Regulated Plus and Minus Power Supply Using Approximate 2DOF Robust Digital Control

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Abstract— **A plus and minus switching power supplies are needed for many applications, for example audio power supply. Since the output voltage changes of such power amplifiers which are loads are large, the power supply voltagse are changed largely. Usually, in order to suppress the change, a capacitor with large capacity is used at the output end. If the capacities are made small, the power supplies can be compacted. In this paper, it is shown that the capacities can be made small using a robust digital control using an approximate 2DOF. The derived controller is actually implemented on a DSP. It is demonstrated from experiments that the power supplies can be compacted by the robust controller.**

I. Introduction

The DC-DC converter is used for a switching power supply of an plus and minus power amplifier for many applications, for example an audio amplifier. This power supply is needed to be compacted. In usual power supply, in order to suppress an output voltage change for load change and input voltage change, a capacitor with large capacity is used for the output end. It can be considered that that the capacity is made small as one means of a compact the audio power amplifier. The authors proposed the method of designing a approximate 2-degree-of-freedom (2DOF) controller of DC-DC converter[1] as a robust digital controller. The digital controller makes the control bandwidth wider, and at the same time makes the change of the output voltage very small at sudden changes of load and the input voltage. This robust digital controller was applied to control one side power supply[2]. In this paper, it is shown that the output voltage change of the both sides plus and minus power supply can fully be suppressed and the power supply can be compacted by the capacitor with small capacity using this type robust digital control. The robust digital controller is actually implemented on a DSP. It is demonstrated from experiments that the capacity can be made small by the robust controller.

II. Plus and Minus power supply

The configuration of the audio power supply is shown in Fig. 1. The part except for the controller is a controlled object. The triangular wave carrier is adopted for the PWM switching signal. The switching frequency is set at 100[KHz] and E is 14.5[V], $n_1 : n_2 = 4 : 13$. The LC circuit is a filter for removing carrier and switching noises. C_0 is 1000[μF] and L_0 is 70[μH]. In Fig.1, a plus side and a minus side are symmetrical, and the plus side model is shown in Fig. 2. The state equation of Fig. 2 is derived. If the frequency of control signal u is smaller enough than that of the carrier, the state equation of the DC-DC converter at a resistive load in Fig.1 except for the controller in DSP can be expressed from the state equalizing method as follows :

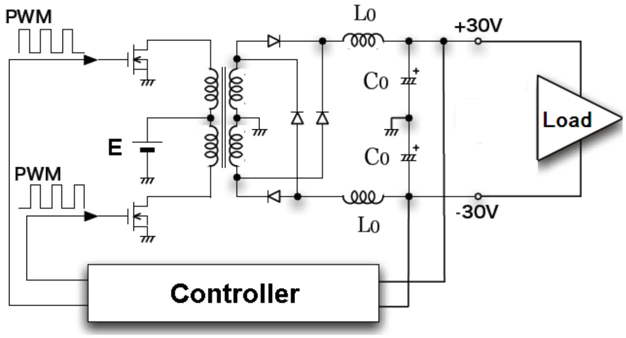

Fig. 1. Plus and Minus power supply

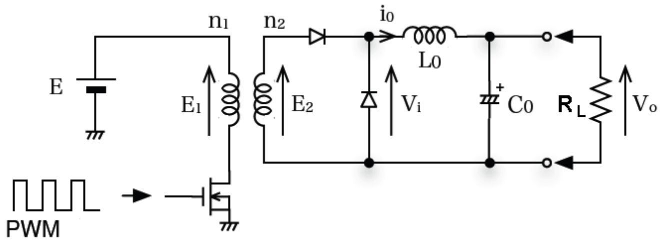

Fig. 2. Plus side model of power supply

$$\begin{cases} \dot{x} = A_c x + B_c u \\ y = Cx \end{cases} \quad (1)$$

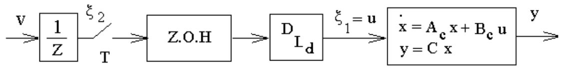

Fig. 3. Controlled object with input dead time $L_d (\leq T)$

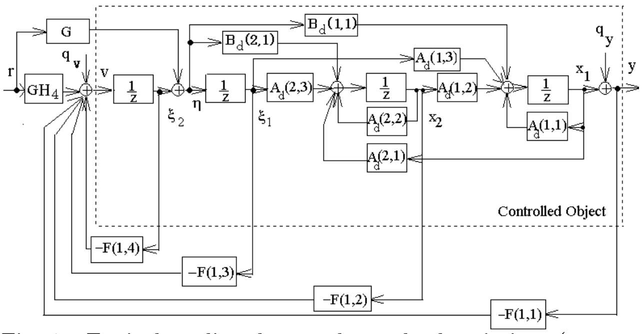

Fig. 4. Equivalent disturbances due to load variations (parameter variations) and model matching system with state feedback

where

$$
\begin{aligned}
x &= \begin{bmatrix} e_o \\ i \end{bmatrix} \quad A_c = \begin{bmatrix} -\frac{1}{C_0 R_L} & \frac{1}{C_0} \\ -\frac{1}{L_0} & -\frac{R_0}{L_0} \end{bmatrix} \quad B_c = \begin{bmatrix} 0 \\ \frac{K_p}{L_0} \end{bmatrix} \\
C &= \begin{bmatrix} 1 & 0 \end{bmatrix} \quad u = e_i \quad y = e_0 \quad K_p = \frac{V_i N_2}{C_m N_1}
\end{aligned}
$$

R_0 is the total resistance of coil and ON resistance of FET, etc., and the value is $0.015[\Omega]$. When realizing a digital controller by a DSP, a delay time exists between the starting time of the sampling operation and the outputting time of the control signal due to the calculation and AD/DA conversion times. This delay time is considered to be equivalent to the input dead time which exists in the controlled object as shown in Fig. 3. Then the state equation of the system in Fig.3 is expressed as follows :

$$
\left\{ \begin{array}{l} x_{dw}(k+1) = A_{dw} x_{dw}(k) + B_{dw} v(k) \\ y(k) = C_{dw} x_{dw}(k) \end{array} \right. \tag{2}
$$

where

$$
\begin{aligned}
x_{dw}(k) &= \begin{bmatrix} x_d(k) \\ \xi_2(k) \end{bmatrix} \quad x_d(k) = \begin{bmatrix} x(k) \\ \xi_1(k) \end{bmatrix} \\
A_{dw} &= \begin{bmatrix} A_d & B_d \\ 0 & 0 \end{bmatrix} \quad B_{dw}(k) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
A_d &= \begin{bmatrix} e^{A_c T} & e^{A_c (T-L_d)} \int_0^{L_d} e^{A_c \tau} B_c d\tau \\ 0 & 0 \end{bmatrix} \\
B_d &= \begin{bmatrix} \int_0^{T-L_d} e^{A_c \tau} B_c d\tau \\ 1 \end{bmatrix} \\
C_{dw} &= \begin{bmatrix} C_d & 0 \end{bmatrix} \quad C_d = \begin{bmatrix} C & 0 \end{bmatrix} \quad \xi_1(k) = u(k)
\end{aligned}
$$

It can be considered that the input voltage E and the load change by change of the amplitude of a voice signal etc. is parameter changes for the controlled object.

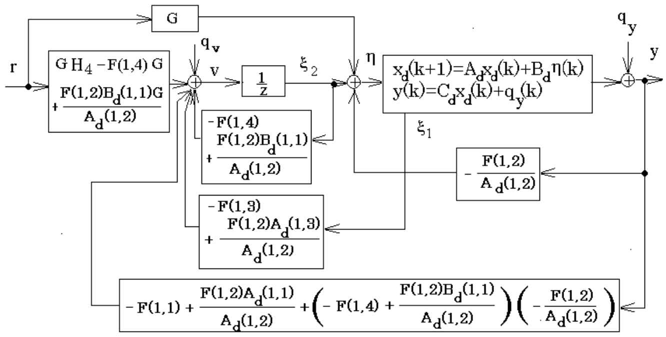

Fig. 5. Model matching system using only voltage (output) feedback

These parameter changes can be considered to be equivalent disturbances q_u and q_y inputted into the controlled object from the exterior as shown in Fig. 4. If the transfer functions between the equivalent disturbances and the output voltage can be made small, it will become the control system hardly influenced of the input voltage change and the load changes. Therefore, if the controller is designed so that this transfer function may become small, the compact audio power supply will be attained even if the capacity C_0 is small.

III. Robust digital controller

A. *Configuration of control system*

First, the transfer function between the reference input r and the output y is specified as follows :
$W_{ry}(z)$

$$
= \frac{(1+H_1)(1+H_2)(1+H_3)(z-n_1)(z-n_2)(z+H_4)}{(1-n_1)(1-n_2)(z+H_1)(z+H_2)(z+H_3)(z+H_4)} \tag{3}
$$

where, n_1 and n_2 are the zeros for the discrete-time control object (2). It shall be specified that the relation of H_1 and H_2, H_3 becomes $|H_1| \gg |Re(H_2)|$, $|H_1| \gg |Re(H_3)|$. Then $W_{ry}(z)$ can be approximated to the following first-order model:

$$
W_{ry}(z) \approx W_m(z) = \frac{1+H_1}{z+H_1} \tag{4}
$$

This target characteristics $W_{ry}(z) \approx W_m(z)$ is specified so that the bandwidth of the audio power supply becomes as wide as possible.

Applying a state feedback

$$
\begin{aligned}
v &= -Fx^* + GH_4 r \\
x^* &= [y \;\; x_2 \;\; \xi_1 \;\; \xi_2]^T
\end{aligned} \tag{5}
$$

and feedforward

$$
\xi_1(k+1) = Gr \tag{6}
$$

to the discrete-time controlled object as shown in Fig. 4, we decide $F = [F(1,1) \; F(1,2) \; F(1,3) \; F(1,4)]$ and G so

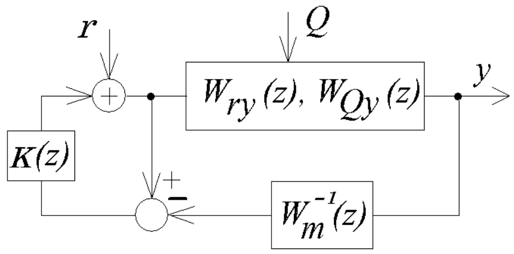

Fig. 6. System reconstituted with inverse system and filter

that $W_{ry}(z)$ becomes eq.(3). Th current feedback is used in Fig. 4. This is transformed to voltage and control input feedbacks without changing the pulse transfer function between $r-y$ by an equivalent conversion. The following relation is obtained from Fig. 4:

$$\begin{aligned} -F(1,2)x_2(k) &= -\frac{F(1,2)}{A_d(1,2)}(x_1(k+1) \\ -A_d(1,1)x_1(k) &- A_d(1,3)\xi_1 - B_d(1,1)\eta) \end{aligned} \quad (7)$$

If the current feedback is transformed equivalently using the right-hand side of this equation, the control system with only voltage feedback as shown in Fig. 5 will be obtained. The transfer function $W_{Qy}(z)$ between this equivalent disturbance $Q=[q_v \quad q_y]^T$ and y of the system in Fig. 5 is desfined as

$$W_{Qy}(z) = \left[\; W_{q_v y}(z) \quad W_{q_y y}(z) \;\right] \quad (8)$$

The system added the inverse system and the filter to the system of Fig.5 is constituted as shown in Fig. 6. In Fig. 6, the transfer function $K(z)$ becomes

$$K(z) = \frac{k_z}{z-1+k_z} \quad (9)$$

The transfer functions between $r-y$ and $Q-y$ of the system in Fig. 6 are given by

$$y = \frac{1+H_1}{z+H_1}\frac{z-1+k_z}{z-1+k_zW_s(z)}W_s(z)r \quad (10)$$

$$y = \frac{z-1}{z-1+k_z}\frac{z-1+k_z}{z-1+k_zW_s(z)}W_{Qy}(z)Q \quad (11)$$

where

$$W_s(z) = \frac{(1-H_2)(1+H_3)(z-n_1)(z-n_2)}{(z-H_2)(z+H_3)(1-n_1)(1-n_2)} \quad (12)$$

Here, if $W_s(z) \approx 1$, then Eqs.(9) and (10) become, respectively,

$$y \approx \frac{1+H_1}{z+H_1}r \quad (13)$$

$$y \approx \frac{z-1}{z-1+k_z}W_{Qy}(z)Q \quad (14)$$

From eqs.(12) and (13), it turns out that the characteristics from r to y can be specified with H_1, and the characteristics from Q to y can be independently specified with k_z. That is, the system in Fig. 6 is an approximate 2DOF, and its sensitivity against disturbance becomes lower with the increase of k_z.

If an equivalent conversion of the controller in Fig. 6 is carried out, the approximate 2DOF digital integral-type control systems will be obtained as shown in **Fig.6**. In Fig. 7, the parameters of the controller are as follows :

$$\begin{aligned} k_1 &= F(1,1+F(1,2)FF(1,1)+((-F(1,4) \\ &- F(1,2)FF(1,4))(-F(1,2)/FF(1,2))) \\ &+ (GH_4+GF_z)(k_z/(1+H_2)) \\ k_2 &= F(1,2)/FF(1,2)+G(k_z/(1+H_2)) \\ k_3 &= F(1,3)+F(1,2)(FF(1,3)) \quad k_4 = -F_z \\ k_{i1} &= Gk_z \quad k_{i2} = (GH_4+GFz)k_z \\ k_{r1} &= G \quad k_{r2} = GH_4+GF_z \quad (15) \\ where & \\ FF(1,1) &= -A_d(1,1)/A_d(1,2) \\ FF(1,2) &= A_d(1,2) \\ FF(1,3) &= -A_d(1,3)/A_d(1,2) \\ FF(1,4) &= -B_d(1,1)/A_d(1,2) \\ F_z &= -F(1,4)-F(1,2)FF(1,4) \end{aligned}$$

In the audio amplifier power supply, since a plus side and a minus side is symmetrical, the controller which controls only the plus side is designed. A model for the controlled object of the plus side becomes as shown in Fig. 2. The robust control system to this controlled object becomes as shown in Fig. 7. This system is an approximate 2DOF system and can specify independently the characteristics between the reference value and the controlled output, and the characteristics between the disturbance and the controlled output. If the parameter of the controller is set up so that the transfer function between the disturbance and the controlled output may become small, the control system will become low sensitivity very much. Therefore, even if the capacity is small, the large load changes can be suppressed. The same controller of Fig. 7 is applied to the minus side of Fig. 1. The averaged control input is switched to the plus side and minus side.

IV. Experimental studies

The sampling period T are set at $10[\mu s]$ and the input dead time L_d is about $0.999T[\mu s]$. The nominal value of R_L is $4[\Omega]$. We design a control system so that all the specifications are satisfied. First of all, in order to satisfy the specification on the rising time of startup transient reponse, H_1, H_2, H_3 and H_4 are specified as

$$H_14 = -0.9 \quad H_2 = 0.1+o.1i \quad H_3 = 0.1-0.1i$$

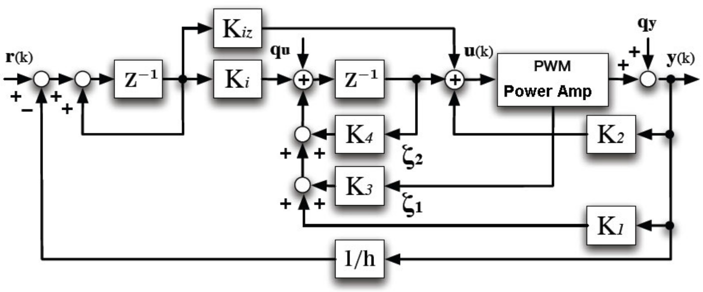

Fig. 7. Approximate 2DOF robust digital control system

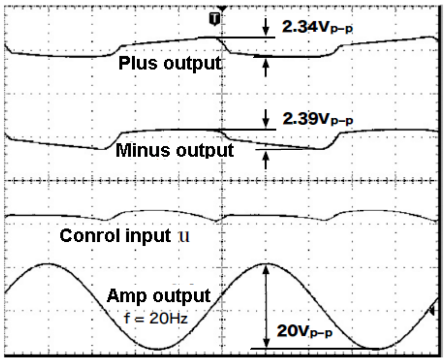

Fig. 8. Experimental results by the approximate 2DOF robust digital control at $C_0 = 1000[\mu\text{F}]$

$$H_4 \quad = \quad -0.999 \quad k_z = 0.15 \tag{16}$$

Then the parameters of controller become

$$\begin{aligned} k_1 &= 14.063 \quad k_2 = -11.605 \quad k_3 = -0.022209 \\ k_4 &= -0.20448 \; k_i = -1.3752 \; k_{iz} = 1.1427 \end{aligned} \tag{17}$$

We used the DSP(TI TMS320LF2808). Fig. 8 and Fig. 9 show the output voltage changes by the approximate 2DOF robust control at $C_0 = 1000[\mu]$ and $C_0 = 220[\mu]$, respectively. From these results, it turns out that output voltage change hardly changes even if capacity becomes very small. Fig. 10 and Fig. 11 show the output voltage changes by PI control at $C_0 = 1000[\mu]$ and $C_0 = 1000[\mu]$, respectively. It turns out that the output voltage changes are very large compared with the robust control. Moreover, the output voltage changes are oscillating at $C_0 = 220[\mu]$. These results show that the robust control by approximate 2DOF is effective in compact audio power supply.

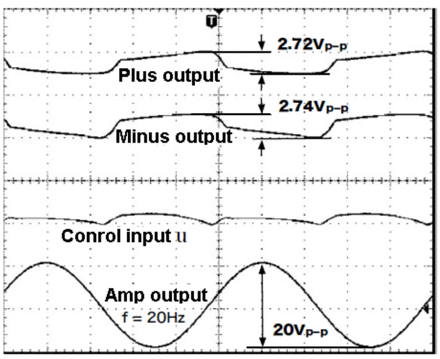

Fig. 9. Experimental results by the approximate 2DOF robust digital control at $C_0 = 220[\mu\text{F}]$

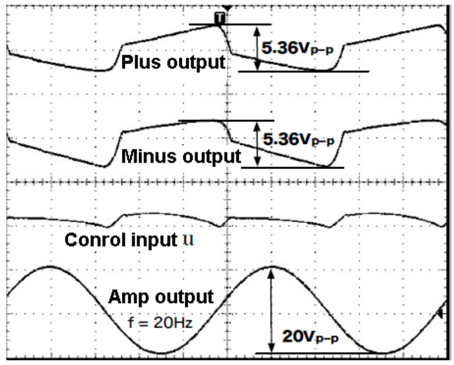

Fig. 10. Experimental results by PI control at $C_0 = 1000[\mu\text{F}]$

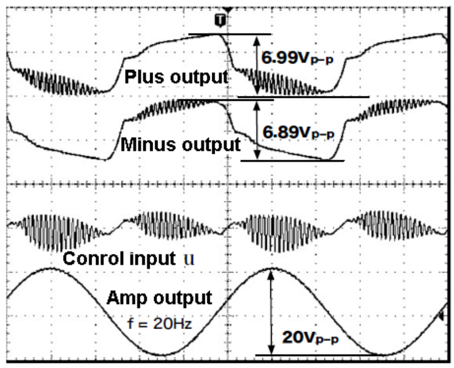

Fig. 11. Experimental results by PI control at $C_0 = 220[\mu\text{F}]$

V. Conclusion

In this paper, the robust digital control was performed by DSP. Consequently, even if it made capacity small, it was shown that output voltage changes of the both sides of the plus and minus power supply can be suppressed almost like the time when capacities are large. A future subject is designing the robust digital controller which can make the value of output voltage change smaller.

References

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