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A Bearing-Only Localization Solved by an Unscented Rauch-Tung-Striebel Smoothing

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Abstract: The unscented Kalman filter (UKF) has become an alternative in nonlinear estimation problems to overcome the limitation of Taylor series linearization used by the extended Kalman filter (EKF). It uses a deterministic sampling approach known as sigma points to propagate nonlinear systems and has been discussed in many literature. However, a nonlinear smoothing problem has received less attention than the filtering problem. Therefore, in this article we examine an unscented smoother based on Rauch-Tung-Striebel form for discrete-time dynamic systems. This smoother has advantages available in unscented transformation over approximation by Taylor expansion as well as its benefit in derivative free. This smoothing technique has been implemented and evaluated through a bearing-only localization problem.

Keywords: Unscented transformation, Rauch-Tung-Striebel smoother, Bearing-only localization problem.

I. INTRODUCTION

The nonlinear filtering problem has been deeply studied and various methods are provided in literature. Among them, the most useful ones are the extended Kalman Filter (EKF), the ensemble Kalman Filter (EnKF), the unscented Kalman Filter (UKF), and the Particle Filter (PF). Historically, the EKF is still the most widely adopted approach to solve the nonlinear estimation problem. It is based on the assumption that nonlinear system dynamics can be accurately modeled by a first-order Taylor series expansion as proved by van der Merwe [1]. The EnKF introduced by Evensen [2] is a reduced rank filter which propagates the states through nonlinearity and updates a relatively small ensemble of samples from which an assumed Gaussian distribution captures the main characteristics in the uncertainty. The PF also uses a sampling approach to estimate the higher-order moments of the posterior probability distribution by propagating and updating a number of particles, but without assuming Gaussian statistics as explained by Arulampalam et al. [3].

The UKF, which is a derivative free alternative to EKF, overcomes the differentiation problem by using a deterministic sampling approach demonstrated by Julier and Uhlmann [4] and Wan and van der Merwe [5]. The state distribution is represented using a minimal set of carefully chosen sample points, called sigma points. This technique is used to linearize a nonlinear function of a random variable through a linear regression between n points drawn from the prior distribution of the random variable. Since we are considering the spread of the random variable during linearization, the technique tends to be more accurate than the Taylor series linearization used in the EKF, particularly in the presence of strong nonlinearities as proved by van der Merwe [1]. The $2n+1$ sigma points, are chosen based on a square-root decomposition of the prior covariance, where n is the state dimension. These sigma points are propagated through the true nonlinear function, without approximation, and then a weighted mean and covariance is taken. This approach results in approximations that are accurate to the third order Taylor series expansion for Gaussian inputs in all nonlinearities.

However, the nonlinear smoothing problem has received less attention than the filtering problem in the literature. Therefore, in this article we investigate the unscented smoother based on Rauch-Tung-Striebel form [6], [7] for discrete-time dynamic systems studied by Särkkä [8] and Saifudin et al. [9], [10]. This smoother takes a benefit over unscented transformation to the limitation of Taylor approximation as well as its derivative free advantages. To evaluate the performance of this smoother, the algorithm is applied for a bearing-only localization problem. In what follows, note that we will use the abbreviations URTSS for the unscented Rauch-Tung-Striebel smoother.

The structure of this paper is as follows: In section 2 we briefly describe the derivation of unscented Rauch-Tung-Striebel smoother and its summary of an implemented algorithm. A bearing-only localization problem is presented in section 3 as an application example of this algorithm, as well as discussions on the simulation results. The paper is concluded in section 4.

II. UNSCENTED RAUCH-TUNG-STRIEBEL SMOOTHER

Consider a state space model of the form,

$$\begin{aligned} x_k &= F_{k-1}(x_{k-1}, u_{k-1}, w_{k-1}) \\ y_k &= H_k(x_k, v_k) \end{aligned} \tag{1}$$

where $x_k \in \mathbf{R}^n$ is the state, $y_k \in \mathbf{R}^m$ is the measurement at time t_k, u_{k-1} is the control action, $w_{k-1} \sim \mathrm{N}(0, Q_{k-1})$ is the Gaussian process noise, $v_k \sim \mathrm{N}(0, R_k)$ is the Gaussian measurement noise, $F_{k-1}(\cdot)$ is the process model function and $H_k(\cdot)$ is the measurement model function. The time step k runs from 0 to T and at time step 0 there is no measurement, only the prior distribution $x_0 \sim \mathrm{N}(m_0, P_0)$.

The purpose of the smoothing algorithm is to find approximations to the smoothing distributions $p(x_k \mid y_{1:T})$ for all $k = 1, 2, \ldots, T$. The approximations are chosen to be Gaussian:

$$p(x_k \mid y_{1:T}) \sim \mathrm{N}(x_k \mid m_k^s, P_k^s). \tag{2}$$

The *optimal smoothing equations* of the model can be written in two options as mentioned by Klaas et al. [11], named as two filter smoother and forward-backward smoother. For the purpose of deriving the Rauch-Tung-Striebel form of smoother, the forward-backward smoothing will be used and it can be written as follows:

$$p(x_k \mid y_{1:T}) = p(x_k \mid y_{1:k}) \times \int \frac{p(x_{k+1} \mid x_k)\, p(x_{k+1} \mid y_{1:T})}{p(x_{k+1} \mid y_{1:k})} dx_{k+1} \tag{3}$$

where $p(x_k \mid y_{1:k})$ is the filtering distribution of the time step k and $p(x_{k+1} \mid y_{1:k})$ is the predicted distribution of the time step $k+1$, which can be computed by the prediction step of the optimal filtering. The smoothing recursion is started from last time step $k = T$ and proceeded backwards in time.

From Eq. (3), the Rauch-Tung-Striebel smoother can be derived as shown by Särkkä [8]. Assumed that the (approximate) mean and covariance of the filtering distributions

$$p(x_k \mid y_{1:k}) \approx \mathrm{N}(x_k \mid m_k, P_k)$$

for the model in Eq. (1) have been computed by the unscented Kalman filter or a similar method.

Further assume that the smoothing distribution of time step $k+1$ is known and Gaussian

$$p(x_{k+1} \mid y_{1:T}) \approx \mathrm{N}\left(x_{k+1} \mid m_{k+1}^s, P_{k+1}^s\right).$$

This smoothing algorithm can be summaried as following steps:

1) Form the matrix of sigma points of the augmented random variable $\widetilde{x}_k = \left(x_k^T \; w_k^T\right)^T$ such that

$$\widetilde{X}_k = [\widetilde{m}_k \quad \cdots \quad \widetilde{m}_k] + \sqrt{c}\left[0 \quad \sqrt{\widetilde{P}_k} \quad -\sqrt{\widetilde{P}_k}\right]$$

where $\widetilde{m}_k = \begin{bmatrix} m_k \\ 0 \end{bmatrix}$ and $\widetilde{P}_k = \begin{bmatrix} P_k & 0 \\ 0 & Q_k \end{bmatrix}$.

2) Propagate the sigma points through the dynamic model

$$\widetilde{X}_{k+1,i}^- = F_k\left(\widetilde{X}_{k,i}^x,\; \widetilde{X}_{k,i}^w\right), \quad i = 1, \ldots, 2n+1$$

where $\widetilde{X}_{k,i}^x$ and $\widetilde{X}_{k,i}^w$ denote the parts of the augmented sigma point i, which correspond to x_k and w_k, respectively.

3) Compute the predicted mean m_{k+1}^-, the predicted covariance P_{k+1}^- and the cross-covariance C_{k+1}:

$$\begin{aligned} m_{k+1}^- &= \sum_i W_{i-1}^{(m)} \widetilde{X}_{k+1,i}^- \\ P_{k+1}^- &= \sum_i W_{i-1}^{(c)} \left(\widetilde{X}_{k+1,i}^- - m_{k+1}^-\right) \\ &\quad \times \left(\widetilde{X}_{k+1,i}^- - m_{k+1}^-\right)^T \\ C_{k+1} &= \sum_i W_{i-1}^{(c)} \left(\widetilde{X}_{k,i}^x - m_k\right) \\ &\quad \times \left(\widetilde{X}_{k+1,i}^- - m_{k+1}^-\right)^T \end{aligned}$$

where the definitions of the weights $W_{i-1}^{(m)}$ and $W_{i-1}^{(c)}$ are the same as in [5].

4) Compute the smoother gain D_k, the smoothed mean m_k^s and the covariance P_k^s:

$$\begin{aligned} D_k &= C_{k+1}\left[P_{k+1}^-\right]^{-1} \\ m_k^s &= m_k + D_k\left(m_{k+1}^s - m_{k+1}^-\right) \\ P_k^s &= P_k + D_k\left[P_{k+1}^s - P_{k+1}^-\right] D_k^T \end{aligned}$$

The above procedure is a recursion, which can be used for computing the smoothing distribution of step k from the smoothing distribution of time step $k+1$. Because the smoothing distribution and filtering distribution of the last time step T are the same, we have $m_T^s = m_T$, $P_T^s = P_T$, and thus the recursion can be used for computing the smoothing distributions of all time steps by starting from the last step $k = T$ and proceeding backwards to the initial step $k = 0$.

III. AN EXAMPLE APPLICATION

In this section we consider the problem of bearing only localization as used by Bailey [12].

1. Process and measurement state

The discrete time vehicle state is given by:

$$\boldsymbol{x}_k = \boldsymbol{f}(\boldsymbol{x}_{k-1}, \boldsymbol{u}_{k-1}, w_{k-1}) \tag{4}$$

where $\boldsymbol{x}_k = [x_k \; y_k \; \phi_k]^T$ are the vehicle position coordinates and its orientation in time step k , respectively. $\boldsymbol{u}_{k-1} = [V \; G]^T$ is control action in which V is the vehicle velocity, and G is the vehicle steering angle. w_{k-1} is a zero-mean Gaussian process noise with covariance Q.

This vehicle is equipped with a range and bearing sensor. It can sense an object bounding in ± 30 degree semi-circle with the maximum range of 30 meter. Only bearing measurement data will be used in this example. The measurement equation is as follows:

$$\begin{aligned} z_k &= h(\boldsymbol{x}_k, \boldsymbol{f}_i) + v_k \\ &= \left[\tan^{-1}\left(\frac{f_{i,y} - y_k}{f_{i,x} - x_k}\right) - \phi_k\right] + v_k \end{aligned} \tag{5}$$

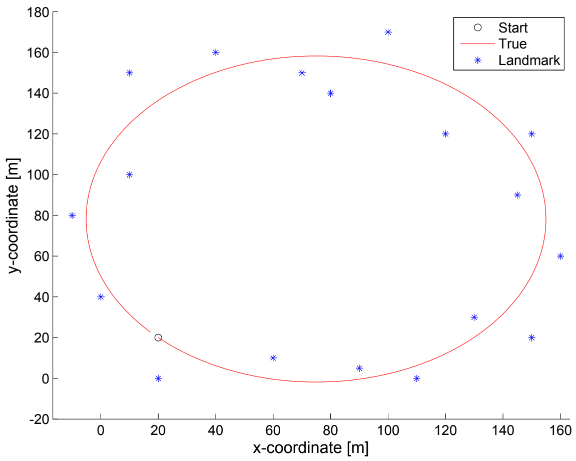

Fig. 1. True trajectory and landmark.

where x_k, y_k and ϕ_k are the vehicle position coordinates and its orientation in time step k, respectively. $\boldsymbol{f}_i$ is a landmark feature available at time when the sensor takes a measurement. This landmark feature is assumed to be static and represented as a Cartesian coordinate system as $(f_{i,x}, f_{i,y})$. v_k is assumed as zero-mean Gaussian white measurement noise with covariance R.

2. Simulation setup

Fig. 1 shows the landmark and true vehicle trajectory setup for this simulation. The vehicle starts at a known location (20 m, 20 m, -0.8 rad) and travels with a nominal speed and a steering angle of 3 m/s and 0.05 rad, respectively. The nominal control values are corrupted with Gaussian noise with standard deviations 0.3 m/s and 0.05 radian, respectively for each 0.5 s sampling interval. The sensor takes bearing measurement and its value is assumed to be corrupted with Gaussian noise with standard deviation 0.09 radian. All simulation parameters and their values are shown in Table 1.

The initial conditions for the filter are set to

$$\hat{\boldsymbol{x}}_0 = \boldsymbol{x}_0 = \begin{pmatrix} 20 \\ 20 \\ -0.8 \end{pmatrix}$$

and

$$P_0 = \begin{pmatrix} 10^{-10} & 0 & 0 \\ 0 & 10^{-10} & 0 \\ 0 & 0 & 10^{-10} \end{pmatrix}$$

which basically means that the vehicle initial position and its orientation are known.

3. Result and discussion

Fig. 2 shows the result of both UKF and URTSS. For more reliable result, we calculated the root mean square (rms) error of x-axis, y-axis, ϕ and also its position for every iteration step and they are shown in Figs. 3, 4, 5 and 6 respectively. Figs. 3 and 4 show that the rms errors of the URTSS are always comparable to or lower than the error values produced by the UKF. This situation also can been seen in rms vehicle position errors as shown in Fig 6. However, the vehicle orientation rms errors for both methods did not show a significant difference. Futhermore, we took root mean square values for 1000 Monte Carlo runs. The results of filtering and smoothing estimation in x-axis, y-axis, and ϕ are shown in Table 2 and it is proved that the URTSS has a better performance over the UKF.

Table 1. Simulation setup

Parameter	Description	Value	Unit
V	Velocity	3	m/s
G	Steering angle	0.05	radian
WB	Wheel-base	4	m
σ_V	Standard deviation of V	0.3	m/s
σ_G	Standard deviation of G	0.05	radian
σ_B	Standard deviation of bearing	0.09	radian

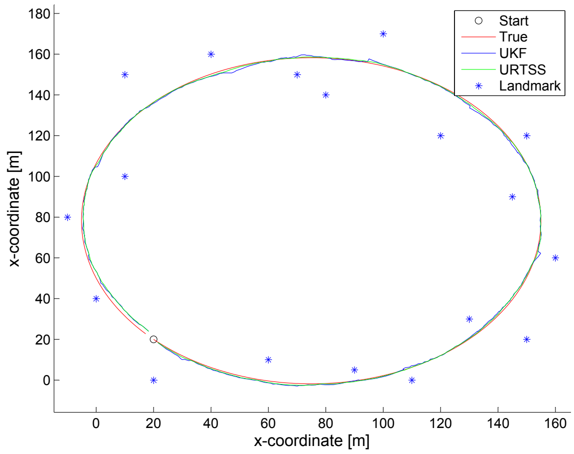

Fig. 2. Estimates by UKF and URTSS.

IV. CONCLUSION

In this paper, an unscented Rauch-Tung-Striebel smoother (URTSS) has been applied to a bearing-only localization problem and its performance has been also evaluated in simulations. It was assumed that the vehicle was equipped with a range and bearing sensor. To compare the performance of both UKF and URTSS, the rms errors were calculated. It was then found the URTSS has a better performance over the UKF.

Table 2. RMS errors

Method	x_{RMSE}	y_{RMSE}	ϕ_{RMSE}
UKF	0.8730	0.8361	0.0941
$URTSS$	0.7438	0.7367	0.0771

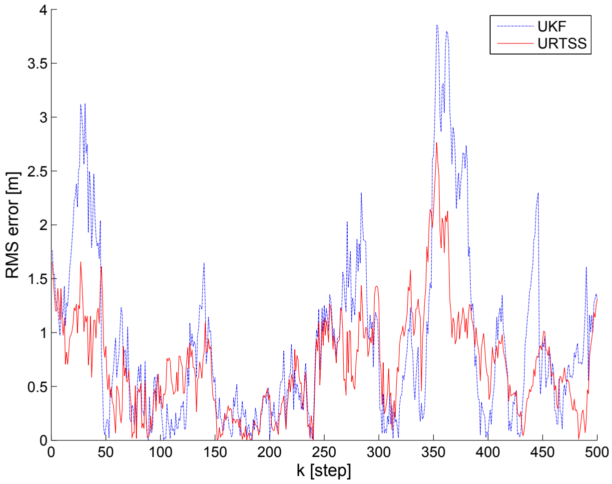

Fig. 3. RMS error of x-axis.

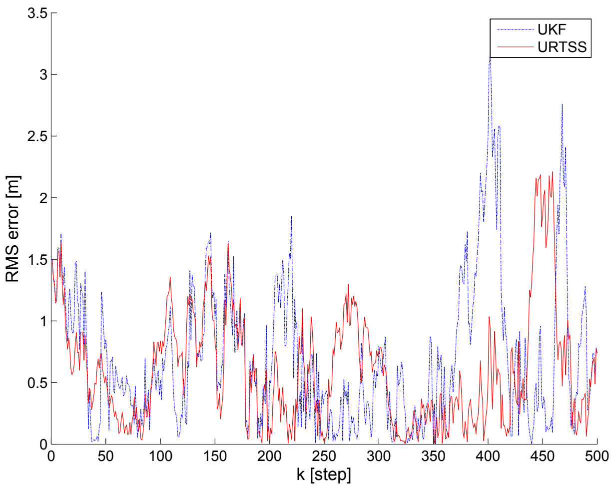

Fig. 4. RMS error of y-axis.

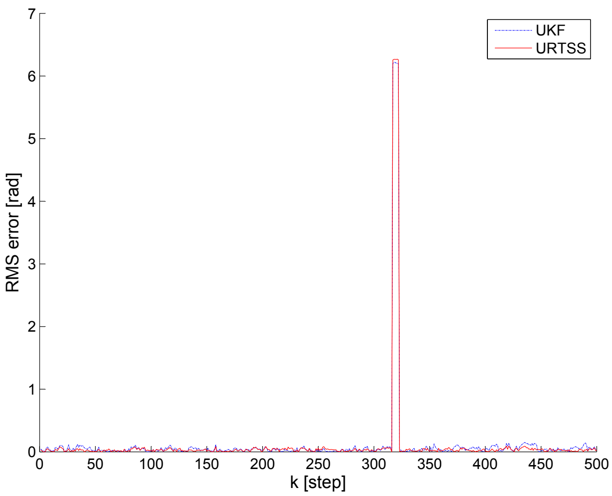

Fig. 5. RMS error of ϕ-axis.

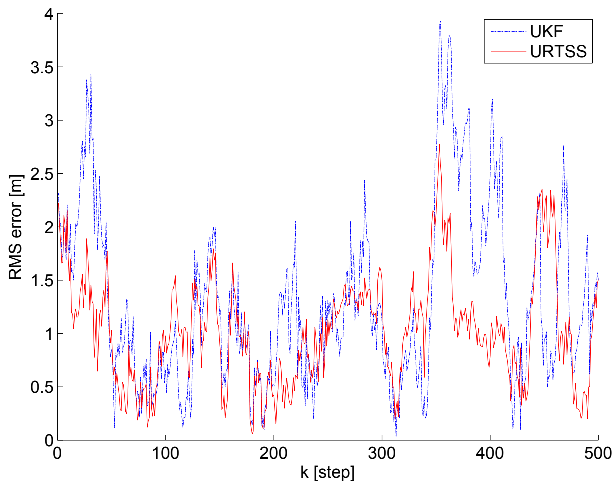

Fig. 6. RMS vehicle position error estimates by UKF and URTSS.

REFERENCES

[1] van der Merwe R (2004), Sigma point Kalman filter for probabilistic inference in dynamic state-space models. PhD Thesis,Oregon Health & Science University, Portland.

[2] Evensen G (2003), The ensemble Kalman filter: Theoretical formulation and practical implementation. Ocean Dynamics, 53: 343–367.

[3] Arulampalam MS , Maskell S, Gordon N, and Clapp T (2002), A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking. IEEE Trans. on Signal Processing, 50(2): 174–188.

[4] Julier SJ and Uhlmann JK (2004), Unscented filtering and nonlinear estimation. Proceedings of the IEEE, 92(3): 401–422.

[5] Wan E and van der Merwe R (2004), The Unscented Kalman Filter, New York: Wiley.

[6] Rauch HE, Tung F and Striebel CT (1965), Maximum likelihood estimates of linear dynamic systems. AIAA Journal 3(8): 1445–1450.

[7] Rauch HE (1963), Solutions to linear smoothing problem. IEEE Trans. on Automatic Control 8(4): 371–372.

[8] Särkkä S (2008), Unscented Rauch-Tung-Striebel smoother. IEEE Trans. on Automatic Control AC-53(3): 845–849.

[9] Saifudin R, Watanabe K, Maeyama S and Izumi K (2010), An unscented Rauch-Tung-Striebel smoother for a bearing only tracking problem. Int. Conf. Control, Automation and Systems 2010, Gyeonggi-do, Korea.

[10] Saifudin R, Watanabe K, Maeyama S and Izumi K (2010), An unscented Rauch-Tung-Striebel smoother for a vehicle tracking problem. Int. Conf. on Soft Computing and Intelligent Systems and Int. Sympsium on Advanced Intelligent Systems, Okayama, Japan.

[11] Klaas M, Briers M, de Freitas N, Doucet A, Maskell S, and Lang D (2006), Fast particle smoothing: If I had a million particles. Proceedings of ICML, 25–29.

[12] Bailey T (2003), Constrained initialization for bearing only SLAM. IEEE Int. Conf. on Robotics and Automation (2): 1966–1977.