Linear Estimation Method for Position and Heading with RDOA Measurements

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Abstract: Passive localization in the sensor network has been studied in many areas. Especially, the estimation of the position and heading for the target is an important subject in the navigation problems. We estimate the position and the heading information only with range difference of arrival measurements. The proposed algorithms are based on the pseudo linear measurement equation transformed from the nonlinear one and uses the instrumental variable method to remove estimation errors. It does not need additional computational burden so that it will be advantageous to real time applications. To show the usefulness of the proposition, we simulate it in the various positions and headings comparing with a nominal least squares method and the robust least squares method.

Keywords: RDOA, Localization, Heading estimation, Least squares, Instrumental Variable.

I. INTRODUCTION

Passive localization in sensor network has been studied in many areas such as control, communication, signal processing and etc. For the localization, the utilized measurements of sensor network are such as TOA (time of arrival), TDOA (time difference of arrival), RDOA (range difference of arrival), or RSS (received signal strength). TDOA or RDOA-based localization methodologies have been applied for finding target's position because it does not need synchronization between the target and the network. Additionally, if one can get two different positions concerned on-board of the target, heading estimation is possible. This method has been used in a navigation problem [1]. In this paper, we are focusing on the heading and position estimation with RDOA measurements.

In a plane, the localization problem with RDOA measurements is regarded as an MLE (maximum likelihood estimation) problem which decides a crossing point between two parabolic functions [2]. However, there is always a possibility that the MLE method is not converged to the global minimum but converged to a local minimum depending on the initial point. Additionally, its solution is often derived with numerical analysis methods which can be a burden for computation.

To convert nonlinear estimation problem to linear one, an intermediate variable method has been proposed with adding a new state variable [3]. This transformed linear equation is called as a pseudo linear equation. With this equation, it can be relaxed for the initial guess problem. However, there are still two problems for the linear equation. The one is that there is an uncertainty in the measurement matrix which causes an estimation error by correlation with itself or with measurement noise [5-7]. The other problem is a bias of the measurement noise. These problems are concerned with the RDOA measurement noise and they may be neglected under the assumption that the variance of the noise is small [2-5]. However, if the condition of SNR is not good, the estimation error can be increased rapidly.

The IV (instrument variable) method can be a proper solution, because the method uses an instrumental variable in the measurement matrix for fleeing from the correlation [8]. It does not require additional computational burden so that it will be advantageous to real time applications.

Therefore, we propose the position and heading estimators based on the IV method and assume the stochastic information is unknown. To treat the bias of measurement noise of the pseudo linear equation, we estimate the bias by augmenting to the state variables based on the bias common model [9]. The useful aspects of the proposition are shown by simulation in the various positions and headings and by comparison with a nominal least squares method and the robust least squares method in [6] which can be adapted to this linear uncertain problem.

II. POSITION AND HEADING ESTIMATION MODEL WITH RDOA MEASUREMENTS

1. *Position Estimation Model*

We assume the target has two transmitters. The transmitters generate some signals which the sensor nodes of a network can realize the target. The network does not know the burst time of the signal but it can measure the arrival time. In this case, one can use a time difference to localize the target's position and the time difference can be transformed to the RDOA by multiplying a propagation velocity.

$$r_{1,2} = v_p(t_1 - t_2) \tag{1}$$

where v_p is the propagation velocity, t_1 and t_2 are arrival time at each sensor node, and $r_{1,2}$ is the RDOA.

To make this problem simple, we derive it in the 2-D case as Fig. 1.

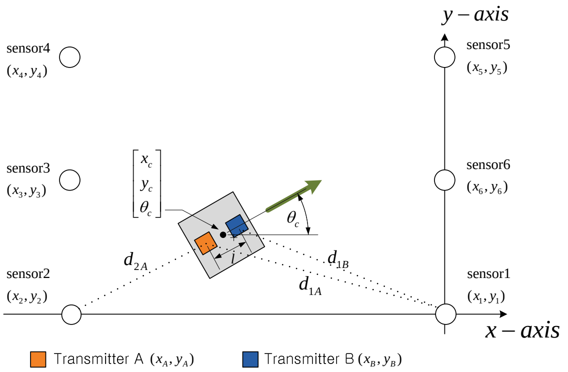

Fig. 1. Positions of the sensor nodes and the target in the 2-D plane

By using the intermediate variable method [4], the measurement model is given by

$$\underbrace{(\tilde{r}_{1,2}^A)^2 - d_{02}^2 + d_{01}^2}_{y_1^A} =$$

$$\underbrace{-2\begin{bmatrix} (x_2 - x_1) \\ (y_2 - y_1) \\ -\tilde{r}_{1,2}^A + \delta r_{1,2}^A \end{bmatrix}^T}_{H_1^A} \underbrace{\begin{bmatrix} x_A \\ y_A \\ d_{1A} \end{bmatrix}}_{X^A} + \underbrace{2\tilde{r}_{1,2}^A \delta r_{1,2}^A - (\delta r_{1,2}^A)^2}_{v_1^A} \tag{2}$$

where

$$\begin{aligned} d_{1A} &= \sqrt{(x_A - x_1)^2 + (y_A - y_1)^2}, \\ d_{01} &= \sqrt{(x_1 - 0)^2 + (y_1 - 0)^2} \end{aligned} \tag{3}$$

, and $\delta r_{1,2}^A$ is the RDOA measurement noise and its stochastic property is a white and zero mean. The measurement noise of (2), v_1^A can be assumed to be a white noise but it cannot be assumed zero mean.

$$E\left[v_1^A\right] = (\sigma^2)_1^A \tag{4}$$

where $(\sigma^2)_1^A$ is the variance of the RDOA measurement noise, $\delta r_{1,2}^A$. In a low SNR condition, this factor can cause an estimation error, so that it needs to be removed from the measurements y_1^A or to be estimated by setting it as a new state variable.

For the other transmitter, B, the measurement equation can be derived likewise.

2. *Position and Heading Estimation Model*

The intermediate variable method makes the nonlinear localization problem to the linear problem so that a linear estimator can be applied to find the transmitters' positions. If one can obtain two positions for the target, its center position and heading can be derived by using the positions. The center position is

$$x_c = \frac{x_A + x_B}{2}, \quad y_c = \frac{y_A + y_B}{2} \tag{5}$$

, and the heading is

$$\theta_c = \cos^{-1}\left(\frac{x_B - x_A}{l}\right) = \sin^{-1}\left(\frac{y_B - y_A}{l}\right) \tag{6}$$

By using this idea, we can build a linear model for the position and heading. Using (5) and (6), the linear equation is given by

$$\underbrace{\begin{bmatrix} y_1^B + y_1^A \\ y_1^B - y_1^A \end{bmatrix}}_{z_1} = -2 \underbrace{\begin{bmatrix} 2(x_2 - x_1) & 0 \\ 2(y_2 - y_1) & 0 \\ 0 & l(x_2 - x_1) \\ 0 & l(y_2 - y_1) \\ -\tilde{r}_{1,2}^A + \delta r_{1,2}^A & \tilde{r}_{1,2}^A - \delta r_{1,2}^A \\ -\tilde{r}_{1,2}^B + \delta r_{1,2}^B & \tilde{r}_{1,2}^B - \delta r_{1,2}^B \end{bmatrix}^T}_{\Pi_1} \underbrace{\begin{bmatrix} x_c \\ y_c \\ \cos(\theta_c) \\ \sin(\theta_c) \\ d_{1,A} \\ d_{1,B} \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} v_1^B + v_1^A \\ v_1^B - v_1^A \end{bmatrix}}_{n_1} \tag{7}$$

, and its generalized form when there are n+1 sensors is

$$\underbrace{\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}}_{z} = \left(\underbrace{\begin{bmatrix} \tilde{\Pi}_1 \\ \tilde{\Pi}_2 \\ \vdots \\ \tilde{\Pi}_n \end{bmatrix}}_{\tilde{\Pi}} - \underbrace{\begin{bmatrix} \Delta\Pi_1 \\ \Delta\Pi_2 \\ \vdots \\ \Delta\Pi_n \end{bmatrix}}_{\Delta\Pi} \right) X + \underbrace{\begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_n \end{bmatrix}}_{n} \tag{8}$$

where

$$\tilde{\Pi}_j = -2\begin{bmatrix} 2(x_j - x_1) & 0 \\ 2(y_j - y_1) & 0 \\ 0 & l(x_j - x_1) \\ 0 & l(y_j - y_1) \\ -\tilde{r}_{1,j}^A & \tilde{r}_{1,j}^A \\ -\tilde{r}_{1,j}^B & \tilde{r}_{1,j}^B \end{bmatrix}^T, \Delta\Pi_j = -2\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\delta r_{1,j}^A & \delta r_{1,j}^A \\ -\delta r_{1,j}^B & \delta r_{1,j}^B \end{bmatrix}^T \tag{9}$$

III. Linear Estimation Methods for Position and Heading

1. *Instrumental Variable Algorithm*

The stochastic compensation solution, the RoLS (robust least squares) in [6] is an useful method because its formulation resembles with the general least squares method and computational burden is low. However, if the sensor network has a characteristic of stochastic information being varying, the estimation results may be incorrect. To overcome this problem when the stochastic properties are unknown, the IV method can be an alternative solution. The IV method uses instrument variable for avoiding the correlations which are between the measurement matrix uncertainties,

$cor(\Delta\Pi, \Delta\Pi)$ or between the measurement matrix uncertainties and the measurement noise, $cor(\Delta\Pi, n)$ [8]. This method is unlike the RoLS algorithm which removes the correlations by using the scale-factor error compensation term, W and the biased error compensation term, V but flees from the correlations by replacing the measurement matrix with an IV matrix. The IV matrix can be built simply with one-step delayed measurement matrix because we assume the measurement noise is white.

$$\begin{aligned} &cor\left[\Delta\Pi(k-1), \Delta\Pi(k)\right] = 0_{2n\times 2n} \\ &cor\left[\Delta\Pi(k-1), n(k)\right] = 0_{2n\times 1} \end{aligned} \tag{10}$$

Therefore, the position and heading estimator with the IV algorithm is

$$\hat{X}^{IV}(k) = \left\{(\tilde{\Pi}^{k-1})^T \tilde{\Pi}^k\right\}^{-1} \left\{(\tilde{\Pi}^{k-1})^T z^k\right\} \tag{11}$$

However, an error still exists in the estimation result by the measurement noise bias in (4). Because we assumed the stochastic properties of the RDOA measurement are not available, the bias which is the variance of the RDOA noise cannot be removed from the measurements. If the sensor network is able to assume that the RDOA noise variances are identical for all measurements as the bias common model in [9], it is possible to estimate the common bias. To estimate the bias, we set the bias as a new state variable and derive the measurement equation as follows:

$$z(k) = \left(\underbrace{\left[\tilde{\Pi}(k) \quad I\right]}_{\tilde{\Pi}_{Aug}(k)} - \left[\Delta\Pi(k) \quad 0\right]\right) \underbrace{\begin{bmatrix} X(k) \\ b(k) \end{bmatrix}}_{X_{Aug}(k)} + \bar{n}(k) \tag{12}$$

where I is a 2(n-1) dimensional vector which is

$$I_{2(n-1)\times 1} = \left[\left[1 \quad 0\right] \quad \left[1 \quad 0\right] \quad \cdots \quad \left[1 \quad 0\right]\right]^T \tag{13}$$

and $\bar{n}(k)$ is the zero mean and white noise which eliminates the bias from n in (8). With (12), the augmented position and heading estimator is

$$\hat{X}^{IV}_{Aug}(k) = \left\{(\tilde{\Pi}^{k-1}_{Aug})^T \tilde{\Pi}^k_{Aug}\right\}^{-1} \left\{(\tilde{\Pi}^{k-1}_{Aug})^T z^k\right\} \tag{14}$$

Theorem 1. (Convergence to true position and heading of the proposed algorithm) If there are enough sensors to observe the position, heading and the biases, and the noises of sensor network are i.i.d. (independent and identically distributed), then the estimation results of the proposed method converge to true position and heading in probability.

$$\operatorname*{plim}_{k\to\infty} \hat{X}^{IV}_{Aug}(k) = X_{Aug} \tag{15}$$

□

Proof:

Since the measurement noises are i.i.d., the auto-correlation of the measurement matrix uncertainties and the cross-correlation between the measurement matrix uncertainties and the measurement are derived as follows:

$$E\left\{\tilde{\Pi}_{Aug}(k-1)\right\}^T \left\{\tilde{\Pi}_{Aug}(k)\right\} = \Pi^T_{Aug}\Pi_{Aug} \tag{16}$$

$$E\left\{\tilde{\Pi}_{Aug}(k-1)\right\}^T \left\{z(k)\right\} = \Pi^T_{Aug}\Pi_{Aug} X \tag{17}$$

Therefore, each correlation can be rewritten as convergence in probability.

$$\operatorname*{plim}_{k\to\infty} \frac{1}{k} (\tilde{\Pi}^{k-1}_{Aug})^T (\tilde{\Pi}^k_{Aug}) = \Pi^T_{Aug}\Pi_{Aug} \tag{18}$$

$$\operatorname*{plim}_{k\to\infty} \frac{1}{k} (\tilde{\Pi}^{k-1}_{Aug})^T (z^k) = \Pi^T_{Aug}\Pi_{Aug} X_{Aug} \tag{19}$$

With these convergences, (18) and (19), the estimation result, (14) is also able to converge to true position and heading by Slutsky's theorem [8].

$$\begin{aligned} &\operatorname*{plim}_{k\to\infty} \hat{X}^{IV}_{Aug}(k) \\ &= \operatorname*{plim}_{k\to\infty} \left[\left\{(\tilde{\Pi}^{k-1}_{Aug})^T \tilde{\Pi}^k_{Aug}\right\}^{-1} \left\{(\tilde{\Pi}^{k-1}_{Aug})^T z^k\right\}\right] \\ &= \operatorname*{plim}_{k\to\infty} \left\{\frac{1}{k}(\tilde{\Pi}^{k-1}_{Aug})^T \tilde{\Pi}^k_{Aug}\right\}^{-1} \times \operatorname*{plim}_{k\to\infty} \left\{\frac{1}{k}(\tilde{\Pi}^{k-1}_{Aug})^T z^k\right\} \\ &= \left\{(\Pi_{Aug})^T \Pi_{Aug}\right\}^{-1} \left\{(\Pi_{Aug})^T X_{Aug}\right\} = X_{Aug} \end{aligned} \tag{20}$$

■

IV. SIMULATION RESULTS

To show the performances of the position and heading estimators, we compare them with the nominal least squares method through simulations. The sensor network contains 6 fixed sensor nodes at (0, 0), (-1.015, 0), (-1.015, 1.015), (-1.015, 2.03), (0, 2.03), and (0, 1.015) $[m]$. The variances of the RDOA measurements are all set to $0.02[m^2]$. For batch type comparison, we accumulate 5000 measurements and calculate the error performance by using 100 iterations. In this simulation, we fix the transmitter A and rotate the transmitter B every 10 degree to check the estimation performances at different target's center position and heading.

The mean error (ME) for the position is shown in Fig. 2 and its RMSE is shown in Fig. 3. As referred in [6], the NLS (nominal least squares) method has a large estimation error by the measurement matrix uncertainty and the bias of the measurement noise. On the contrary, the RoLS, IV and augmented IV methods have low estimation error within $0.02[m]$. The bias of the IV method caused by non-zero mean of the measurements noise is not relatively serious. However, the augmented IV method compensates properly the error by estimating it.

The estimation results for the heading are shown in Fig. 4 and Fig. 5. The trend of heading results of the NLS resembles with the position results. The heading error from the cosine is bigger than the one from the sine like the position error. The reason for this is that the heading from the cosine depends on the x-position of the target and the other is vice versa as shown in (6). The heading estimation by the augmented IV method also shows good performance as the RoLS estimator. Therefore, the augmented IV method can be a practical method because it does not need any stochastic information and additional computational burden.

V. CONCLUSION

The target's position and heading can be estimated in the RDOA measured sensor network with linear estimator. However, since a nominal least squares estimator causes estimation error by the measurement matrix uncertainty and the bias of the measurement noise, there needs a proper compensation algorithm. We propose the position and heading estimator and it does not need an additional compensation procedure and the stochastic information of the RDOA measurement noise. The proposition is based on the instrumental variable method and estimates the bias of the measurement noise under the bias common model assumption. The estimator works well in noisy environment like the robust least squares method and shows better performance than the nominal least squares estimator. The useful aspects that it has low computational burden and does not need any stochastic information make it utilized for the practical applications.

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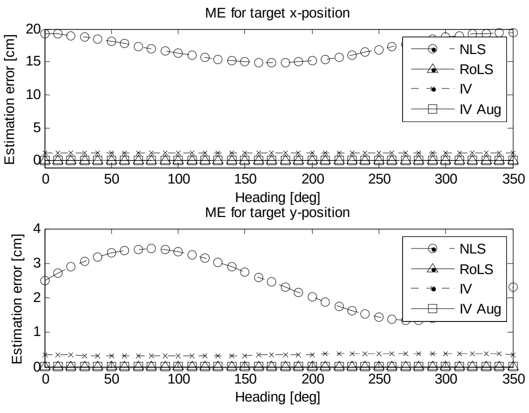

Fig. 2. Mean error for the target's center position

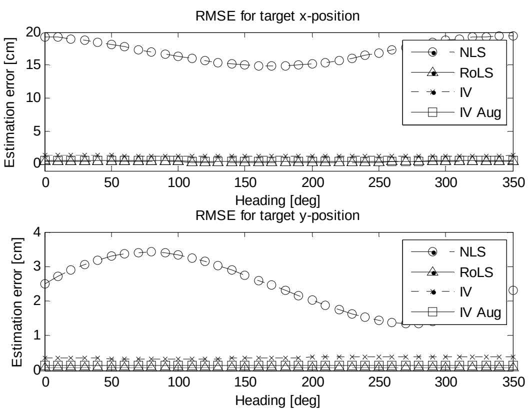

Fig. 3. RMSE for the target's center position

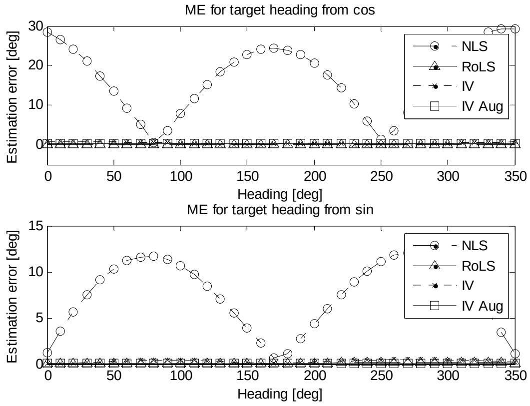

Fig. 4. Mean error for the target's heading

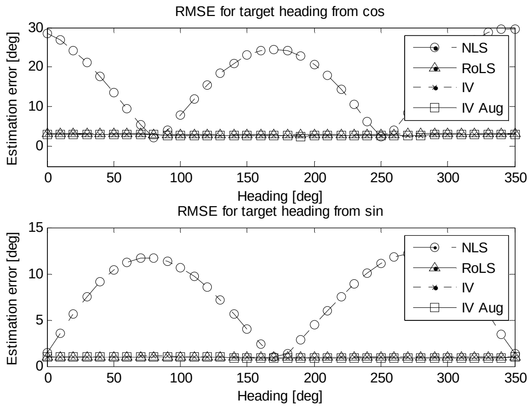

Fig. 5. RMSE for the target's heading

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