A Novel Variable Structure Theory Applied in Design for Wheeled Mobile Robots

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Abstract: This paper is concerned with the control of wheeled mobile robots (WMRs) using a modified variable structure theory. Firstly, we introduce the dynamic characteristic of a WMR. Secondly, the conventional variable structure control is reviewed. To remarkably improve the transient response during the reaching phase, a modified variable structure control is proposed. The validity of the proposed variable structure theory is verified by means of a simulation testing on a homemade wheeled mobile vehicle. The simulation results validate the superiority and practicality of the modified variable structure for WMRs.

Keywords: Variable Structure theory, Sliding mode control, Wheeled mobile robot.

I. INTRODUCTION

Mobile robots have a wide background of application, and motion control of WMRs has found considerable attentions over the last decades. The path-tracking control problem of WMRs has received sustained attention [1][2][14]. However, the above researches are mainly based on kinematic models of nonholonomic mobile robots. Moreover, the velocity commands must be converted into the actual control input for vehicles. Hence, dynamic models of systems should be considered. Recently, several authors also consider the dynamic model of the WMR [7][13].

Variable structure control was initiated in Russia by many researchers, like Barbashin [3], Utkin [10], Emelyanov [8]. The control scheme has successfully been applied to many engineering problems including automatic flight control, chemical processes, helicopter stability augmentation systems, electric motors, robots, etc. Variable structure control law is deliberately changed according to some defined rules which depend on the state of the system. The scheme has been mainly considered for continuous-time systems in the form of sliding mode control.

Sliding mode control is known to be very robust against parameter variations and external disturbances and has been widely accepted as an efficient method for tracking control of uncertain nonlinear systems. It has been shown to be able to achieve 'perfect' performance in principle in the presence of parameter uncertainties, bounded external disturbance, etc [6]. However, in order to account for the presence of parameter uncertainties and bounded disturbances, a discontinuous switching function is inevitably incorporated into the control law to achieve so-called sliding condition [11]. Due to imperfect switching in practice it will raise the issue of chattering, which is usually undesirable. To suppress chattering, a continuous approximation of the discontinuous sliding control is usually employed in the literature. Though, the chattering can be made negligible if the width of the boundary layer is chosen large enough, the guaranteed tracking precision will deteriorate if the available control bandwidth is limited. To reach a better compromise between small chattering and good tracking precision in the presence of parameter uncertainties, various compensation strategies have been proposed. For example, integral sliding control [4], [5], [9], sliding control with time-varying boundary layers [5] etc., were presented. Alternatively, applying so-called reaching law approach, Gao et al. [12] proposed sliding controllers such that the trajectories are forced to approach the sliding surface faster when they are far away from the sliding surface. This approach seems to be an efficient method capable of increasing the approaching speed to the sliding surface; however, the behavior of the system dynamics, governed by the transformed first-order equation, can only be predicted through the measurement of the generalized error; hence the transient response during the reaching phase may not be remarkably improved.

In this paper, it is concerned with the control of a WMR using a modified variable structure theory in the boundary layer. Then we will review the conventional variable structure control scheme and present the modified variable structure control theory. The effectiveness of the newly developed control scheme will be demonstrated through the control of the WMR. We'll show that the transient response during the reaching phase has been remarkably improved by the proposed control.

II. HOME-MADE WMR

The home-made WMR is shown in Fig.1. It consists of a vehicle with two driving wheels mounted on the same axis and a free front wheel. The motion and orientation are achieved by independent actuators. Hence in the WMR model, we assume the coordinates of the mass center of the WMR is located in the middle of the hind driving wheel.

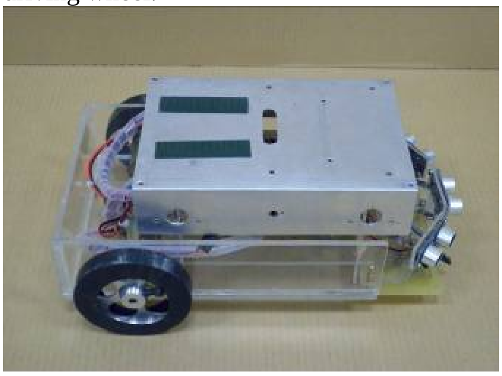

Fig.1. Home-made WMR

◎ Dynamic Equations of a WMR

The 2-D figure of a WMR is shown in Fig. 2. Fig. 3 shows 2-D of the car motion. The dynamic equations are described as follows,

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix}=\begin{bmatrix}\cos\theta & 0\\ \sin\theta & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v_l\\ v_r\end{bmatrix} \quad (1)$$

where v_l is the linear velocity and v_r is the angle velocity.

Let (x_r, y_r, θ_r) be the reference coordinate, $(x_e, y_e, \theta_e) = (x_r - x, y_r - y, \theta_r - \theta)$ be the coordinate errors and $\dot{y}\cos\theta - \dot{x}\sin\theta = 0$ be the non-holonomic constraint. By mathematical processing, we have the error equations

$$\begin{aligned}\dot{e}_1 &= -v_l + e_2 v_r + v_l^R \cos e_3\\ \dot{e}_2 &= -e_1 v_r + v_l^R \sin e_3\\ \dot{e}_3 &= -v_r + v_r^R\end{aligned} \quad (2)$$

where v_l^R is the reference linearly velocity and v_r^R is the reference angle velocity. If e_1 and e_3 converge to zero, the e_2 will also converge to zero. So we concern the stability of the error equations.

$$\begin{aligned}\dot{e}_1 &= v_l^R \cos e_3 + e_2 v_r - v_l\\ \dot{e}_3 &= v_r^R - v_r\end{aligned} \quad (3)$$

The object is to design v_l and v_r such that $e_1 = e_3 = 0$.

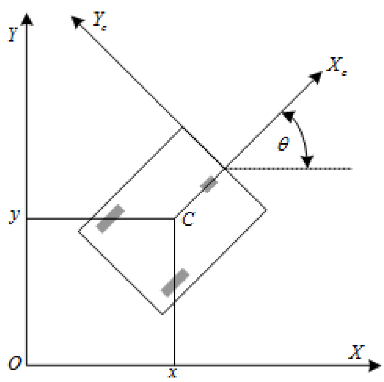

Fig.2. 2-D of a wheeled mobile

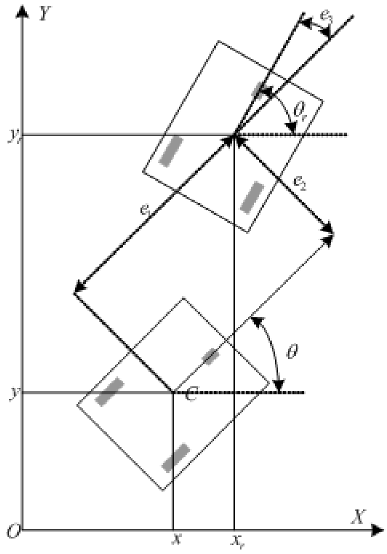

Fig.3. 2-D of the car motion

III. CONVENTIONAL VARIABLE STRUCTURE CONTROL

Consider the double integrator given by

$$\ddot{y}(t) = u(t) \quad (4)$$

Let the feedback control law be

$$u(t) = -ky(t) \quad (5)$$

where k is a strictly positive scalar. We have

$$\dot{y}\ddot{y} = -k\dot{y}y \quad (6)$$

Integrating this expression gives the following relationship between velocity and position

$$\dot{y}^2 + ky^2 = c \quad (7)$$

where c is a strictly positive and constant value.

Since y and $\dot{y}$ remain bounded for all time, the closed-loop systems are stable. For asymptotic stability, the control law of the form given in (5) is not appropriate. Consider instead the control law

$$u(t) = \begin{cases}-k_1 y(t), \text{ if } y\dot{y} < 0\\ -k_2 y(t), \text{ otherwise}\end{cases} \quad (8)$$

where $0 < k_1 < 1 < k_2$. Then the phase portrait must spiral in towards the origin and an asymptotically stable motion result. Next, consider a second-order system

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= 2x_2 - x_1 + u\end{aligned} \quad (9)$$

with

$$u = \begin{cases}-4x_1, \text{ if } s(x_1, x_2) > 0\\ 4x_1, \text{ if } s(x_1, x_2) < 0\end{cases} \quad (10)$$

where the switching function is defined by

$$\begin{aligned}s(x_1, x_2) &= x_1\sigma\\ \sigma &= 0.5x_1 + x_2\end{aligned} \quad (11)$$

The system structure varies along the switching lines: $x_1 = 0$ and $\sigma = 0$. Figure 4 shows the phase portrait of the subsystem under $u = -4x_1$. Similarly, Figure

5 shows the phase portrait of the subsystem under $u = 4x_1$. Evidently, both subsystems are unstable. However, the origin can be made asymptotically stable by the switching law, as shown in Fig. 6. Note that the phase portrait under variable structure control consists of a reaching mode during which trajectories starting off $s = 0$ move toward it and reach it in finite time, followed by a sliding mode ($\sigma = 0.5x_1 = \dot{x}_1 = 0$) during which the motion will be confined to $s = 0$. During the sliding mode, trajectory dynamics are of a lower order than the original mode. The sliding mode is a trajectory that is not inherent in either one of the two subsystems.

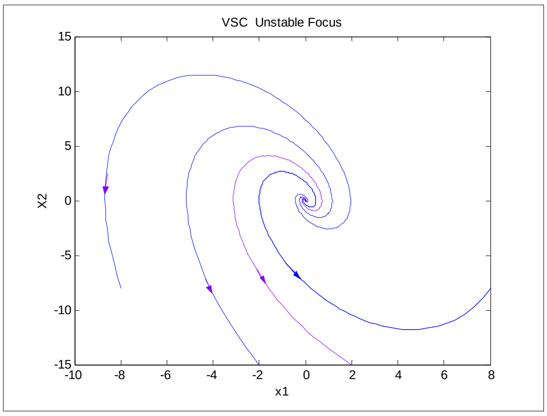

Fig.4. Phase portrait of an unstable focus

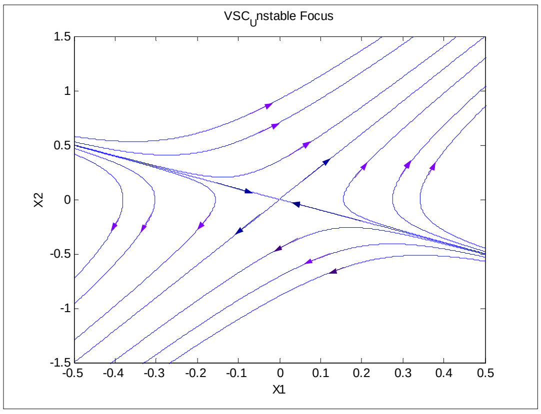

Fig.5. Phase portrait of an unstable saddle

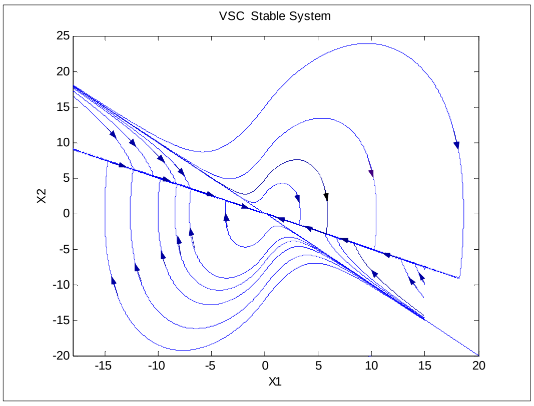

Fig.6. Phase portrait of the VSC stable system

IV. MODIFIED VARIABLE STRUCTURE CONTROL

Consider a simple n^{th}-order SISO nonlinear system

$$\overset{(n)}{y} = f(t, y, \ldots, \overset{(n-1)}{y}) + g(t, y, \ldots, \overset{(n-1)}{y})u \tag{12}$$

Assume that $f = \hat{f} + \Delta f$, where $\hat{f}$ is the known part and Δf is the uncertain part, which includes the internal noise bounded in in $|\Delta f| \le F$. The object is to find a sliding control u such that the output y of (12) will approximately track a desired signal, y_d which is assumed to be n^{th}-order continuously differentiable and all of its derivatives are uniformly bounded. Given the tracking error

$$e(t) = y(t) - y_d(t) \tag{13}$$

For any $q > 0$, define the following transformation

$$s(t) = \left(\frac{d}{dt} + q\right)^{n-1} e(t) \tag{14}$$

Then

$$\dot{s} = \hat{f} + \hat{w} + u \tag{15}$$

where

$$\hat{w} = \sum_{k=1}^{n-1} \binom{n-1}{k-1} e^{(k)} \lambda^{n-k} - y_d^{(n)} \tag{16}$$

An n^{th}-order tracking problem can be transformed into an equivalent 1st-order stabilization problem. It is easy to show that the control law

$$u = \frac{1}{g}\left(\hat{u} + u_f\right) \tag{17}$$

with

$$\begin{cases} \hat{u} = -\hat{f} - \hat{w} \\ u_f = -\Gamma sat\left(\dfrac{s}{\Phi}\right) \end{cases} \tag{18}$$

will result in a closed-loop system satisfying the reaching condition:

$$s\dot{s} \le -\eta |s|, \ \forall |s| \ge \Phi , \tag{19}$$

Provided $\Gamma \ge F + \eta$, for some $\eta > 0$.

This controller ensures that starting from any initial state the error trajectory will reach the boundary layer, $|s| \le \Phi$ in finite time.

V. SIMULATION RESULTS

To illustrate the performance of the above, the WMR is considered, given as the section II. According to the notations given above, we assume initial values are chosen as $(x_0, y_0, \theta_0) = (3,1,\pi)$ and expected values are $(x_r, y_r, \theta_r) = (0,0,\pi/4)$. By setting the parameters as $\Phi_y = 1$, $\lambda_y = 7$, $\Phi_a = 15$, and $\lambda_a = 25$.

Figure 7 and figure 8 indicate the tracking errors vi a CVSC and MVSC, respectively.

VI. CONCLUSION

We have presented a modified variable structure control scheme in this paper for a WMR. Not only the conventional variable structure control has exhibited good responses but also the proposed control law has shown to be capable of improving the transient response as well as the steady state response. Simulation results showed good responses to any initial conditions.

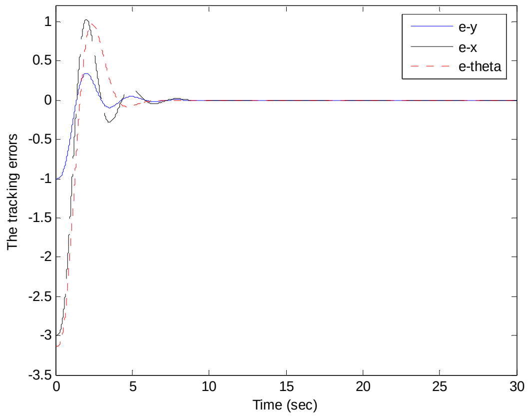

Fig.7. Tracking errors via CVSC

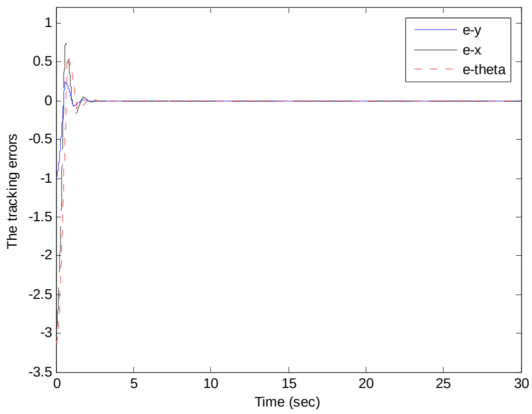

Fig.8. Tracking errors via MVSC

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