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Evaluation of a competitive particle swarm optimizer in multimodal functions with complexity

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Abstract: In this paper, we present a simple competitive particle swarm optimizer (CPSO) for finding plural solutions. In the CPSO, particles are divided into groups corresponding to the required number of solutions. Each group simultaneously searches solutions having a priority search region. This region affects to prohibit that different groups search the same solutions. The CPSO can effectively find desired plural acceptable solutions with a high accuracy and with a low computation cost, and can easily control combinations of these solutions by adjusting a parameter. This paper evaluates the CPSO in complex global optimization benchmarks. Through the numerical experiments, searching performances of the CPSO are clarified.

Keywords: Particle Swarm Optimization, Optimization Problems, Plural Solutions.

I. INTRODUCTION

Particle Swarm Optimizers (PSOs) are known as a kind of metaheuristic algorithms [1]-[4]. Swarms such as birds and fishes decide actions to consider not only status information of each individual but also status information as whole of their swarms. The PSO expresses such actions by simple arithmetic operations. In the PSO, particles search solutions in an objective problem. Each particle has velocity and position information, and moves in a multidimensional search space considering a personal best solution which each particle memorizes and a global best solution which all particles share. The PSO can fast solve various optimization problems with a low computation cost.

On the other hand, in the actual engineering optimization problems, there exist a lot of design variables and constrained conditions to be considered. Then, the exact modeling for these problems can be hard. Also, solutions obtained from approximated models are not always available in the actual problems. Therefore, it is needed that plural acceptable solutions as design candidates can be provided in reasonable computation time. Many methods along this line have been proposed [5]-[9]. These methods can sequentially find plural acceptable solutions by behaving like general tabu search. However, by the effect of competitive search, the quality of each solution obtained from these methods is often lower than that obtained from the original PSO. In addition, these methods have many parameters; it is hard to control them.

In our previous works, a simple Competitive PSO (CPSO) for finding plural solutions has been proposed [10]. In the CPSO, particles are divided into groups corresponding to the required number of solutions. Each group simultaneously searches solutions having a priority search region. This region affects to prohibit that different groups search the same solution. The CPSO can effectively find desired plural acceptable solutions and can easily control combinations of these solutions by adjusting a parameter. Also, quality of some solutions obtained from the CPSO are almost the same as or better than that obtained from the original PSO. This means that the competitive search in the CPSO does not suppress the searching performances of the original PSO, and realize to effectively search plural acceptable solutions. In addition, the CPSO can fast find plural solutions without repeating many trials.

The CPSO has been evaluated for basic global optimization benchmarks and has been applied to a problem in sensor networks [10]. However, in practical problems such as the sensor networks, objective functions can be complex shapes and can include complex dependencies between design variables. The detailed evaluations of the CPSO in such functions have not been sufficient so far. This paper evaluates the CPSO in complex global optimization benchmarks. Through the numerical experiments, searching performances of the CPSO are clarified.

II. PARTICLE SWARM OPTIMIZERS

Swarms such as birds and fishes decide actions depending on not only status information of each individual but also status information as whole of their swarms. PSO is an optimization method that imitates behavior of the swarms. In PSO, particles efficiently search solutions in a target problem, by updating their positions and velocities based on personal best solutions which each particle has and a global best solution which all the particles have. Basic algorithm of PSO is described as follows (see Fig.1).

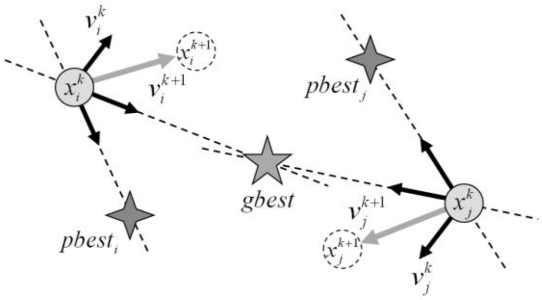

Fig.1. Movements of particles

(step1) Set positions and velocities of each particle at random.
(step2) Update the positions of each particle by Equation (1).

$$x_i^{k+1} = x_i^k + v_i^k \tag{1}$$

where x_i^k and v_i^k are position and velocity of the i-th particle at the k-th iteration, respectively.
(step3) Calculate evaluation values of each particle and update each personal best solution ($pbest_i$).
(step4) Update global best solution ($gbest$).
(step5) Update the velocities of each particle by Equation (2).

$$\begin{aligned} v_i^{k+1} = w \cdot v_i^k + c_1 \cdot r_1 \cdot (pbest_i - x_i^k) \\ + c_2 \cdot r_2 \cdot (gbest - x_i^k) \end{aligned} \tag{2}$$

where w is an inertia coefficient for the previous velocity vector. c_1 is a weight coefficient for the personal best position vector. c_2 is a weight coefficient for the global best position vector. r_1 and r_2 are uniform random numbers from 0 to 1.

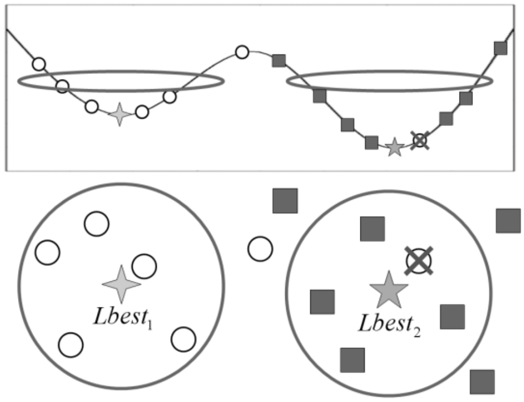

Fig.2. Groups of particles with priority search region s

(step6) Repeat from step2 to step5 until the number of iterations or an evaluation value of a solution reaches a predetermined value.

III. COMPETITIVE PSO

In general, it will be difficult to model design variables and constraint conditions exactly when optimization algorithms are applied to real problems. Then, it is more practical to obtain plural acceptable solutions as design candidates and to select the best solution from them rather than to obtain a single exact optimum solution. This paper presents a competitive PSO (CPSO) that can efficiently find the plural different acceptable solutions by dividing particles into plural groups. In the original PSO, it is difficult to find plural solutions because all the particles converge to a single solution by moving toward a global best solution. In the CPSO, it is considered that particles are divided into arbitrary m groups. In addition, these groups have own local best solution instead of global best solution as shown in Fig.2. As a result, plural solutions can be found because particles move toward each own local best solution. But, only dividing into plural groups, they may converge to the same solution. Therefore, priority search regions from each local best solution are introduced. In the region, the particles of a group can search more preferential than other groups. If a particle moves into the regions of the other groups, the CPSO excepts for the particle from the candidate in updating own local best solution. Then, it is possible to search the plural different solutions efficiently because each group does not approach to the regions of the other groups to each other. The algorithm of the CPSO is described as follows.

(step1) Set positions and velocities of each particle at random and set the parameter r corresponding to priority search region.

(step2) Divide p particles into arbitrary m groups.

(step3) Update positions of all the particles regardless of groups by Equation (1).

(step4) Calculate evaluation values of each particle and update each personal best solution ($pbest_i$).

(step5) Let $lbest_j$ be the local best solution which particles in the j-th group have. Calculate Euclidean distances between positions of each particle in the j-th group and positions of $lbest_{j'}$ in the j'-*th* group ($j' \neq j$).

(1)Euclidean distance is shorter than the range r:
The particle is excepted from a candidate of $lbest_j$.

(2)Euclidean distance is longer than the range r:
The particle is included as a candidate of $lbest_j$ in the same way as $gbest$ in the original PSO.

(step6) Update each $lbest_j$ that are chosen from the particles of each group.

(step7) If each position of $lbest_j$ and $lbest_{j'}$ ($j' \neq j$) overlaps to each other for the priority search regions, values of $lbest_j$ and $lbest_{j'}$ are compared. If $lbest_j$ is better, the positions of the particles in the j-th group are left. Otherwise, their positions and $lbest_j$ are reset at random.

(step8) Update the velocities of each particle by Equation (3).

$$v_i^{k+1} = w \cdot v_i^k + c_1 \cdot r_1 \cdot (pbest_i - x_i^k) + c_2 \cdot r_2 \cdot (lbest_j - x_i^k) \quad (3)$$

(step9) Repeat from step3 to step8 until the number of iterations or an evaluation value of a solution reaches a predetermined value.

When a group always overlaps priority search regions of other groups, this group can obtain no solution because local best solution of the group is reset every time.

IV. SIMULATION RESULTS

The CPSO is applied to some benchmark problems and the performances are confirmed. In all the experiments, some parameter values are fixed as follows: $p = 500$, $w = 0.9$, $c_1 = c_2 = 1.0$, $m = 5$. Varying the parameter of range r and the number of dimensions n, typical results are presented.

The CPSO is applied to Modified Rastrigin function defined by Equation (4).

$$F(x) = \sum_{i=1}^{n} (z_i^2 - 10\cos(2\pi z_i) + 10) \quad (4)$$

$$z_i = R(\alpha)(x_i - x_i^*)$$
$$x_i^* = U(-4.0, 4.0),\ \alpha = \pi/4,\ -5.0 < x_i < 5.0$$

x^* is a random optimal solution and $U(a,b)$ denotes uniform random numbers from a to b. And rotation matrix R is given by Equations (5a) and (5b).

$$R = T^{12} \times T^{13} \times \cdots \times T^{1n} \times T^{23} \times \cdots \times T^{(n-1)n} \quad (5a)$$

$$T_{kl}^{ij} = \begin{cases} \cos\alpha & k = i, l = i \\ -\sin\alpha & k = i, l = j \\ \sin\alpha & k = j, l = i \\ \cos\alpha & k = j, l = j \\ 1 & k = l \neq i, j \\ 0 & \text{not above} \end{cases} \quad (5b)$$

This function has a complex shape. This function gives minimum value 0 when design variables are x^*. It has many suboptimum solutions in arrangement like a lattice around the optimum solution.

First, we show the simulation results for $n = 2$. As $r = 0.8$, the CPSO can find the optimum and better suboptimum solutions as shown in Fig.3. Therefore, when the range is set appropriately, it is possible to find desired plural acceptable solutions. As $r = 1.3$, the CPSO can find optimum and better suboptimum solutions as shown in Fig.4. In addition, the discovered suboptimum solutions are more distant from the optimum solution than those obtained in $r = 0.8$. This reason is that the suboptimum solutions obtained in $r = 0.8$ are contained in priority search region of the group at the optimum solution as $r = 1.3$. As a result, the CPSO can accurately search for better solutions outside the range r. This means that the CPSO can easily control the distance of each solution by adjusting the range r.

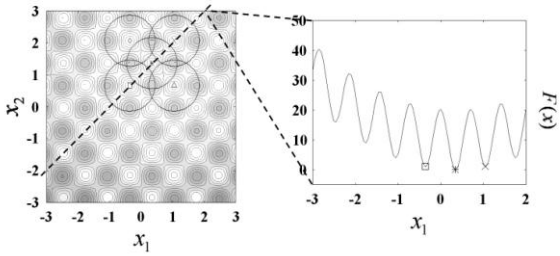

Fig.3. Simulation results ($n = 2,\ r = 0.8$)

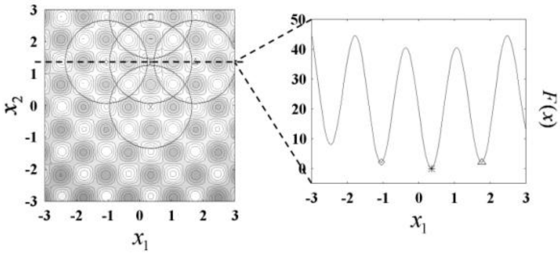

Fig.4. Simulation results ($n = 2,\ r = 1.3$)

Next, simulation results for higher dimensional Modified Rastrigin function ($n = 20$) are shown. The trials are repeated 100 times and the average values are presented. Table 1 shows the results for $r = 0.8$. Although the optimum solution cannot be found, plural acceptable suboptimum solutions can be found as shown in Table 1. Therefore, the CPSO can obtain plural solutions easily also in high dimensional and complex problems. In addition, quality of some solutions obtained from the CPSO are almost the same as or better than that obtained from the original PSO.

V. CONCLUSION

This paper has been evaluated a simple competitive PSO for multimodal functions with complexity. Adjusting a parameter of a priority search range, desired plural acceptable solutions can be effectively found and can be easily controlled. In the practical engineering optimization problems, not only a single exact optimum solution for the problems but also plural acceptable solutions for them are often required. The CPSO will be a simple and powerful tool to effectively solve these problems. Future problems include (1) adaptive control of parameters in the CPSO, (2) investigations of performances for various higher-dimension problems, and (3) applications to various engineering optimization problems.

Table 1. Evaluation values ($n = 20,\ r = 0.8$)

Group	F_2		
	average	best	worst
1	23.10	9.95	38.80
2	29.42	17.91	50.74
3	34.96	22.88	60.69
4	40.49	26.86	61.69
5	51.63	28.85	95.52
Original PSO	37.08	13.93	71.58

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