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# Trajectory Tracking Control for Nonholonomic Mobile Robots by an Image-Based Approach

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***Abstract***:
The vision-based control that uses cameras for observing a robot environment has been researched widely. Especially, a method called image-based control has high robust properties, because it can control a target on an image plane coordinate without using a robot position. Kurashiki et al. have already studied on an image-based control method that can realize a robust trajectory tracking. Although their objective was to control a nonholonomic mobile robot, the problem setting used there was to be little affected from the nonholonomic constraint. Further more, there were unclear points in deriving a control law. In this paper, such unclear points are explained from an geometric relation and other problem settings, which are clearly affected by a nonholonomic constraint, are proposed.

***Keywords***: Vision-based control, Image-based control, Trajectory tracking, nonholonomic mobile robot.

## I. INTRODUCTION

Unmanned vehicles such as automated driving cars require high robustness against any disturbances for safety. On the other hand, with the popularization of inexpensive cameras, vision-based control has been researched in the domain of robot control. Vision-based control is classified roughly into two methods that are called a "position-based" method and an "image-based" method [1], [2], [3], [4]. In the position-based method, the control errors are calculated from the position and the pose of the camera estimated from captured images. Although the method can control these states directly, the camera calibration is very important to the state estimation. The image-based method does not consider the position of the camera. The control errors are calculated on the coordinate which is attached directly to the captured 2D image. The control input is determined according to the control errors such as the amount of features, the location of the target on the image coordinate and so on. In general, the image-based method is known to be robust not only camera but also robot calibration errors.

The purpose of our research is controlling mobile robots with the image-based method, because it has higher robustness than the position-based method. As the earlier study of the image-based method, Kurashiki [5] developed a system consisting of a nonholonomic robot and a camera so that the robot can track the line drawn on the floor, with controlling the gradient and the intercept of the line on the captured image to their desired values. However, a nonholonomic constraint does not affect its problem setting. Additionally, there is a mistake of the derivation of a control low. In this paper, these two problems are explained and the control low is checked with a simulation experiment. Then, a new problem setting which is influenced by a nonholonomic constraint is described.

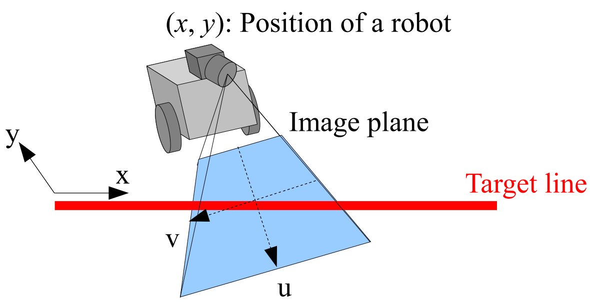


Fig. 1. Definition of coordinates

## II. PROBLEM SETTING

Fig. 1 shows the environment of the trajectory tracking system. A camera is attached on the robot to observe the target line drawn on the floor. The objective of the control is that the robot tracks the target line autonomously based on the captured image.

### 1. Coordinates

The world coordinate is set such that x-axis is along the target line and y-axis is perpendicular to the x-axis shown in Fig. 1. The v-u coordinate, whose origin corresponds to the center of the captured image plane, is attached on the image plane as shown in Fig. 2. For simplification, the camera is assumed to be equipped at the center of the robot with its downward direction. Thus, the origin of v-u coordinate corresponds to the position of the robot $(x,\ y)$ on the world coordinate. An anticlockwise rotation is to be positive for the angle between target line and u-axis $\theta$ (i. e., $\theta$ has a negative value in Fig. 2).

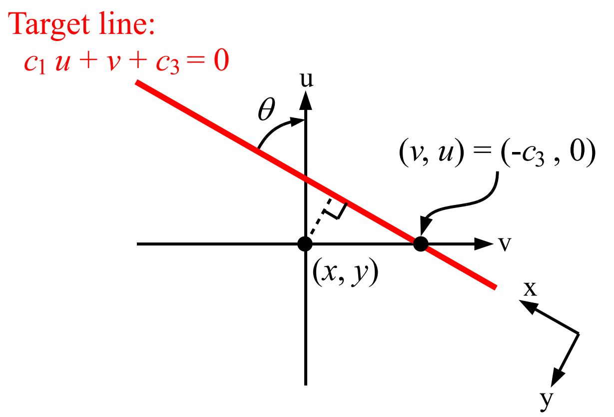


Fig. 2. Coordinate of the image plane

In general, the equation of a straight line on v-u coordinate is denoted as follows:

$$a_1 u + a_2 v + a_3 = 0 \tag{1}$$

Assuming that this strait line is not parallel to v-axis, $a_2$ is not 0. Thus, Eq. (1) can be divided by $a_2$ to obtain Eq. (2),

$$c_1 u + v + c_3 = 0, \left(c_1 = \frac{a_1}{a_2}, c_3 = \frac{a_3}{a_2}\right) \tag{2}$$

where $c_1$ is a factor related to the gradient of the line and $c_3$ is the reversal sign of v-coordinate value at the intersection of the line and v-axis. Thus, the parameters of the target line are $c_1$ and $c_3$. The relationship between $c_1$ and $\theta$ is written as follows:

$$c_1 = -\tan\theta, \left(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\right) \tag{3}$$

On the other hand, the distance $y$ between the target line and the robot should converge to 0, to track the target line. On the world coordinate, it is just as the $y$ value of the robot position $(x, y)$. The relationship between $c_3$ and $y$ is obtained geometrically such as

$$y = -c_3 \cos\theta \cdot \frac{h}{f} \tag{4}$$

where $f$ is a focal length of a camera and $h$ is an altitude of a camera position. Then the above equation is rewritten as follows:

$$c_3 = -\frac{1}{\cos\theta} \cdot \frac{yf}{h} \tag{5}$$

Thus, controlling $c_1$ and $c_3$ to zero on the image plane is equivalent to tracking the target line on the world coordinate. The observing equation to obtain $c_1$ and $c_3$ from the captured image is written as follows:

$$\begin{bmatrix} c_1 \\ c_3 \end{bmatrix} = \frac{1}{u_2 - u_1} \begin{bmatrix} v_1 - v_2 \\ -u_2 v_1 + u_1 v_2 \end{bmatrix} \tag{6}$$

where the points of $(v_1, u_1)$ and $(v_2, u_2)$ are arbitrary points on the target line on the captured image. Assuming that the target line is not parallel with v-axis, it follows that $u_2 - u_1 \neq 0$.

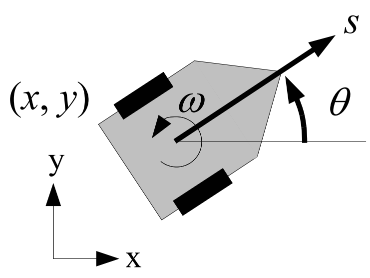


Fig. 3. Robot model

### 2. Question of Earlier Research

Kurashiki et al. derived the relationship between $y$ and $c_3$ as follows:

$$y = \sqrt{u_3^2 + v_3^2} \cdot \mathrm{sign}(c_3 c_1) \cdot \frac{h}{f} \tag{7}$$

where the point of $(v_3, u_3)$ is the nearest point to the origin of v-u coordinate on the target line. These $v_3$ and $u_3$ are calculated by the following equations:

$$v_3 = -\frac{c_3}{c_1^2 + 1} \tag{8}$$

$$u_3 = -\frac{c_1 c_3}{c_1^2 + 1} \tag{9}$$

Although Kurashiki et al. said that Eq. (5) was able to be derived with Eqs. (7), (8) and (9), the signum function still remains. Thus, it is necessary to split the case where $c_3 c_1$ is positive or negative, but there is no explanation about it. Additionally, when $c_1 > 0$ and $c_3 < 0$ as shown in Fig. 2, Eq. (7) gives a negative value, though $y$ is positive. Thus, Eq. (7) seems to be a wrong relationship equation. However, note that somehow they derived a right relationship given in Eq. (5).

## III. ROBOT MODEL

In this paper, a robot is to be a two-wheel independent driven type shown in Fig. 3. The position of the robot is $(x, y)$ on the world coordinate. The pose of the robot is the angle $\theta$ between the direction of forward movement and x-axis. The kinematic model of this robot is denoted by

$$\frac{d}{dt}\begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s \\ \omega \end{bmatrix} \tag{10}$$

where $s = \sqrt{\dot{x}^2 + \dot{y}^2}$ is the translational velocity and $\omega = \dot{\theta}$ is the angular velocity.

## IV. CONTROLLER

In this section, a controller is designed based on Liapunov's theory. Since an image-based method is proposed, a control target is not a robot but coefficients of the target line, i. e., parameters $c_1$ and $c_3$ on the image plane. The goal of the control is to be the convergence of these values to 0.

To derive a control low, Eqs. (3) and (5) are differentiated with respect to time and they are rearranged to obtain

$$\frac{d}{dt}\begin{bmatrix} c_1 \\ c_3 \end{bmatrix} = \frac{fs}{h}\begin{bmatrix} 0 \\ c_1 \end{bmatrix} - \begin{bmatrix} c_1^2+1 \\ c_1 c_3 \end{bmatrix}\omega \tag{11}$$

The next equation is one candidate of a Liapunov function:

$$V = \frac{K_1}{2}c_1^2 + \frac{K_3}{2}c_3^2 \tag{12}$$

where $K_1$ and $K_3$ are positive gains. Differentiating it with respect to time gives

$$\frac{dV}{dt} = \frac{fs}{h}K_3 c_1 c_3 - c_1\{K_1(c_2^2+1) + K_3 c_3^2\}\omega \tag{13}$$

Assume that the input value $\omega$ is taken as follows:

$$\omega = \{K_1(c_1^2+1) + K_3 c_3^2\}^{-1}\left(\frac{fs}{h}K_3 c_3 + K_2 c_1\right) \tag{14}$$

where $K_2$ is a positive gain. Then, substituting Eq. (14) into Eq. (13) yields

$$\frac{dV}{dt} = -K_2 c_1^2 \le 0 \tag{15}$$

Using the Barbalat's lemma [6], it is proved that $\dot{V} \to 0$ and $c_1 \to 0$. Also, note that

$$\lim_{t\to\infty}\dot{c}_1 = -\lim_{t\to\infty}(K_1 + K_3 c_3^2)^{-1}\frac{fs}{h}K_3 c_3 = 0 \tag{16}$$

Thus, assuming that $s \neq 0$, it is seen that $c_3 \to 0$, so that the convergence to a desired state is ensured.

## V. SIMULATION

A simulation experiment is conducted to test the designed controller. The initial state of a robot is set to $(x,\ y,\ \theta)$ = (0 [m], $-0.71$ [m], $-0.78$ [rad]) so as to obtain $(c_1,\ c_3)$ = (1, 1). The function of a target line is to be $y$ = 0 [m]. The focal length of a camera is $f$ = 1 [m] and its altitude from the floor is $h$ = 1 [m]. The translational velocity is fixed as $s$ = 0.5 [m/s]. The control gains are set as $(K_1,\ K_2,\ K_3)$ = (0.1, 3, 10).

Fig. 4 shows the experimental result. It is confirmed that controlling $c_1$ and $c_3$ to 0 was able to be accomplished by tracking the target line.

## VI. CONSIDERATION

Although the mobile robot has nonholonomic characteristics, the problem setting is not affected by such features. In the problem setting explained previously, the robot tracks its target line by controlling only $y$ and $\theta$, without controlling $x$. Since the states to be controlled are only two, it need not use any crosscut of steering. Therefore, it is necessary to set other problems in which there exist influences due to nonholonomic characteristics.

## VII. OTHER PROBLEM SETTINGS

In what follows, two problem settings affected by nonholonomic characteristics are proposed.

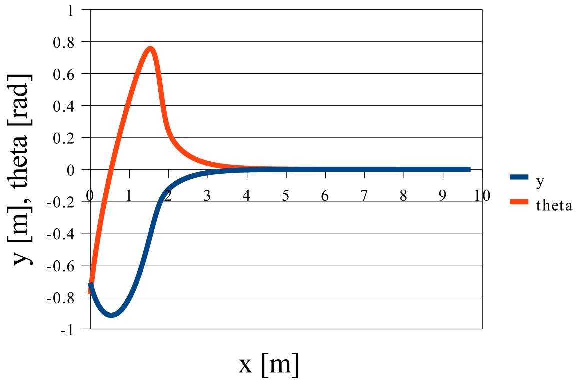


Fig. 4. Simulation result

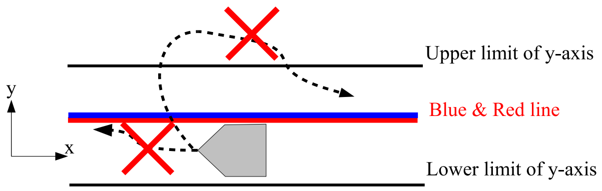


Fig. 5. Situation when the y-axis is given the upper and lower limits

### 1. Problem Setting 1: Specifying Direction

The target line is to be two colored to specify the robot direction as shown in Fig. 5. Since the problem setting explained previously assumes that the pose of the robot is within $-\pi/2 < \theta < \pi/2$, the robot cannot turn around. When applying any controllers to real robots, specifying the direction of the robot movement is useful for widespread purposes. Additionally, since the controller designed in this paper shows an overshoot shown in Fig. 5, the robot needs to use any crosscut motion by limiting y-axis value.

### 2. Problem Setting 2: Specifying Endpoint

The controller explained previously makes the robot with nonholonomic features track a target line by ignoring the value of $x$. This problem setting is given the end point of a target line. The goal of this control is to position the end point to the center of image plane. The forementioned controller cannot accomplish such an objective because the controller ignores $x$ value and the translational velocity is set to be constant. Thus, it needs to set the translational velocity as a variable, instead of setting it as a constant.

## VIII. CONCLUSION

In this paper, the paper given in Kurashiki et al. [5] has been questioned and a designed controller has been checked on a simulation experiment. It was confirmed from the simulation experiment that the robot can trace a target line with the designed controller. Two problem settings affected by nonholonomic characteristics were

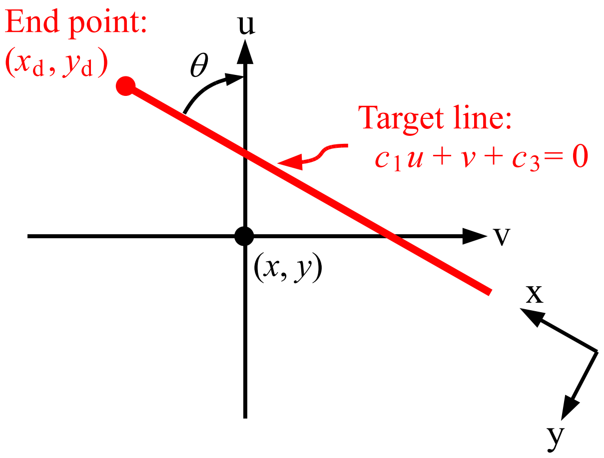


Fig. 6. Situation when the target line has an end point

also proposed to demonstrate the ability of the current image-based control method. In the future, suitable controllers will be designed and checked for the two proposed problems.

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