A three-variable silicon neuron circuit

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Abstract: The silicon neuron is a type of artificial neuron implemented with electronic circuit. Previously a design approach based on mathematical structures under neuronal dynamics was proposed. It is based on the mathematical techniques such as phase plane and bifurcation analysis. These methods allow us to implement silicon neuron with smaller circuit area and to strategically adjust the bias parameter voltages without losing variety of output patterns. In this study we demonstrate a mathematical-structure-based silicon neuron, which operates a three-dimensional system. This silicon neuron can generate a firing pattern called square-wave bursting. In this report we show the experimental results of this silicon neuron. We are planning to make pattern generating network using this silicon neuron and silicon synapses.

Keywords: Silicon neuron, MOSFET, Phase plane, Bifurcation analysis

I Introduction

The neuromorphic hardware is an electronic system that mimics functions of nerve systems. Biological studies revealed that nervous systems process information in fundamentally different ways from digital computers. They have good adaptability to their environment and high robustness. To investigate the mechanism of such splendid information processing ability and to reproduce it in artificial systems, many biophysical and theoretical studies have been done about the neuron and the synapse which are the basic component of the nerve system. Silicon neurons have been produced through the efforts to reproduce various properties elucidated by these studies using electronic circuits.

There have been two major types of approaches, one is phenomenological approach and the other is conductancebased one. Phenomenological silicon neurons are based on extremely simplified neuron models such as the leaky integrate-and fire model [1]. They could be implemented by relatively simple and compact circuit because phenomenological neuron models focus on the specific properties of neurons, thus they are suitable for investigating large silicon neural networks. However, they reproduce limited aspects of neuronal dynamics because these models ignore the ionic dynamics in neurons. On the other hand, conductancebased silicon neurons, such as [2] are intended to emulate the dynamics of ionic channels in neurons. They have the ability to generate various firing patterns by adjusting the externally applied parameter voltages to the circuit. However, they have drawbacks of complexity in their circuitry and a number of parameter voltages to be adjusted. These points raise difficulty in circuit implementation and operation under the presence of device mismatch and noises. Thus it is hard to compose large silicon neural networks of the conductance-based silicon neurons.

previous studies [3], Kohno proposed a In mathematical-structure-based approach to design silicon neurons. Qualitative neuron models, such as the FitzHugh-Nagmo [4] and the Hindmarsh-Rose models [5] can generate various firing patterns with fewer parameters. However, their equations are not suitable for implementation by electronic circuit. In the mathematicalstructure-based approach, the equations of the qualitative models are modified so that they can be implemented by electronic circuit effectively while preserving their mathematical structures utilizing phase plane and bifurcation analyses. In this study we present a mathematical-structurebased silicon neuron, which has three dimensional system equations and designed to produce a burst firing pattern called square-wave bursting when all of the three variables are activated.

II System equations of our silicon neuron

The system equations of our silicon neuron contain three variables, v, n and q. Variable v corresponds to the membrane potential of the neuron, n and q are the variables that represent an ionic channel and a negative feedback cur-



Figure 1: The *n*-*v* phase plane of silicon neuron model (simulation). *n*-nullcline and *q*-nullcline are respectively the set of $\frac{dn}{dt} = 0$ and $\frac{dq}{dt} = 0$

rents. They are written as follows:

$$C_v \frac{dv}{dt} = f_m(v) - g(v) - n - q + I_a + I_{stim} \quad (1)$$

$$\frac{dn}{dt} = \frac{f_n(v) - n}{T_n} \tag{2}$$

$$\frac{dq}{dt} = \frac{f_q(v) - q}{T_q} \tag{3}$$

where I_a is an ionic current which is independent of the membrane potential. Current I_{stim} is an externally applied stimulus current. Constant C_v is the membrane capacitance. Constant T_n and T_q are the time constants for n and q.

The functions $f_x(v)$ (x = m, n, q) and g(v) are the sigmoidal characteristic curves of the differential-pair circuitries, which are expressed in the following forms:

$$f_x(v) = M_x \frac{1}{1 + \exp(-\frac{\kappa}{U_T}(v - \delta_x))}$$
(4)

$$g(v) = S \frac{1 - \exp(-\frac{\kappa}{2U_T}(v -))}{1 + \exp(-\frac{\kappa}{2U_T}(v -))}$$
(5)

Equations (1) and (2) comprise the basic excitable system, which can reproduce the same mathematical structures as various non-bursting neuron models, such as Hodgikin-Huxley and Morris-Lecar models. Equation (3) comprises the negative feedback system that generates positive current q into the membrane capacitor while the potential v is high, and negative current while v is low, whose mechanism cause the square-wave bursting when the basic excitable system has a kind of bistability. Figure 1 shows the v-n phase plane of the basic excitable system of our silicon neuron. When no stimulus current is applied, the membrane potential stays at the stable equilibrium (S) (resting point). If the stimulus current is small, the system state cannot move over a stable manifold of the saddle point (T)



Figure 2: Block diagram of our silicon neuron



Figure 3: Schematics of f(v) (left) and g(v) (right) generator circuits.

and go back to (S). However, if the stimulus is sufficiently strong, the system state travels around the unstable node (U) and come back to the resting state. In the next section, we present the circuit experiment results of our basic excitable system.

III Circuit of our silicon neuron

In Fig.2 the block diagram of our silicon neuron is shown. It is composed of differential pair (Fig.3), current mirror, and current-mode integrator (Fig.4) circuits. The differential pair circuits make the functional curves of the nullclines, and the current-mode integrator integrates the system equations. Each of function modules generates output current which depends on membrane potential v, and the output currents flow into the membrane capacitor whose voltage represents v through current mirror or current-mode integrator.



Figure 4: Schematic of the current-mode integrator circuit where MOSFETs are operated in the subthreshold condition. This circuit realize the integration in Eqs. (2) and (3).



Figure 5: The v- and the n-nullclines drawn by the voltage clamp system implemented in the same chip with silicon neuron.

IV Experimental results

1 Drawing phase plane

Our silicon neuron has the voltage-clamp measurement system inside the same VLSI chip, which can generate the output currents of each functional modules while clamping the membrane potential to a specific voltage. Figure 5 shows the v-n phase plane of our silicon neuron, which was drawn by this system. In the following subsections we present experimental results of the basic excitable system in our silicon neuron circuit. The values of parameter voltages are shown in Table 1.

2 Responses to singlet pulse stimuli

Figure 6 shows the behaviour of the membrane potential v in response to the singlet pulse stimuli. The duration of the input pulse is 1.0 ms and the amplitude $\delta_{I_{stim}}$ is varied from -90 mV to -30 mV. The value of the amplitude is

 Table 1: Operating parameter set

Parameters	Values[V]	Parameters	Values [V]
V_{DD}	3.3	V_{δ_n}	0.09
V_{SS}	0	V_{M_n}	0.395
V_{δ_m}	-0.01	$V_{\tau b_n}$	0.243
V_{M_m}	0.31	V_{ofst}	0.39
$V_{ heta}$	-0.13	$V_{ au}$	0
V_S	0.3	$V_{\tau d}$	0
$V_{\delta_{Ia}}$	0	$V_{\delta_{Istim}}$	0
$V_{S_{Ia}}$	0.35	$V_{S_{Istim}}$	0.35



Figure 6: Response of the membrane potential to singlet pulse stimuli. Stimulus is applied to the membrane capacitor through a V-I transmitter circuit in the VLSI chip, which converts negative voltage to positive current. The duration of the stimulus pulse is 1.0 ms, and the strength is varied from -30 mV to -90 mV.

negative voltage because stimulus current is applied via a V-I transmitter circuit that converts negative input voltage into positive current. These results demonstrate that our silicon neuron has the first four out of the five properties of silent neurons described by Zeeman [6]. (1) a stable equilibrium point exists that corresponds to the resting state. (2) and action potential can be generated in response to an external stimulus, and the size of the response is absolutely larger than that of the stimuli. (3) a threshold of the stimulus magnitude exists for the generation of action potential. (Fig.6 shows that the threshold voltage of our silicon neuron exists between -50 mV and -60 mV.) (4) an action potential returns to the resting state more slowly than its rising phase. (5) refractoriness exists after generation of an action potential.

3 Responses to doublet pulse stimuli

Figure 7 shows the responses to the doublet pulse stimuli. The duration of the input pulse is 1.0 ms and their in-



Figure 7: Responses to doublet pulse stimuli. The duration of the input pulse is 1.0 ms and their interval is 20 ms. Both of the pulses have the same amplitude of -60 mV, -70 mV, or -80 mV. This graph shows that the responses to second pulse is smaller than first one's. This result indicates the existence of refractory period after the firing.

terval is 20 ms. Both of the pulses have the same amplitude of -60 mV, -70 mV, or -80 mV. In each amplitude, membrane potential v was less responsive to the second stimulus in comparison to the first one. This indicates the existence of refractory period which is the fifth property in the Zeeman's characterization listed above.

V Square-wave burster mode

When the parameters are selected appropriately, a saddle-loop homoclinic orbit bifurcation emerges in the basic excitable system when q is varied. In this situation, there exists a bistability between a stable limit cycle that represents a tonic firing state and a stable equilibrium that represents a silent state (see Fig.8). When the system state is in the left side of the q-nullcline, $\frac{dq}{dt}$ is positive, thus the state point moves to the right direction generating tonic firing. At the point of the saddle-loop homoclinic orbit bifurcation, the minimum potential of the stable limit cycle reaches to the saddle point and the state point is attracted to the stable node along the unstable manifold of saddle node. Then $\frac{dq}{dt}$ becomes negative, thus the state point moves to the left until jumps to the limit cycle to generate tonic firing again. These are the mechanism of generating square-wave bursting in our silicon neuron.

VI Conclusion

We introduced a mathematical-structure-based silicon neuron circuit with 3 variables and reported the experimental results of its basic excitable system. The mathematical structure which dominates the circuit operation was repre-



Figure 8: The v-q plane of the system equations of our silicon neuron. At the point of q = 33.47 pA, minimum of the stable limit cycle reaches to the saddle point.

sented by the phase plane. We determined the externally applied parameter voltages utilizing the structures in the phase plane. In HSpice circuit simulation, our silicon neuron circuit successfully produced burst firing patterns including square-wave bursting (not shown). We are working on the circuit experiments of the total system in our silicon neuron, which will be presented in our future publications.

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