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Parameter Identification of Lorenz System Using RBF Neural Networks with Time-Varying Learning Algorithm

Chia-Nan Ko[1], Yu-Yi Fu[2], Cheng-Ming Lee[3], and Guan-Yu Liu[4]

[1,2]*Department of Automation Engineering,*
[3]*Department of Computer and Communication Engineering,*
[4]*Department of Electronic Engineering,*
[1,2,3,4]*Nan-Kai University of Technology, Tasotun, Nantou 542, Taiwan*
(*E-mail:* [1]t105@nkut.edu.tw; [2] t098@nkut.edu.tw; [3] t104@nkut.edu.tw; [4] guanyu@nkut.edu.tw)

Abstract: A hybrid evolutionary algorithm is proposed to identify parameters for Lorenz chaotic system. In the proposed algorithm, time-varying learning algorithm based on annealing robust concept (ARTVLA) is adopted to optimize a radial basis function neural network (RBFNN) for parameter identification of Lorenz system. In the ARTVLA, the determination of the learning rate would be an important work for the trade-off between stability and speed of convergence. A computationally efficient optimization method, particle swarm optimization (PSO) method, is adopted to simultaneously find a set of promising learning rates and optimal parameters of RBFNNs. The proposed RBFNN (ARTVLA-RBFNN) has good performance for identifying parameters of Lorenz system. Simulation results are illustrated the effectiveness and feasibility of the proposed ARTVLA-RBFNN.

Keywords: Parameter identification, time-varying learning algorithm, particle swarm optimization, Lorenz chaotic system.

I. INTRODUCTION

A chaotic system is a nonlinear deterministic system that has some special features of sensitive dependence on initial conditions and unstable bounded trajectories in the phase space. Due to their characteristics sensitivity to initial conditions, chaotic systems are not easy to identify. Recently, some researchers have endeavored to improve the identification of chaotic systems[1-3].

Recently, RBFNNs have received considerable applications in various fields, such as function approximation, prediction, recognition, etc[4,5]. Since RBFNNs have only one hidden layer and have fast convergence speed, they are widely used for nonlinear system identifi- cation recently. Besides, the RBFNNs are often referred to as model-free estimators since they can be used to approximate the desired outputs without requiring a mathematical description of how the outputs functionally depend on the inputs[6,7] .

When utilizing RBFNNs, a learning rate serves as an important role in the procedure of training RBFNs. Generally, the learning rate is selected as a time-invariant constant by trial and error. However, there still exist several problems of unstable or slow convergence. Some researchers have engaged in exploring the learning rate to improve the stability and the speed of convergence[8,9]. In this article, time-varying learning algorithm (TVLA) is then applied to train the RBFNN (TVLA-RBFNN), in which PSO method[10] is adopted to find optimal learning rates during learning procedure. A typical system, Lorenz chaotic system, will be given to illustrate the feasibility and efficiency of the proposed TVLA-RBFNNs for parameter identification of the chaotic system.

II. PROBLEM FORMULATION

Considering the following n-dimensional chaotic system:

$$\dot{\boldsymbol{X}} = F(\boldsymbol{X}, \boldsymbol{X}_0, \boldsymbol{Q}_0) \quad (1)$$

where $\boldsymbol{X} \in R^n$ denotes the state vector, $\boldsymbol{X}_0$ denotes the initial state, and $\boldsymbol{Q}_0$ is a set of original parameters.

When estimating the parameters, suppose the structure of the system is known in advance, and thus the estimated system can be described as follows:

$$\dot{\hat{\boldsymbol{X}}} = F(\boldsymbol{X}, \boldsymbol{X}_0, \boldsymbol{Q}) \quad (2)$$

where $\dot{\hat{\boldsymbol{X}}} \in R^n$ denotes the state vector, and $\boldsymbol{Q}$ is a set of estimated parameters. Therefore, the problem of parameter estimation can be formulated as the following optimization problem:

$$\text{Min} \quad J = \frac{1}{M}\sum_{K=1}^{M}\left\|\boldsymbol{X}_K - \hat{\boldsymbol{X}}_K\right\|^2 \quad (3)$$

where M denotes the length of data used for parameter identification, $\boldsymbol{X}_K$ and $\hat{\boldsymbol{X}}_K$, denote state vectors of the original and the identified systems at time K, respectively.

Obviously, the parameter identification for chaotic systems is a multi-dimensional continuous optimization problem, where the decision vector is $\boldsymbol{Q}$ and the optimization goal is to minimize J.

III. ARTVLA-BASED RBFNNs USING PSO

1. Architecture of RBFNNs

In general, the input-output relation of a nonlinear system can be expressed as

$$\mathbf{y}(t+1) = \mathbf{f}(\mathbf{y}(t), \mathbf{y}(t-1), \cdots, \mathbf{y}(t-n_y), \mathbf{x}(t), \mathbf{x}(t-1), \cdots, \mathbf{x}(t-n_u)) \quad (4)$$

where $\mathbf{x}(t) \in R^m$ is the input vector, $\mathbf{y}(t) \in R^p$ is the output vector, n_u and n_y are the maximal lags in the input and output, respectively, and $\mathbf{f}(t) \in R^p$ denotes the nonlinear relation to be estimated. One can use a neural network to estimate the input-output relation of a nonlinear system.

A radial basis function neural network (RBFNN) consists of three layers, the input layer, the hidden layer, and the output layer. The transformation from the input layer to the hidden layer is nonlinear. The output layer is linear and gives a summation at the output neurons. The structure of an RBFNN is shown in Fig. 1. When the Gaussian function is chosen as the radial basis function, an RBFN can be expressed in the form

$$\hat{y}_j(t+1) = \sum_{i=1}^{l} G_i w_{ij} = \sum_{i=1}^{l} w_{ij} \exp\left(-\frac{\|\hat{\mathbf{x}} - \mathbf{m}_i\|^2}{2\sigma_i^2}\right) \quad \text{for } j = 1, 2, \cdots, p \quad (5)$$

where $\hat{\mathbf{x}}(t) \in R^m$ is the input vector, $\hat{y}_j(t+1)$ is the jth output, w_{ij} is the synaptic weight between the ith hidden neuron and the jth output neuron, G_i is the Gaussian function at the ith neuron in the hidden layer, $\mathbf{m}_i$ and σ_i are the center and width of G_i, respectively, and l is the number of the Gaussian functions, which is also equal to the number of hidden layer nodes.

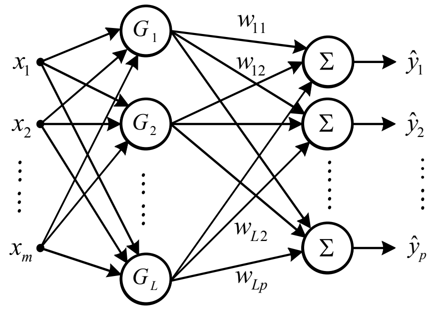

Fig. 1. The structure of an RBFNN.

2. PSO-Based ARTVLA

In the training procedure of the proposed RBFNNs, the annealing concept in the cost function of robust back-propagation learning algorithm was adopted to overcome the existing problems in robust back-propagation learning algorithm, such as slow convergence rate and getting into local minimum[11]. A cost function for the ARTVLA is defined here as

$$J_j(h) = \frac{1}{N}\sum_{k=1}^{N} \rho\left[e_j^{(k)}(h); \beta(h)\right] \quad \text{for } j = 1, 2, \cdots, p \quad (6)$$

where

$$e_j^{(k)}(h) = y_j^{(k)} - \hat{y}_j(\mathbf{x}^{(k)}) \quad (7)$$

h is the epoch number, $e_j^{(k)}(h)$ is the error between the jth desired output and the jth output of the RBFNN at epoch h for the kth input-output training data in a nonlinear system, $\beta(h)$ is a deterministic annealing schedule acting like the cut-off point, and $\rho(\cdot)$ is a logistic loss function defined as

$$\rho[e_j^{(k)}; \beta] = \frac{\beta}{2}\ln\left[1 + \frac{(e_j^{(k)})^2}{\beta}\right] \quad \text{for } j = 1, 2, \cdots, p \quad (8)$$

Based on the gradient-descent kind of learning algorithms, the synaptic weights w_{ij}, the centers $\mathbf{m}_i$, and the widths σ_i of Gaussian functions are updated as

$$\Delta w_{ij} = -\eta_{w_j}\frac{\partial J_j}{\partial w_{ij}} = -\frac{\eta_{w_j}}{N}\sum_{k=1}^{N}\varphi_j(e_j^{(k)}; \beta)\frac{\partial e_j^{(k)}}{\partial w_{ij}} \quad (9)$$

$$\Delta \mathbf{m}_i = -\eta_c\frac{\partial J_j}{\partial \mathbf{m}_i} = -\frac{\eta_c}{N}\sum_{j=1}^{p}\sum_{k=1}^{N}\varphi_j(e_j^{(k)}; \beta)\frac{\partial e_j^{(k)}}{\partial \mathbf{m}_i} \quad (10)$$

$$\Delta \sigma_i = -\eta_\sigma\frac{\partial J_j}{\partial \sigma_i} = -\frac{\eta_\sigma}{N}\sum_{j=1}^{p}\sum_{k=1}^{N}\varphi_j(e_j^{(k)}; \beta)\frac{\partial e_j^{(k)}}{\partial \sigma_i} \quad (11)$$

$$\varphi_j(e_j^{(k)}; \beta) = \frac{\partial \rho(e_j^{(k)}; \beta)}{\partial e_j^{(k)}} = \frac{e_j^{(k)}}{1 + (e_j^{(k)})^2 / \beta(h)} \quad (12)$$

where η_{w_j}, η_c, and η_σ are the learning rates for the synaptic weight w_{ij}, $j = 1, 2, \cdots, p$, the center $\mathbf{m}_i$, and the width σ_i, respectively, $\varphi(\cdot)$ is usually called the influence function.

In (9) to (11), when the learning rates are constant, the work for selecting an appropriate learning rate is tedious; moreover, there exists a tendency to get stuck in a near-optimal solution or to converge slowly. To overcome the stagnation in searching a globally optimal solution, TVLA is proposed to approach the optimal solution closely in this paper. In the TVLA, a nonlinear time-varying evolution concept is adopted over iterations, in which the learning rates η_w, η_c, and η_σ with a high value $\eta_{\max}$ and nonlinearly decreases to $\eta_{\min}$ at the maximal number of epochs, respectively. This means that the mathematical expressions are given as shown as

$$\eta_w = \eta_{\min} + \left(1 - \frac{h}{epoch_{\max}}\right)^{pw}(\eta_{\max} - \eta_{\min}) \quad (13)$$

$$\eta_c = \eta_{\min} + \left(1 - \frac{h}{epoch_{\max}}\right)^{pc}(\eta_{\max} - \eta_{\min}) \quad (14)$$

$$\eta_\sigma = \eta_{\min} + \left(1 - \frac{h}{epoch_{\max}}\right)^{ps}(\eta_{\max} - \eta_{\min}) \quad (15)$$

where $epoch_{max}$ is the maximal number of epochs and h is the current number of epochs. In the updated procedure, appropriate functions for the learning rate η_w, η_c, and η_σ can promote the performance of RBFNNs. However, simul-taneously determining the optimal combination of pw, pc, and ps is a time-consuming work.

In this paper, the PSO method is adopted to find the optimal combination (pw, pc, ps) of learning rates in (13) to (15) and optimal parameters of RBFNNs for parameter identification of Lorenz system. In the system identi-fication, the goal is to minimize the error between the desired outputs and the trained outputs. Therefore, the root mean squares error (*RMSE*) should be used to define a fitness function. This means that the fitness function will be defined as

$$RMSE = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(y^{(k)} - \hat{y}^{(k)}\right)^2} \quad (16)$$

where $y^{(k)}$ is the desired output, $\hat{y}^{(k)}$ is the trained output for N sampling data.

IV. SIMULATION RESULTS

The identification scheme of a chaotic system is depicted in Fig. 2, training input-output data are obtained by feeding a signal $\mathbf{x}(k)$ to the system and measure its corresponding output $\mathbf{y}(k+1)$ Then subject to the same input signal, the objective of identification is to construct an ARTVLA-RBFNN using PSO method, which produces an output $\hat{\mathbf{y}}(k+1)$ to approximate $\mathbf{y}(k+1)$ as closely as possible.

In this section, Lorenz system is used to verify the feasibility of the proposed ARTVLA-RBFNNs. When applying the proposed algorithm, the population size, the maximal iteration number, and the maximal epoch number are chosen to be 30, 200, and 200, respectively. The variables pw, pc, and ps in learning rate functions (13) to (15) are all chosen as real numbers in the range $[0.1, 5]$. Meanwhile, the values of η_{max} and η_{min} are set as 3.0 and 0.5, respectively.

Two problems are investigated for Lorenz system. First, the impact on efficiency of annealing robust learning algorithms (ARLA) for various learning rates is studied, in which the best learning rate will be determined by trial and error. Secondly, the comparison between the proposed ARTVLA-RBFNN with nonlinear time-varying learning rates and the ARLA-based RBFNN (ARLA-RBFNN) with a fixed learning rate is illustrated. The *RMSE* (16) of the training data is adopted to evaluate the performance of the RBFNNs.

Example: A typical chaotic system, Lorenz system, is considered as an example described as follows[2,12]:

$$\begin{cases} \dot{x} = q_1(y - x) \\ \dot{y} = q_2 x - y - xz \\ \dot{z} = xy - q_3 z \end{cases} \quad (17)$$

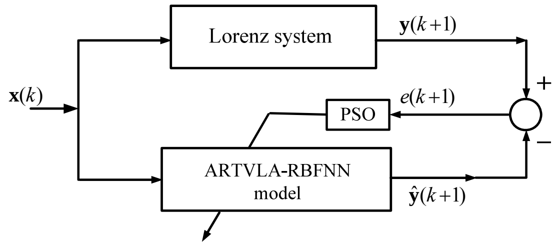

Fig.2. The proposed PSO-based ARTVLA-RBFNN scheme for parameter identification of Lorenz system.

where $q_1 = 1,\ q_2 = 5.46,\ q_3 = 20$ are the original parameters. The initial values of the system are $x(0) = -0.3,\ y(0) = 0.3,\ z(0) = 0.2$.

With the 2000 training data, two annealing robust algorithms are then applied to train the RBFN, respectively.

Problem 1:

In the ARLA-RBFNNs, various learning rates, $0.5 \le \eta \le 3.0$, are used to train the RBFNNs. After 200 training epochs, the *RMSE* values for various learning rates are obtained, respectively. The details of the simulation results are shown in Table 1.

Problem 2:

With the nonlinear learning rates, the ARTVLA is adopted to train the RBFNNs, in which the optimal learning rates are determined by linear time-varying evolution PSO[13] method. The optimal sets in (13) to (15) are obtained as follows: $(pw,\ pc,\ ps) = (1.3379,\ 0.0547,\ 1.0273)$, $(pw,\ pc,\ ps) = (4.4722,\ 0.0309,\ 3.2897)$, and $(pw,\ pc,\ ps) = (1.4503,\ 0.3081,\ 4.1117)$ for $x(t)$, $y(t)$, and $z(t)$, respectively. Meanwhile, the final values of *RMSE* with ARTVLA-RBFNNs are found to be 0.02313, 0.02733, and 0.05276 shown in Table 1. Figure 3 shows the values of *RMSE* for $x(t)$, $y(t)$, and $z(t)$ using the proposed algorithm with optimal learning rates. To show the feasibility of the ARTVLA-RBFNNs, the errors of training data after 200 epochs are illustrated in Fig. 4.

V. CONCLUSIONS

This paper presented PSO-based ARTVLA to train RBFNNs for parameter identification of Lorenz chaotic system. In the proposed ARTVLA-RBFNNs, time-varying learning rates and optimal parameters of RBFNNs are simultaneously determined by PSO method. Then the optimal RBFNNs are adopted to identify the chaotic system. From the simulation results, the effectiveness and the feasibility of the proposed ARTVLA-RBFNNs identifying parameter of Lorenz system has been verified. Meanwhile, the superiority of the proposed ARTVLA-RBFNNs with nonlinear learning rates over the ARLA-RBFNNs with fixed learning rates for parameter identification has been illustrated.

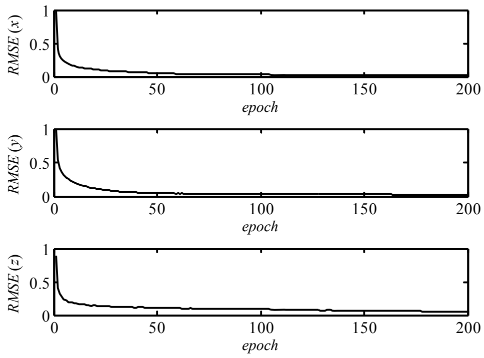

Fig. 3. The values of *RMSE* after 200 training epochs using ARTVLA-RBFNN with the optimal learning rates.

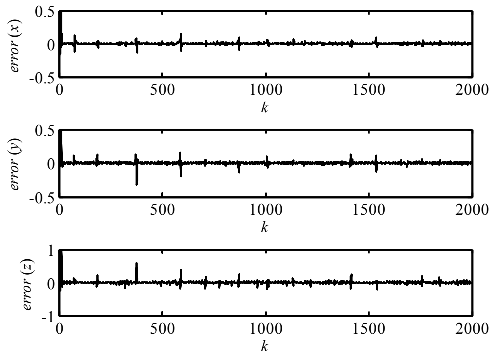

Fig. 4. The plots of $error = y(k) - \hat{y}(k)$ for 2000 training data after 200 training epoch.

Table 1. The values of *RMSE* (16) for Lorenz chaotic system after 200 training epochs, in which ARLA with various learning rates and ARTVLA with time-varying learning rates are applied to train the RBFNNs.

	ARTVLA	ARLA (learning rate η)								
		4.0	3.5	3.0	2.5	2.0	1.5	1.0	0.5	0.2
$x(t)$	**0.0231**	0.0657	0.0392	**0.0356**	0.0386	0.0419	0.0389	0.0492	0.0727	0.0925
$y(t)$	**0.0273**	0.0929	0.0927	0.0584	0.0750	0.0431	**0.0393**	0.0824	0.1077	0.1560
$z(t)$	**0.0528**	0.1024	0.1008	0.0945	0.0944	0.0880	**0.0623**	0.0675	0.0703	0.0976

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