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A Nonholonomic Control Method for Stabilizing an X4-AUV

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Abstract: A nonholonomic control method is considered for stabilizing all attitudes and positions (x, y or z) of an X4-AUV with four thrusters and six degrees-of-freedom (DOF), in which the positions were stabilized according to the Lyapunov stability theory. The derived model is dynamically unstable, so a sequential nonlinear control strategy is implemented for the X4-AUV, composed of translational and rotational subsystems. A controller for the translational subsystem stabilizes one position out of x-, y-, and z-coordinates, whereas controllers for the rotational subsystems generate the desired roll, pitch and yaw angles. Thus, the rotational controllers stabilize all the attitudes of the X4-AUV at a desired (x-, y- or z-) position of the vehicle.

Keywords: AUV, Underactuated control system, Nonholonomic systems.

I. INTRODUCTION

Nowadays control problems of underactuated vehicles motivate the development of new design methodologies in nonlinear control. The control system for such vehicles has fewer number of control inputs than the number of generalized state variables to be controlled. Hence, the dynamical equations of the vehicle exhibit so-called second-order nonholonomic constraints, i.e. non-integrable conditions imposed on the acceleration in one or more DOFs, because the vehicle lacks capability to command instantaneous accelerations in these directions of the configuration space [1]. As pointed in a celebrated paper of Brockett in1983, such nonholonomic systems cannot be stabilized by usual smooth, time-invariant, and state feedback control algorithms. For the control of underactuated underwater systems, many researches proposed nonlinear feedback control [2], among many others. Nonholonomic property of a mechanical systems can be seen in underactuated systems, even though the connection between nonholonomic control systems and underactuated systems is not completely understood. To control nonholonomic systems, many control approaches have been studied. Kolmanovsky and Clamroch present a vast literature and overview on nonholonomic control in their work [1].

In this paper, we present a model of an underactuated X4-AUV with 6-DOF and four control inputs and propose a control scheme based on Lyapunov approach to stabilize all attitudes and positions of the vehicle. A sequential nonlinear control strategy is implemented for the derived 6-DOF vehicle model, constituted of translational and rotational subsystems. The controller for the translational subsystem stabilizes the position and the controllers for the rotational subsystems generate the desired roll, pitch and yaw angles.

II. DYNAMICAL MODEL

1. Coordinate System of AUV

In order to describe the underwater vehicle's motion, a special reference frame must be established. There have two coordinate systems: i.e., inertial coordinate system (or fixed coordinate system) and motion coordinate system (or body-fixed coordinate system). The coordinate frame {E} is composed of the orthogonal axes {E_x E_y E_z} and is called as an inertial frame. This frame is commonly placed at a fixed place on Earth. The axes E_x and E_y form a horizontal plane and E_z has the direction of the gravity field. The body fixed frame {B} is composed of the orthonormal axes {X, Y, Z} and attached to the vehicle. The body axes, two of which coincide with principle axes of inertia of the vehicles, are defined in Fossen [3] as follows:

X is the longitudinal axis (directed from aft to fore)
Y is the transverse axis (directed to starboard)
Z is the normal axis (directed from top to bottom)

Fig. 1 shows the coordinate systems of AUV, which consist of a right-hand inertial frame {E} in which the downward vertical direction is to be positive and right-hand body frame {B}.

Letting $\boldsymbol{\xi} = [x \quad y \quad z]^T$ denote the mass center of the body in the inertial frame, defining the rotational angles of X-, Y- and Z-axis as $\boldsymbol{\eta} = [\phi \quad \theta \quad \psi]^T$, the rotational

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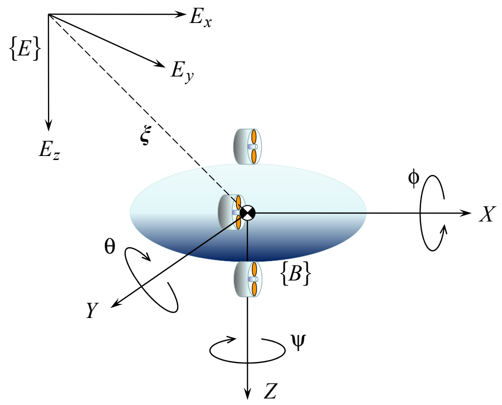

Fig. 1. Coordinate systems of AUV

matrix R from the body frame {B} to the inertial frame {E} can be reduced to:

$$R = \begin{bmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\ c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \quad (1)$$

where $c\alpha$ denotes $\cos\alpha$ and $s\alpha$ is $\sin\alpha$.

2. Derivation of a Dynamical Model

Following a Lagrangian method, this section describes the dynamic model of the X4-AUV with the assumption of balance between buoyancy and gravity. The kinetic energy formula is

$$T = T_{trans} + T_{rot} \quad (2)$$

where T_{trans} and T_{rot} are the translational kinetic energy and the rotational kinetic energy, which are given by

$$T_{trans} = \frac{1}{2}\dot{\boldsymbol{\xi}}^T M \dot{\boldsymbol{\xi}} \quad (3)$$

$$T_{rot} = \frac{1}{2}\dot{\boldsymbol{\eta}}^T J \dot{\boldsymbol{\eta}} \quad (4)$$

in which M is the total mass matrix of the body, and J is the total inertia matrix of the body. From the characteristics of added mass, they can be written as

$$M = \text{diag}\,(m_1, m_2, m_3) = m_b I + M_f \quad (5)$$

$$J = \text{diag}\,(I_x, I_y, I_z) = J_b + J_f \quad (6)$$

Here, m_b is a mass of the vehicle, J_b is an inertia matrix of the vehicle and I is a 3×3 identity matrix. Letting ρ denote a density of the fluid and using the formulation of the added mass and inertia [4] under the assumption of $r_1 = 5r$ and $r_2 = r_3 = r$ where r_1, r_2 and r_3 are the lengths of the semi axes of the ellipsoidal vehicle, the added mass matrix M_f and the added inertia matrix J_f are obtained by

$$M_f = \text{diag}\,(0.394\rho\pi r^3, 5.96\rho\pi r^3, 5.96\rho\pi r^3) \quad (7)$$

$$J_f = \text{diag}\,(0, 24.2648\rho\pi r^5, 24.2648\rho\pi r^5) \quad (8)$$

From the assumption of the balance between the buoyancy and the gravity, i.e., the potential energy $U = 0$, the Lagrangian can be written as

$$\begin{aligned} L &= T - U \quad (9) \\ &= T_{trans} + T_{rot} \quad (10) \end{aligned}$$

The resultant dynamical model for the X4-AUV is summarized by [5]

$$\begin{aligned} m_1\ddot{x} &= \cos\theta\cos\psi u_1 \\ m_2\ddot{y} &= \cos\theta\sin\psi u_1 \\ m_3\ddot{z} &= -\sin\theta u_1 \\ I_x\ddot{\phi} &= \dot{\theta}\dot{\psi}(I_y - I_z) + u_2 \\ I_y\ddot{\theta} &= \dot{\phi}\dot{\psi}(I_z - I_x) - J_t\dot{\psi}\Omega + lu_3 \\ I_z\ddot{\psi} &= \dot{\phi}\dot{\theta}(I_x - I_y) + J_t\dot{\theta}\Omega + lu_4 \end{aligned} \quad (11)$$

Here, the values of m_1, m_2, m_3, I_x, I_y, and I_z are defined as in (5) and (6). u_1, u_2, u_3, u_4 are the control inputs for the translational (x, y and z-axis) motion, the roll (ϕ-axis) motion, the pitch (θ-axis) motion, and yaw (ψ-axis) motion, respectively. Defining that b is a thrust factor, d is a drag factor, taken from $\tau_{Mi} = d\omega_i^2$, Ω, u_1, u_2, u_3 and u_4 are given by

$$\begin{aligned} \Omega &= (\omega_2 + \omega_4 - \omega_1 - \omega_3) \\ u_1 &= f_1 + f_2 + f_3 + f_4 \\ &= b\left(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2\right) \\ u_2 &= d\left(-\omega_2^2 - \omega_4^2 + \omega_1^2 + \omega_3^3\right) \\ u_3 &= f_1 - f_3 = b\left(\omega_1^2 - \omega_3^2\right) \\ u_4 &= f_2 - f_4 = b\left(\omega_2^2 - \omega_4^2\right) \end{aligned} \quad (12)$$

III. STABILIZATION CONTROL FOR X4-AUV

The model (11), developed in the previous section, can be rewritten in a state-space form $\dot{X} = f(X, U)$ by introducing $X = (x_1 \ldots x_{12})^T \in \Re^{12}$ as state vector of the system as follows:

$$\begin{aligned} x_1 &= x \\ x_2 &= \dot{x}_1 = \dot{x} \\ x_3 &= y \\ x_4 &= \dot{x}_3 = \dot{y} \\ x_5 &= z \\ x_6 &= \dot{x}_5 = \dot{z} \\ x_7 &= \phi \\ x_8 &= \dot{x}_7 = \dot{\phi} \\ x_9 &= \theta \\ x_{10} &= \dot{x}_9 = \dot{\theta} \\ x_{11} &= \psi \\ x_{12} &= \dot{x}_{11} = \dot{\psi} \end{aligned} \quad (13)$$

where the inputs $U = (u_1 \ldots u_4)^T \in \Re^4$.

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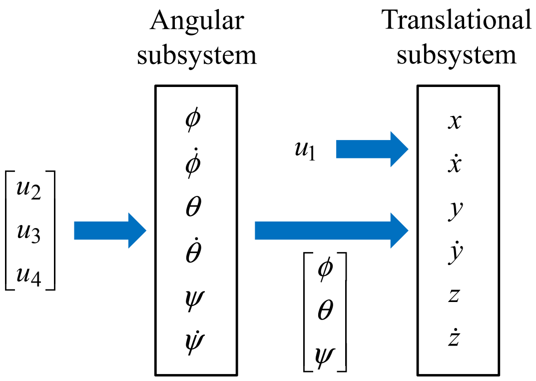

Fig. 2. Connections of rotational and translational subsystems

From (11) and (13) we obtain:

$$f(X,U) = \begin{pmatrix} x_2 \\ (\cos\theta\cos\psi)\frac{1}{m_1}u_1 \\ x_4 \\ (\cos\theta\sin\psi)\frac{1}{m_2}u_1 \\ x_6 \\ (-\sin\theta)\frac{1}{m_3}u_1 \\ x_8 \\ x_{10}x_{12}(\frac{I_y-I_z}{I_x}) + \frac{1}{I_x}u_2 \\ x_{10} \\ x_8x_{12}(\frac{I_z-I_x}{I_y}) - \frac{J_t}{I_y}x_{12}\Omega + \frac{l}{I_y}u_3 \\ x_{12} \\ x_8x_{10}(\frac{I_x-I_y}{I_z}) + \frac{J_t}{I_z}x_{10}\Omega + \frac{l}{I_z}u_4 \end{pmatrix} \quad (14)$$

It is interesting to note that in the dynamics of the latter system how the angles and their time derivatives do not depend on the translational components, whereas the translations only depend on the angle. It can ideally imagine the overall system described by (14) as constructed by two subsystems: i.e., the angular rotations and the linear translations (see Fig. 2). The subsystem of the angular rotations has the restriction X_α of X to the last 6 components as state, which is regarded as the roll, pitch, yaw and their time derivatives. The dynamics related to these variables are described by $f_\alpha(X,U)$ corresponding to the last 6 components of the mapping (14). Observe that the mapping $f_\alpha(X,U)$ is a function only of X_α and of $(u_2,u_3,u_4)^T$, and does not depend on the translational components. On the other hand, the translational subsystem (with state X_Δ) consists of the first 6 elements of the state X, which are the x, y, z and their time derivatives, where the dynamics are described by the first 6 rows $f_\Delta(X,U)$ of the mapping (14). The translational subsystem mapping $f_\Delta(X,U)$ is not independent of the angular variables but depends only on the roll, pitch and yaw and not on their time derivatives. Note that these discussions are the same as those for a Quadrotor studied in [6].

1. Rotation Control

Consider first the control for the subsystem of the angular rotations because of its complete independence from the other subsystem. A stabilization of the X4-AUV angles is considered for $X_\alpha^d = (x_7^d, 0, x_9^d, 0, x_{11}^d, 0)^T$.

Let us consider the Lyapunov function $V(X_\alpha)$ that is positive definite around the desired position X_α^d:

$$V(X_\alpha) = \frac{1}{2}(x_7 - x_7^d)^2 + \frac{1}{2}x_8^2 + \frac{1}{2}(x_9 - x_9^d)^2 + \frac{1}{2}x_{10}^2 + \frac{1}{2}(x_{11} - x_{11}^d)^2 + \frac{1}{2}x_{12}^2 \quad (15)$$

The time derivative of (15), $\dot{V} = (\nabla V)^T f_\alpha$, in the case of X4-AUV ($I_y = I_z$) is drastically reduced to:

$$\dot{V} = x_8(x_7 - x_7^d) + x_{10}(x_9 - x_9^d) + x_{12}(x_{11} - x_{11}^d) + \frac{1}{I_x}x_8u_2 + \frac{l}{I_y}x_{10}u_3 + \frac{l}{I_z}x_{12}u_4 \quad (16)$$

By simply choosing:

$$\begin{aligned} u_2 &= -I_x(x_7 - x_7^d) - k_1x_8 \\ u_3 &= -\frac{I_y}{l}(x_9 - x_9^d) - k_2x_{10} \\ u_4 &= -\frac{I_z}{l}(x_{11} - x_{11}^d) - k_3x_{12} \end{aligned} \quad (17)$$

with k_1, k_2 and k_3 of positive constants, we obtain for (16):

$$\dot{V} = -\frac{1}{I_x}k_1x_8^2 - \frac{l}{I_y}k_2x_{10}^2 - \frac{l}{I_z}k_3x_{12}^2 \quad (18)$$

which is only negative semi-definite. By Lyapunov theorem the simple stability for equilibrium is now ensured. By Lasalle invariance theorem we can ensure also that starting from a level curve of the Lyapunov function defined in (15) where $V(X_\alpha)$ is constant, the state evolution is constrained inside the region bounded by the level curve [6]. This is very useful when trying to avoid particular configuration. It is simply necessary to start with a level curve not containing these points and apply the previous defined control. We can also ensure the asymptotic stability by applying the Lasalle theorem because the maximum invariance set of (angular rotations) subsystem under control (17) contained in the set $S = \{X_\alpha^S \in \Re^6 : \dot{V}|_{X_\alpha^S = 0}\}$ is restricted only to the equilibrium point [6].

If $V(X_\alpha) > 0$ when $(X_\alpha \neq X_\alpha^d)$ and $\dot{V}(X_\alpha) \leq 0$ $(X_\alpha \neq X_\alpha^d)$, the system is asymptotically stable. In the case $\dot{V}(X_\alpha) \equiv 0$, the solution of state equation is given by only $X_\alpha = X_\alpha^d$.

By the latter consideration we can ensure an asymptotical stability starting from a point in a set around the equilibrium. To ensure the global stability, it is sufficient that $\lim_{\|X_\alpha\| \to \infty} V(X_\alpha) = \infty$, which is our case.

2. Translation Controller

Let us consider the simple task for the X4-AUV to be translated to a particular position $x = x^d$, $y = y^d$ and $z = z^d$. The dynamics of the x, y and z-position are described by lines 1 and 2, 3 and 4, and 5 and 6 in system (14), i.e., *x-position:*

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ (\cos x_9 \cos x_{11}) \frac{u_1}{m_1} \end{pmatrix} \tag{19}$$

y-position:

$$\begin{pmatrix} \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} x_4 \\ (\cos x_9 \sin x_{11}) \frac{u_1}{m_2} \end{pmatrix} \tag{20}$$

z-position:

$$\begin{pmatrix} \dot{x}_5 \\ \dot{x}_6 \end{pmatrix} = \begin{pmatrix} x_6 \\ (-\sin x_9) \frac{u_1}{m_3} \end{pmatrix} \tag{21}$$

By the previous considerations in the control for the subsystem of the angular rotations, we ensure that starting from an initial condition where $V(X_\alpha) < \pi/2$, the angles and their velocities are constrained in this hypersphere of $\Re^6$. In this case $\cos x_9 \cos x_{11} \neq 0$, $\cos x_9 \sin x_{11} \neq 0$ and $-\sin x_9 \neq 0$ for all the trajectories of the system under the previous control law. Systems (19), (20) and (21) can be linearized by simply compensating the weighted force by

x-position:

$$u_1 = \frac{m_1 \hat{u}_1}{\cos x_9 \cos x_{11}} \tag{22}$$

y-position:

$$u_1 = \frac{m_2 \hat{u}_2}{\cos x_9 \sin x_{11}} \tag{23}$$

z-position:

$$u_1 = \frac{-m_3 \hat{u}_3}{\sin x_9} \tag{24}$$

where $\hat{u}_1$, $\hat{u}_2$ and $\hat{u}_3$ are additional terms. By this partial feedback linearization [7], (19), (20) and (21) become

x-position:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ \hat{u}_1 \end{pmatrix} \tag{25}$$

or

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \end{pmatrix} = \begin{pmatrix} e_2 \\ -\hat{u}_1 \end{pmatrix} \tag{26}$$

y-position:

$$\begin{pmatrix} \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} x_4 \\ \hat{u}_2 \end{pmatrix} \tag{27}$$

or

$$\begin{pmatrix} \dot{e}_3 \\ \dot{e}_4 \end{pmatrix} = \begin{pmatrix} e_4 \\ -\hat{u}_2 \end{pmatrix} \tag{28}$$

z-position:

$$\begin{pmatrix} \dot{x}_5 \\ \dot{x}_6 \end{pmatrix} = \begin{pmatrix} x_6 \\ \hat{u}_3 \end{pmatrix} \tag{29}$$

or

$$\begin{pmatrix} \dot{e}_5 \\ \dot{e}_6 \end{pmatrix} = \begin{pmatrix} e_6 \\ -\hat{u}_3 \end{pmatrix} \tag{30}$$

where $e_i \triangleq x_i^d - x_i$, $i = 1, \ldots, 6$. Adopting a simple linear state feedback stabilization law $\hat{u}_1 = k_4 e_1 + k_5 e_2$, $\hat{u}_2 = k_6 e_3 + k_7 e_4$ and $\hat{u}_3 = k_8 e_5 + k_9 e_6$ we can stabilize the position by placing the poles of the subsystem in any position in the complex left half plane.

IV. DISCUSSIONS

A nonlinear control strategy is implemented to stabilize the X4-AUV. The position and angles of the X4-AUV are stabilized by using control input u_1, u_2, u_3 and u_4 respectively. PD controllers were introduced for controlling each orientation angle such that

$$u_{i+1} = k_{pi} e_{2i+5} + k_{di} \dot{e}_{2i+5}, \quad i = 1, 2, 3 \tag{31}$$

where $k_{p1} \equiv I_x$, $k_{p2} \equiv I_y/l$, $k_{p3} = I_x/l$, $k_{d1} = k_1$, $k_{d2} = k_2$, and $k_{d3} = k_3$.

The angles and their time derivatives of rotational subsystem do not depend on translation components, whereas the translations depend on the angles. Ideally, it can be imagined as two subsystems: the angular rotations and the linear translations. Due to its complete independence from the other subsystem, the angular rotation-related subsystem is tuned first. The rotational control keeps a 3D orientation of the X4-AUV to the desired state and the translational control moves the vehicle to the desired position. The controllers have been implemented on MATLAB and the simulation results for stabilizing the X4-AUV can be found in [8] and [9].

V. CONCLUSION

In this paper, a nonholonomic control method for stabilizing all attitudes and positions (x, y or z) of an underactuated X4-AUV, with four thrusters and 6-DOF has been presented. The controller design was separated into two parts: the rotational and translational dynamics-related controller designs. The stabilization strategy is based on the Lyapunov stability theory. For the future work, an underactuated controller will be constructed by combining such three types of controllers to realize an underactuated control system.

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