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DEA various method used Restricted Multiplier DEA

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Abstract: Data Enveloped Analysis (DEA) is used for evaluating management efficiency of Decision Making Units (DMUs). The traditional DEA has restrictions on weight for multiple objectives. This paper proposes a DEA method that puts restrictions on not the weight but the multi-input and multi-output items in order to incorporate decis ion makers' subjective viewpoint. More DEA variant models can be derived from its basic model. Therefore, the proposed method allows to use evaluation criteria that cannot be used in the traditional method. Thus, the proposed method is able to widely analyze for various efficiency of DMU. The numerical experiments show the performance of the proposed method where teams that participated in the RoboCup 2010 Soccer Simulation 2D are evaluated.

Keywords: Data Envelopment Analysis, Linear Programming, Decision Making Support, RoboCup

I. INTRODUCTION

Data Envelopment Analysis (DEA) is a method for evaluating management efficiency of entities. DEA evaluates entities called DMUs (Decision Making Units). DMUs evaluated by comparing with their competitors. The main characteristics of DEA are the following:

- DEA evaluates DMUs that are characterized by multi-input and multi-output elements.
- The efficiency factors are calculated from multi-input and multi-output elements.

Therefore, DEA can perform evaluation from a lot of aspects and also can be used in various problems such as evaluation of universities, baseball players.

DEA calculates the weights for each of multi input and output elements. Different DMUs have different weights. The weights show the advantages of DMUs. However, the more input and output elements DMU has, the more weights become zero. In this case, it is difficult to clarify the difference between DMUs. Moreover, some elements that do not have a clear meaning for business judgment might cause a significant difference between DMUs.

Restricted Multiplier DEA (RM-DEA) was proposed to solve the above problem by including a priori knowledge in DEA. In this method, all weights are able to have nonzero value. However, it is difficult to apply DEA because RM-DEA is infeasible when the restrictions of weights are too severe.

To remedy the disadvantage of RM-DEA, this paper suggests a method that puts restrictions on not the weight but the multi input and output elements in order to incorporate a priori knowledge. The utility and effectiveness of the proposed method are shown through a series of numerical experiments.

II. DATA ENVELOPMENT ANALYSIS

1. Outline of DEA

DEA was proposed by Charnes et al. in 1978 as a method for management analysis [1]. The applicable field of DEA is widely used in data mining such as the prediction of bankruptcy.

DEA regards each DMU as a production function that produces outputs from inputs. Then the efficiency of a DMU is calculated by comparing with other DMUs. There are two characteristics in DEA; (1) Weights are assigned to each input and output data and virtual input and output are generated. These weights are not fixed but variable so that each DMU can employ suitable weights to be evaluated better. (2) Common index for evaluation is shown as efficiency value. The value of the most efficient DMU is one. On the other hand, the efficiency value is less than one if a DMU is not efficient compared with the others.

2. Formulation of DEA

While DEA has various models, this paper employs efficiency model and inefficiency model [2]. The former model evaluates the relative efficiency by the advantage points. On the other hand, the latter model evaluates the relative inefficiency by the disadvantage points.

Let us assume that there are n DMUs and each DMU is characterized by m input and s output. That is, the input for DMU_k has input expressed as $x_{1k},\ldots,x_{mk}$

and the output expressed as $y_{1k},\ldots, y_{sk}$. Here the efficiency value is calculated by solving the following linear programming:

$$\begin{aligned} &\text{Max} \quad \sum_{r=1}^{s} u_r y_{rk} \\ &\text{s.t.} \quad \sum_{i=1}^{m} v_i x_{ik} = 1 \\ &\quad -\sum_{i=1}^{r} v_i x_j + \sum_{r=1}^{s} u_r y_{rj} \leq 0 \, (j = 1,\ldots,n) \\ &\quad u_r \geq 0 \, (r = 1,\ldots,s), \quad v_i \geq 0 \, (1,\ldots,m) \end{aligned} \tag{1}$$

The above equation signifies that weights are assigned to the input elements of DMU_k so that the weighted sum of the input elements equals to one. This guarantees that the efficiency value of other DMUs does not exceed one. The objective function has the role for maximizing the output of remarkable DMU. Moreover, it is possible to analyze the advantage points of each DMU by the assigned weights. This is because these input and output elements that have nonzero weight are considered in evaluation. In other words, those elements that have nonzero weight are considered advantage points.

The inefficiency value is calculated by solving the following linear programming:

$$\begin{aligned} &\text{Min} \quad \sum_{r=1}^{s} u_r y_{rk} \\ &\text{s.t.} \quad \sum_{i=1}^{m} v_i x_{ik} = 1 \\ &\quad -\sum_{i=1}^{r} v_i x_j + \sum_{r=1}^{s} u_r y_{rj} \geq 0 \, (j = 1,\ldots,n) \\ &\quad u_r \geq 0 \, (r = 1,\ldots,s), \quad v_i \geq 0 \, (1,\ldots,m) \end{aligned} \tag{2}$$

The above equation allows to analyze the disadvantage point of each DMU by weights. Therefore, these input and output elements that have nonzero weight are considered disadvantage points.

3. Problem of DEA

In DEA, the number of evaluation criteria is increased if the number of input and output elements is increased. Therefore, even if the input or output elements have only one advantage point, DMU is evaluated efficient. In this case, other elements becomes weaker. Thus, the elements that are not advantageous are not emphasized as more zero weights are assigned to more input and output. Thus excessive number of input and output lead the following two problems: (1) Many DMUs are evaluated as efficient, which makes the evaluation meaningless. (2) Some advantages of input or output elements that make DMU efficient are not concerned in the standard business judgment. To deal with these problems, RM-DEA was proposed. In the RM-DEA, maximum and minimum of weights are restricted using a priori knowledge. However, the RM-DEA is infeasible when the restrictions of weights are too severe. Due to the above problems, it is difficult for decision makers to adjust the restriction using RM-DEA.

III. PROPOSED METHOD

This section shows the proposed model that unifies the efficiency model and the inefficiency model to solve the problem that is described in the previous section. By controlling the process of transformation from the efficiency model to the inefficiency model, the method is considered so that a priori knowledge can be incorporated.

1. Reformulation of the efficiency model

The dual problem of Equation (1) is written as follows:

$$\begin{aligned} &\text{Min} \quad \theta_k \\ &\text{s.t.} \quad -\sum_{j=1}^{n} x_{ij}\lambda_j + \theta_k x_{ik} \geq 0 \\ &\quad \sum_{j=1}^{n} y_{rj}\lambda_j \geq y_{rk} \\ &\quad \lambda_j \geq 0, \theta_k : \textit{free} \end{aligned} \tag{3}$$

Where, $\boldsymbol{\theta}$ and $\boldsymbol{\lambda}$ are variables for the dual problem. Equation (2) which $(d^+_{x_i}, d^-_{x_i})$, $(d^+_{y_i}, d^-_{y_i})$ are added as constraints for x and y respectively can be transformed as follows:

$$\begin{aligned} &\text{Min} \quad \sum_{i=1}^{m} d^-_{x_i} + \sum_{r=1}^{s} d^-_{y_r} + M\Big(\sum_{i=1}^{m} d^+_{x_i} + \sum_{r=1}^{s} d^+_{y_r}\Big) \\ &\text{s.t.} \quad -\sum_{j=1}^{n} x_{ij}\lambda_j + x_{ik} + d^+_{x_i} - d^-_{x_i} = 0 \\ &\quad \sum_{j=1}^{n} y_{rj}\lambda_j + d^+_{y_r} - d^-_{y_r} = y_{rk} \end{aligned} \tag{4}$$

where M is a very big number, $(d^-_{x_i}, d^-_{y_r})$ are slack vectors to hold the equality, and $(d^+_{x_i}, d^+_{y_r})$ are artificial vectors to unify the model. This equation regards the efficiency value as the distance from efficiency frontier. Thus, if a DMU is efficient, its efficiency value is zero. If a DMU is not efficient, the efficiency value is greater than zero.

2. Reformulation of the inefficiency model

The inefficiency model is reformulated in a similar way to the efficiency model. Reformulation of the inefficiency model is shown in the following:

$$\begin{aligned}
\text{Max} \quad & \sum_{i=1}^{m} d_{x_i}^{+} + \sum_{r=1}^{s} d_{y_r}^{+} - M\left(\sum_{i=1}^{m} d_{x_i}^{-} + \sum_{r=1}^{s} d_{y_r}^{-}\right) \\
\text{s.t.} \quad & -\sum_{j=1}^{n} x_{ij}\lambda_j + x_{ik} + d_{x_i}^{+} - d_{x_i}^{-} = 0 \\
& \sum_{j=1}^{n} y_{rj}\lambda_j + d_{y_r}^{+} - d_{y_r}^{-} = y_{rk}
\end{aligned} \tag{5}$$

where ($d_{x_i}^{+}, d_{y_r}^{+}$) are slack vectors to hold the equality, and ($d_{x_i}^{-}, d_{y_r}^{-}$) are artificial vectors to unify the model.

3. Unification of two models

Both the reformulated models have the same constraint. Thus, it is possible to unify the efficiency model and the inefficiency model by transforming the objective function. In order to unify the models, we define the objective function as follows:

$$\begin{aligned}
\text{Min} \quad & \sum_{i=1}^{m} (d_{x_i}^{+} \sin\varphi_{x_i} - d_{x_i}^{-} \sin\varphi_{x_i}) \\
& + \sum_{r=1}^{s} (d_{y_r}^{+} \sin\varphi_{y_r} - d_{y_r}^{-} \sin\varphi_{y_r}), \quad -\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}
\end{aligned} \tag{6}$$

where $\boldsymbol{\varphi}$ is the Value of Deciding Evaluation Criteria (VDEC). VDEC is used to control the degree of the transformation between the efficiency and the inefficiency models. Thus, the proposed method is shown by calculating the following linear programming:

$$\begin{aligned}
\text{Min} \quad & \sum_{i=1}^{m} (-d_{x_i}^{+} \sin\varphi_{x_i} + d_{x_i}^{-} \sin\varphi_{x_i}) \\
& + \sum_{r=1}^{s} (-d_{y_r}^{+} \sin\varphi_{y_r} + d_{y_r}^{-} \sin\varphi_{y_r}) \\
\text{s.t.} \quad & -\sum_{j=1}^{n} x_{ij}\lambda_j + x_{ik} + d_{x_i}^{+} - d_{x_i}^{-} = 0 \quad (i = 1,2,\ldots,m) \\
& \sum_{j=1}^{n} y_{rj}\lambda_j + d_{y_r}^{+} - d_{y_r}^{-} = y_{rk} \quad (r = 1,2,\ldots,s) \\
& \lambda \ge 0, d_{x_i}^{+} \ge 0, d_{x_i}^{-} \ge 0, d_{y_r}^{+} \ge 0, d_{y_r}^{-} \ge 0, -\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}
\end{aligned} \tag{7}$$

In this equation, optimal DMUs are evaluated with the efficient value of zero. On the other hand, the values for non-optimal DMUs are greater than zero.

Equation (7) is the efficiency model when φ_{xi} and φ_{yr} are equal to π/2 for all i and r. Equation (7) is the inefficiency model when φ_{xi} and φ_{yr} is equal to π/2 for all i and r. Moreover, Equation (7) is regarded as the mixture of efficiency and inefficiency models when given φ_{xi} are distinct primes for each input elements. In addition, when φ_{xi} is less than π/2 and φ_{xi} is greater than 0, the value of $d_{x_i}^{-}$ is not likely to increase compared to the case when φ_{xi} is π/2. Then, input element of i-th number is unlikely to search for an advantage as compared with other input elements. Therefore, the proposed method is able to calculate difference of value for each element.

From the above discussion, we can see that the proposed method is able to show the various models such as the mixture of the efficiency and the inefficiency models. Moreover, it is able to incorporate a priori knowledge by VDEC.

IV. NUMERICAL EXPERIMENTS

1. Experimental conditions and data

In order to show the effectiveness of proposed method (Equation (7)) visually, we apply it to an artificial data set. The data set consists of 16 input-output DMUs that are characterized by one input and two output elements (Table 1). We conducted four experiments using VDEC as follows:

(1) $\boldsymbol{\varphi}$ are π/2 for all elements
(2) $\boldsymbol{\varphi}$ are -π/2 for all elements
(3) φ_{x1} is -π/2 for input1, φ_{x2} is π/2 for input2.
(4) φ_{x1} is π/2 for input1, φ_{x2} is -π/2 for input2.

In the numerical experiments in this section, characteristics of the method clarify to look at differences of efficiency value. Experiments using data of RoboCup will be shown in the presentation at the symposium.

Table 1. Artificial data set

DMU	input1	input2	output
1	1	1	1
2	1	2	1
3	1	3	1
4	1	4	1
5	2	1	1
6	2	2	1
7	2	3	1
8	2	4	1
9	3	1	1
10	3	2	1
11	3	3	1
12	3	4	1
13	4	1	1
14	4	2	1
15	4	3	1
16	4	4	1

2. Experimental Results

The results are shown in Table 2. For instance, the value of the DMU_2 is evaluated value as zero in the case (1), and as six in the case (2).

Table 2. Distance from optimal DMU

DMU	(1)	(2)	(3)	(4)
1	0	6	3	3
2	1	5	4	2
3	2	4	5	1
4	3	3	6	0
5	1	5	2	4
6	2	4	3	3
7	3	3	4	2
8	4	2	5	1
9	2	4	1	5
10	3	3	2	4
11	4	2	3	3
12	5	1	4	2
13	3	3	0	6
14	4	2	1	5
15	5	1	2	4
16	6	0	3	3

3. Discussion

The numerical experiments of case (1), (2), and (3) are shown in the Fig, 1, Fig 2, and Fig. 3 in order to examine the effectiveness of the proposed method viscerally. All X-axes are input1 over output and all Y-axes are input2 over output. The grid points show DMUs and the lines connecting them show DMUs which have the same evaluated value.

A. Case (1)

DMU_1 is the optimal DMU in this case, and its evaluated value is "0". The DMUs which are equidistance form DMU_1 have the same evaluated value. Moreover, as the distance from DMU_1 becomes farther, the evaluated value also become large. This shows that DMU with smaller input and larger output is able to become optimal. Thus, given that φ is $\pi/2$ for all elements, the proposed method is able to be regarded as efficiency model.

B. Case (2)

DMU_{16} is the optimal DMU in this case, and its evaluated value is "0". All evaluated values are inverted in comparison with case (1). Thus, given that φ is $-\pi/2$ for all elements, the proposed method is able to be regarded as inefficiency model.

C. Case (3)

DMU_4 is the optimal DMU in this case, and its evaluated value is "0". DMU_4 is efficient when DEA calculates it for input1 and output. On the other hand, DMU_4 is inefficient for input2/output. Thus, the proposed method is able to evaluate DMUs by mixing the of efficiency and inefficiency.

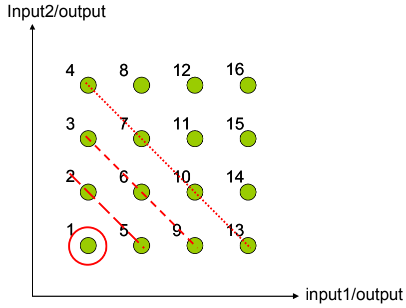

Fig. 1. Visualization of case (1)

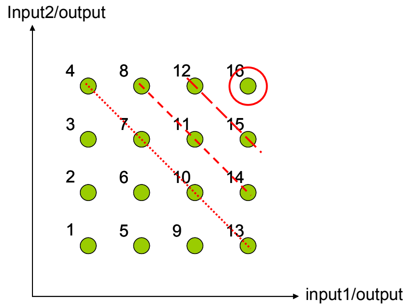

Fig. 2. Visualization of case (2)

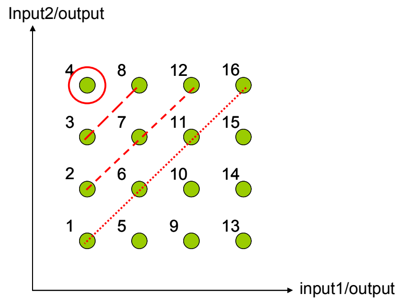

Fig. 3. Visualization of case (3)

V. CONCLUSION

This study proposed a DEA model which unifies the efficiency model and the inefficiency model. The proposed model is able to evaluate various criteria which the traditional model can not evaluate. Moreover, VDEC helps incorporating a priori knowledge. The effectiveness of the proposed method was illustrated in the numerical experiments.

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