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Effects of a membrane formation in Spatial Prisoner's Dilemma

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Abstract: A cooperative relationship has been developed among individuals. However, an altruistic behavior has little a dvantage against selfish behavior in the sense of rational terms. Each individual chooses a selfish behavior pursuing the ir own payoff then the altruistic behavior will vanish. Earlier studies proposed the mechanisms based on game theory w hich explains the problem of the difference between the theoretical prediction and observation. Furthermore, those studi es also considered the mechanisms of protecting a cooperators cluster in a spatial prisoner's dilemma involving spatial s trategies and a spatial generosity, although did not analyze rigorously effects of the membrane for the cooperators. In th is paper, we report the quantitative effect of membrane for protecting the cooperators from the exploitation of the defect ors.

Keywords: Spatial Prisoner's Dilemma, Spatial Strategy, cooperation, membrane formation

I. INTRODUCTION

Cooperation is basic components of animals and society. And a cooperative relationship has been developed among individuals. However, an altruistic behavior has little advantage against a selfish behavior in the sense of rational terms. In stead of an altruistic behavior, each individual should choose a selfish behavior pursuing their profit. The mechanisms are proposed by earlier studies for explain the deference between the theoretical consideration and observational results [2, 3]. The some proposed mechanisms are based on the game theory, especially Prisoner's Dilemma model [4-7].

In the model we proposed that is involving spatial strategy and spatial generosity [8], we observed membrane formation as a mechanism for protects cooperators form invasion of defectors [9]. Furthermore, we revealed the condition for constructing membrane. The constructing membrane only depends upon the spatial generosity k [10]. However, we have not analyzed rigorously effects of the membrane on the cooperators.

In this paper, we report the quantitative effect of membrane on protecting the cooperators form the exploitation of the defection.

II. MODEL

1. Prisoner's Dilemma

The Prisoner's dilemma (PD) is a fundamental model of game theory. It played just once by two players have two behavioral options: C (Cooperation), or D (defection). The players decide behavior simultaneously whether to cooperate or to defect. If both players cooperate, both players receive payoff R (reward), whereas if both players defect, both players receive payoff P (punishment). If a player cooperates and an opponent player defects, the cooperator receives payoff S (sucker) and the defector receives payoff T (temptation) where the payoffs must satisfy T > R > P > S. If an opponent player chooses behavior whatever cooperation or defection, the other should choose defection, because it is better than choses cooperation. Therefore, the both players always choose defection and receive payoff P lower than that when both choose cooperation.

The Iterated Prisoner's Dilemma (IPD) is temporal expansion model of PD. In IPD, PD is carried out repeatedly. Many strategies have been proposed in IPD. Axelrod reported tit-for-tat (TFT) strategy [1]. In Axelrod's round-robin tournaments, TFT strategy was the best strategy. TFT strategy is consists in playing C in the first round and from then on chose action whatever chosen by other player in the previous round. TFT strategy contains temporal generosity as an element.

2. Spatial Prisoner's Dilemma

Spatial prisoner's dilemma is spatio-temporal version of PD. Our model generalized SPD by introducing spatial strategy. Each player placed at each lattice of the two-dimensional lattice. Each player has an action and a strategy, and receives a score. Spatial strategy determines the next action dependent on the spatial pattern of actions in the neighbors. Each player plays PD with the neighbors, and changes its strategy to the strategy that earns the highest total score among the neighbors. Table 1 is the Payoff Matrix of PD. In our simulations, R, S, T, and P are respectively set to 1, 0, b ($1 < b < 2$, a bias for defectors) and 0 in simulations below following the Nowak-May's simulations.

Table 1. The Payoff Matrix of the Prisoner's Dilemma Game. R, S, T, P are payoff for player 1. ($1 < b < 2$)

		Player 2	
		C	D
Player 1	C	1	0
	D	*b*	0

Our SPD model is done in the following way with n players.

1. Initial phase: the action and the strategy of each player are determined randomly.
2. Renewal of action: the next action will be determined by player's strategy based on the neighbors' actions and the player's own action.
3. Calculate score: the score for each player is calculated by summing up all the scores received from PD with neighbor players and itself (self-interaction involved to make compare to the Nowak-May model), and the score added to the current player's score.
4. Renewal of strategy: the nest strategy will be chosen from the strategy with the highest score among the neighbors including the player itself.

3. Spatial Strategy and spatial generosity

The next action will be determined based on the pattern of neighbor's actions. However, the pattern of neighbor's action is a lot. For simplicity, we restrict ourselves to a "totalistic spatial strategy" that only depends upon the number of D (defection) of the neighbors, not on their positions. To represent a strategy, let l be the number of the D action of the neighbors excluding the player itself. We define k-D strategy that chose action D if $l \geqq k$ and C otherwise. This k-D strategy can be regarded as a spatial version of TFT where k indicates the spatial version of the generosity.

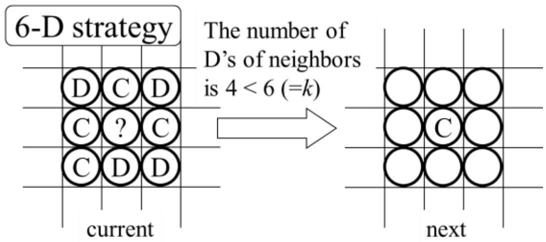

Fig.1. An example of spatial strategy. In this situation, k (spatial generosity) is 6 and neighborhood is *Moore* neighborhood.

III MEMBRANE FORMATION AND MEMBRANE INDEX

We simulated interaction between All-D (always D) vs. k-D instead of All-D vs. All-C (Nowak-May's simulation). In All-D vs. k-D simulation, we already observed the membrane as a mechanism for protects cooperation from invasion of defectors of All-D. The membrane is composed of only D players of k-D. Fig.2 shows an example of the membrane formation. Although we can understand that the membrane protects cooperators from invasion of defectors intuitively, the effects of the membrane on cooperators have not been investigated mathematically.

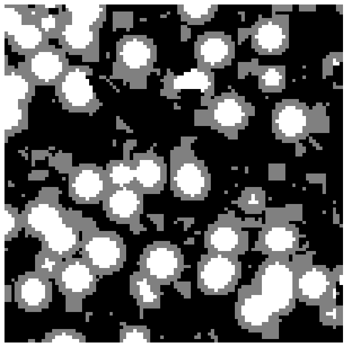

Fig.2. The membrane formation generated by SPD simulation. Black cells indicate All-D players. White and gray cells indicate C and D players of k-D. In this snapshot, k (spatial-generosity) is 6. The C clusters are covered by the membrane (gray color). This snapshot shows typical membrane.

Therefore we consider a membrane index to investigate quantitative effects of the membrane formation on cooperators. The membrane index means how much membranes protect cooperators from invasion of defectors. The following is definition of the player as a membrane.

1. The player is D player of k-D.
2. The player is playing PD game with C player.

3. The interaction between All-D players and C players is nothing within player's neighborhood.

In other words, we define the membrane in terms of the function protecting cooperators from invasion of the defectors. Fig.3 shows the membrane that satisfied a definition.

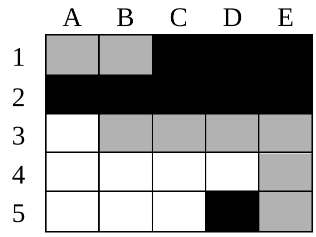

Fig.3. In the *Moore* neighbors, the center cell (C3 player) is a membrane that satisfied a definition. Bl ack cells indicate All-D players. White and gray ce lls indicate C and D of *k*-D. If the immediate left cell of center (B3 player) was white (Cooperator), t he center cell did not satisfy a definition of the me mbrane. Because All-D players and C player (imme diate left cell of center cell) will be interaction eac h other among center cell's neighbors, the 3rd defin ition of the membrane could not satisfy.

IV SIMULATION

We simulate to investigate quantitative effect of membrane on cooperators with the following parameters list in Table 2.

Table 2. Parameters list for simulations.

Parameter Name	Description	Value
L×L	Size of lattice	500 × 500
N	Number of players	250,000
T	Number of steps	1000
r	Neighborhood radius	1
b	Bias for defectors of the payoff matrix in Table.1	1.800001

In our simulation, the membrane is formed within certain scope of *k* [10].

Fig. 4 plots the time evolution of the average score of the C players when the *k* varies. The 6-D earns highest score among other *k*-D strategies. We can consider that the cooperators construct a cluster if the score per player is high. Because, if the cooperators have not constructed a cluster, the cooperators would be exploited by defectors, and the average payoff of the C players would become low.

Fig. 5 plots the time evolution of the membrane indexes divided by the number of D of *k*-D. It means percentage of the D players of *k*-D that are effective as a membrane. If the membrane indexes per player are one, all of D players of *k*-D are action effectively as a membrane.

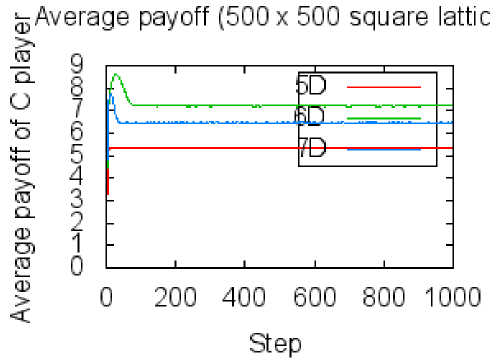

Fig.4. The time evolution of the average payoff of the player C. The 6-D earns highest score among o ther *k*-D strategies.

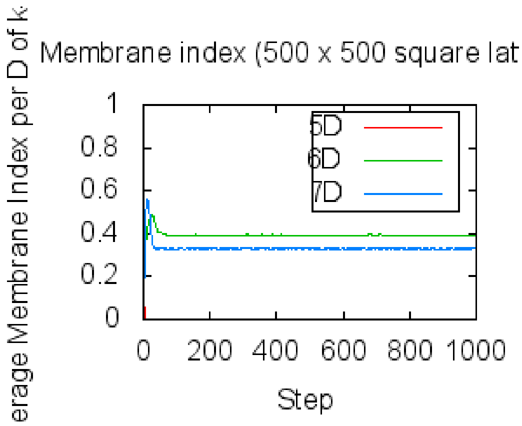

Fig.5. The time evolution of the membrane indexes per D player of *k*-D.

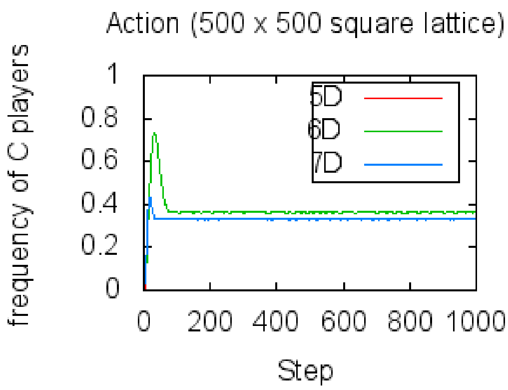

Fig.6 The time evolution of frequency of C players when k varies. The fraction of 6-D is the highest among this three strategies. The fraction of 5-D is the lowest, because the 5-D cluster could not expan d although the cluster is protected by the membran e.

Fig.6 plots the time evolution of frequency of C players. In 5-D vs. All-D simulation, the cooperators have been remained in a small amount. In 6-D or 7-D vs. All-D simulations, the cooperators have been remained in a relative large amount.

We calculate coefficient of correlation between average score of C players and membrane indexes per players to investigate effect of the membrane on cooperators.

Table 3 shows the coefficient of correlation between average score of C players and membrane indexes per D players of *k*-D for each All-D vs. *k*-D simulation.

Table 3. The coefficient of correlation between aver age score of C players and membrane indexes per D players of *k*-D.

k value	Coefficient of correlation
5	-0.7119
6	0.812
7	0.9549

Consequence of calculation of correlation coefficient, we get interesting results. In 5-D vs. All-D simulation, the average score of C players have a negative correlation with the membrane indexes. By contrast, in the simulations of 6-D and 7-D, the average score of C players have a strong positive correlation with the membrane indexes.

We considered that this consequence is related to the conditions the membrane expands. Only 5-D constructs membrane that can not expand among three strategies. Therefore, the amounts of the cooperators do not increase. Thereby a lot of D players of *k*-D do not satisfy the 2nd condition of the definition that the membrane should interact with cooperators. Hence, although the cooperators construct a cluster and the membrane protects cooperators from defectors, the membrane indexes lower. Because cooperators earn high score by a cluster whereas the membrane indexes lower, the negative correlation occurs.

VI. CONCLUSION

We investigate that membrane formation one of a mechanism for protects cooperation from invasion of defectors. The membrane formation has been reported in SPD, however the quantitative effects have not investigated.

We defined membrane index to investigate the membrane. And we denoted the quantitative effect of membrane on cooperators by membrane index. The membrane indexes have a strong correlation with the average score of C players when certain scope of *k*. If the membrane expands, the coefficient of correlation will become positive; otherwise the coefficient of correlation will become negative.

We investigate an effect of membrane on cooperators when Moore neighbors hood. We investigated the simulation in the *Moore* neighborhood; however we need to investigate the simulation in the *Neumann* neighborhood in future to compare both neighborhoods.

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