}}=1 \mid n_i^t)$	0	p_2	0	0
n_i^t	(1,1,1)	(1,1,0)	(1,0,1)	(1,0,0)
$P(b_i^{t+\frac{1}{2}}=1 \mid n_i^t)$	0	p_6	0	0

Table 9: The second phase of the transformed hierarchical rule from the rule of Table 7.

		$(a_{i-1}^t, a_i^t, a_{i+1}^t)$			
	$P(a_i^{t+1} \mid n_i^{t+\frac{1}{2}})$	(0,1,1)	(0,1,0)	(0,0,1)	(0,0,0)
$(b_{i-1}^{t+\frac{1}{2}}, b_i^{t+\frac{1}{2}})$	(0,0)	p_3	1	p_1	p_0
	(0,1)	-	0	-	-
	(1,0)	-	-	-	-
	(1,1)	-	-	-	-

		$(a_{i-1}^t, a_i^t, a_{i+1}^t)$			
	$P(a_i^{t+1} \mid n_i^{t+\frac{1}{2}})$	(1,1,1)	(1,1,0)	(1,0,1)	(1,0,0)
$(b_{i-1}^{t+\frac{1}{2}}, b_i^{t+\frac{1}{2}})$	(0,0)	p_7	1	0	0
	(0,1)	-	0	-	-
	(1,0)	-	-	1	1
	(1,1)	-	-	-	-

5 Conclusions

We addressed the problem of previous method. The problem is losing the conservation of mass which is one of the hidden rule in the spatio-temporal pattern when identifying the rule. We solved the problem by expanding the method to identifying the hierarchical rule. The expanded method can investigate the conservation of mass from the spatio-temporal patterns. Finally, we redefined the ASEP rule as the hierarchical rule.

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