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Properties of Localized Oscillatory Excitation on the Non-Linear Oscillatory Field

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Abstract: We analyze the dynamics of the non-linear oscillatory field composed of Radial Isochron Clocks (RICs) or Stuart-Landau (SL) oscillators, which are the simplest dynamical systems that have one stable limit cycle around one unstable equilibrium. According to our computer simulation, the non-linear oscillatory field with two kinds of Mexican-hat-type connection had the function of several peak ditection of an external input by the localized oscillatory excitation areas. Moreover, this non-linear oscillatory field could also realize in-phase phase-locking within each localized oscillatory excitation area, but maximize the phase difference between different localized oscillatory excitation areas. As Amari (1977) model of the nerve field provided mathematical base for the self-organizing map (SOM) algorithm, the non-linear oscillatory field is expected to provides theoritical base for the oscillatory SOM algorithm.

Keyword: non-linear oscillatory field, localized oscillatory excitation, RIC, SL oscillator, phase-locking, Mexican-hat-type connection

1 Introduction

Many 1970's experimental results suggested that feature-extracting cells are self-organized in visual cortex through postnatal sensory experience [1, 2]. Hirsch and Spinelli (1970), for example, observed such self-organization in the visual cortical cells of young kitten [1].

In response to this, a lot of neural network models were proposed at the time to describe this phenomenon. Malsburg (1973) proposed the earliest model which explained the mechanism of self-organization of the visual cortex [3]. Amari and Takeuchi (1978) formulated the Malsburg model in the mathmatically simple form [4]. In this model, they adopted Amari (1977) model of the nerve field, which had been shown to allow stable localized excitation areas [5]. This stable excitation pattern in the nerve field provides base for neighborhood learning used in Malsburg model and Kohonen's Self-Organizing Map (SOM) algorithm [6]. SOM is one of the simplest learning model of the cerebral cortex. Amari expressed the property of peak detection of the function through the dynamics of the nerve field with the Mexican-hat-type connection pattern, while Kohonen built that of peak detection to the algorithm directly in the SOM.

Meanwhile, recent studies found that the sign and magnitude of synaptic plasticity depend critically on the precise timing of pre- and postsynaptic firing. This phenomenon is called spike timing dependent plasticity (STDP) [7]. Moreover, there is a hypothesis that the property binding should be represented by phase-locking among neuronal oscillatory firing, called synchronous firing hypothesis. Eckhorn et al. (1988) and Gray et al. (1989) discovered synchronous periodic firing of neurons in the visual cortex of the monkey and that of the cat [8, 9].

Regarding periodic neuronal firing as oscillation, and synchronous firing as in-phase phase-locked oscillation, we extend Amari model of the nerve field to the oscillatory field. Kuramoto (1982) first proposed the oscillatory field [10]. The difference of our model from the preceding models such as Kuramoto model is that oscillation occurs in local area in the oscillatory field. We call this excitation pattern *localized oscillatory excitation* [13]. In this paper, we study the mechanism of formation of localized oscillatory excitation areas in the oscillatory field. To this end, we consider a non-linear oscillatory field composed of Radial Isochron Clocks (RIC) or Stuart-Landau (SL) oscillators, which are the simplest dynamical systems that have one stable limit cycle around one unstable equilibrium [11, 12].

Our computer simulation showed that the non-linear oscillatory field with two kinds of Mexican-hat-type connection had the function of several peak ditection of an external input by the localized oscillatory excitation areas, and could also realize in-phase phase-locking within each localized oscillatory excitation area, but maximize the phase difference between different localized oscillatory excitation areas.

This non-linear oscillatory field is expected to provide theoritical base for the oscillatory SOM algorithm, as Amari (1977) model of the nerve field provided mathematical base for the SOM algorithm [14].

2 Model

2.1 RIC (SL Oscillator)

RICs (SL oscillators) are known to be one of the simplest dynamical systems which have one stable limit cycle around one unstable equilibrium [11, 12]. The dynamics of RICs are written in polar form as below.

$$\frac{dr}{dt} = r(1 - r^2) \tag{1}$$

$$\frac{d\theta}{dt} = 1 \tag{2}$$

As shown in Fig.1, any orbit beginning with $r > 0$ approaches the stable limit cycle ($r = 1$) as t increases. According to the equation (2), the behavior of the phase θ does not depend on the amplitude r.

2.2 Non-Linear Oscillatory Field

Let us consider a non-linear oscillatory field consisting of RICs (SL oscillators) as shown in the following equations.

$$\frac{dr(x)}{dt} = r(x)(I_1(x) - r(x)^2) + \delta \tag{3}$$

$$\frac{d\theta(x)}{dt} = 1 + I_2(x) \tag{4}$$

$$I_1(x) = \sum_{\xi} w_1(\xi - x) r(\xi) \tag{5}$$

$$I_2(x) = \sum_{\xi} w_2(\xi - x) r(\xi) \sin(\theta(\xi) - \theta(x)) \tag{6}$$

$$w_1(x) = (1 - 2(\frac{x}{\sigma_1})^2) \exp\{-(\frac{x}{\sigma_1})^2\} \tag{7}$$

$$w_2(x) = \exp\{-(\frac{x}{\sigma_2})^2\} - C \tag{8}$$

Fig.2 illustrates our model of the non-linear oscillatory field. One osillator is connected with all the other oscillators through two kinds of Mexican-hat-type connection, w_1 and w_2. To facilitate visualization, we assume in this section that the oscillators

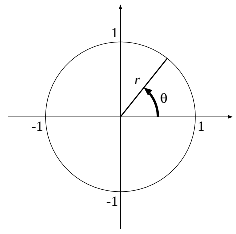

Fig. 1: The structure of a RIC (SL oscillator).

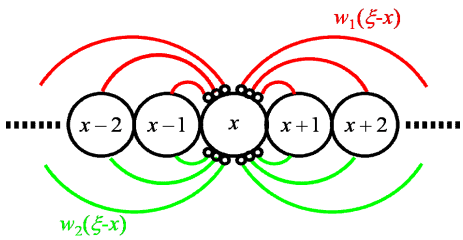

Fig. 2: Our model of the non-linear oscillatory field.

are arranged in one dimension. Moreover, only the connections from the oscillator in the center to the others is discribed in Fig.2, but actually, the other oscillators also have similar connection.

The oscillator located at place x has information of the amplitude $r(x)$ and the phase $\theta(x)$. the positive value of the amplitude $r(x) > 0$ represents neuronal firing, and $r(x) = 0$ represents that the neuron is not firing. The small positive constant δ deviate the amplitude $r(x)$ from an unstable equilibrium ($r(x) = 0$).

The input $I_1(x)$ effects on the amplitude $r(x)$ of the oscillator. One localized oscillatory excitation area exists over a range of the positive value of $I_1(x) > 0$. On the other hand, the input $I_2(x)$ effects the phase $\theta(x)$. The positive value of $I_2(x) > 0$ increases the phase velocity.

As shown in Fig.3, two kinds of Mexican-hat-type connection $w_1(\xi - x)$ and $w_2(\xi - x)$ depend on the distance from one oscillator at place x to another oscillator at place ξ. Although both w_1 and w_2 are Mexican-hat-type connection, these two types of effect on the distant unit are different. While w_1 has no effect on the distant unit as $w_1(\infty) = 0$, w_2 has the negative effect on the distant unit as $w_2(\infty) = -C$, in order to maximize the phase difference between the distant units. For in-phase phase-locking within one localized oscillatory excitation area, the range of positive effect of $w_2(\xi - x) > 0$ must be wider than that of the localized oscillatory excitation area, $I_1(x) > 0$, namely, $\sigma_1 < \sigma_2$.

This non-linear oscillatory field allows one or several localized oscillatory excitation areas, as described in the next chapter.

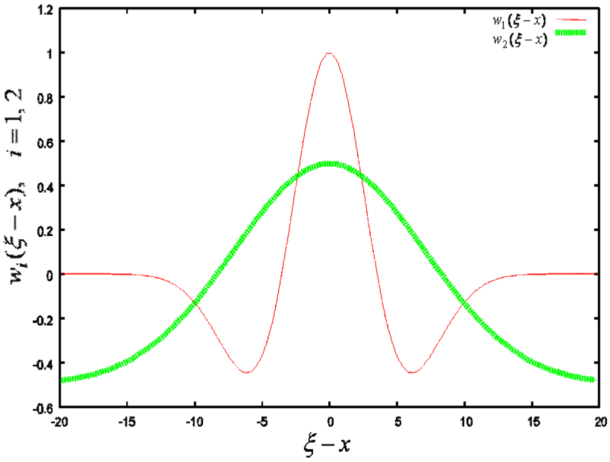

Fig. 3: Two kinds of Mexican-hat-type connection.

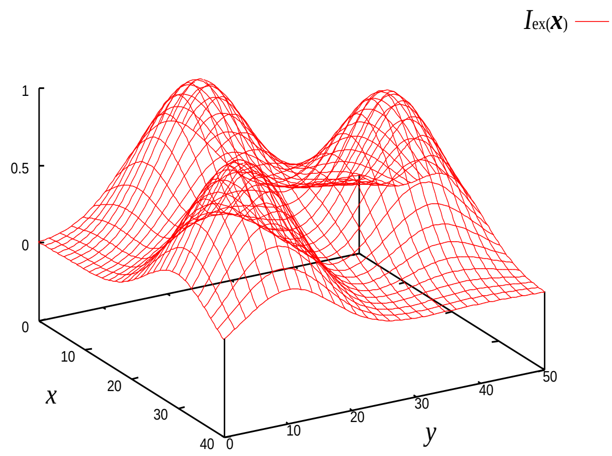

Fig. 4: The external input $I_{ex}(\boldsymbol{x})$ on the computer simulation.

3 Computer Simulation

3.1 Settings

In this chapter, we assume two-dimensional field $\boldsymbol{x} = (x, y)$, $\boldsymbol{x}' = (\xi, \eta)$. Hense, the mathematical expression $\boldsymbol{x}' - \boldsymbol{x}$ represents $\|\boldsymbol{x}' - \boldsymbol{x}\|$. The non-linear oscillatory field consists of 40×50 oscillators, and the distance between one oscillator and the neighboring oscillator is defined as 0.05.

The winner of the ordinary dot-product SOM has the weight vectors which maximize the dot-product between the input [6]. In other words, the ordinary SOM algorithm has the ability of the peak detection. Amari (1977) model justifies the ability of the SOM algorithm by the dynamics of the nerve field [5].

Recently, oscillatory SOMs, the extended version of the SOMs by the oscillators, have been proposed [14]. The oscillatory SOMs allow the several winners in the output layer. In other words, the oscillatory SOMs detect the several maximal points of the function, or the dot-product between the input vectors and the weight vectors. The ability of the oscillatory SOMs to detect the several peaks of function is also required to be justified by the dynamics of the oscillatory field.

Therefore, we carried out the computer simulation of peak detection of function, using our model of the non-linear oscillatory field.

As shown in Fig.4, the constant extranal input $I_{ex}(\boldsymbol{x})$ which had three maximal points was added to $I_1(\boldsymbol{x})$ in the following equation (9). The external input was given by the equation (10), and the three maximal points were located at place $\boldsymbol{x}_1 = (7, 20)$, $\boldsymbol{x}_2 = (20, 40)$, $\boldsymbol{x}_3 = (30, 10)$.

$$I_1(\boldsymbol{x}) = \sum_{\boldsymbol{x}'} w_1(\boldsymbol{x}' - \boldsymbol{x}) r(\boldsymbol{x}') + I_{ex}(\boldsymbol{x}) \qquad (9)$$

Table 1: Parameters on the computer simulation

δ	σ_1	σ_2	C	σ
0.001	0.25	0.5	0.5	0.5

$$\begin{aligned} I_{ex}(\boldsymbol{x}) &= \exp(-\frac{\|\boldsymbol{x} - \boldsymbol{x}_1\|^2}{\sigma^2}) \\ &+ \exp(-\frac{\|\boldsymbol{x} - \boldsymbol{x}_2\|^2}{\sigma^2}) \\ &+ \exp(-\frac{\|\boldsymbol{x} - \boldsymbol{x}_3\|^2}{\sigma^2}) \end{aligned} \qquad (10)$$

The simulation was carried out by using the Runge-Kutta method of which the size of time step was 0.005. Each oscillator was set the initial value of the amplitude to 0, and set that of the phase to the uniform pseudorandom number from 0 to 1. Each parameter was given as Table 1.

3.2 Results

Fig.5 illustrates the non-linear oscillatory field when the external input which has the three local maximal points is given. The circle represents the activated oscillator $r(\boldsymbol{x}) > 0$. The direction of the radus represents the phase $\theta(\boldsymbol{x})$ of the oscillator, or the timig of neuronal firing. Three framed rectangles denote the maximal points of the external input.

As shown in Fig.5, the non-linear oscillatory field kept stably three localized oscillatory excitation areas around the same number of maximal points of the external input.Therefore, the non-linear oscillatory field had the ability of detection of several

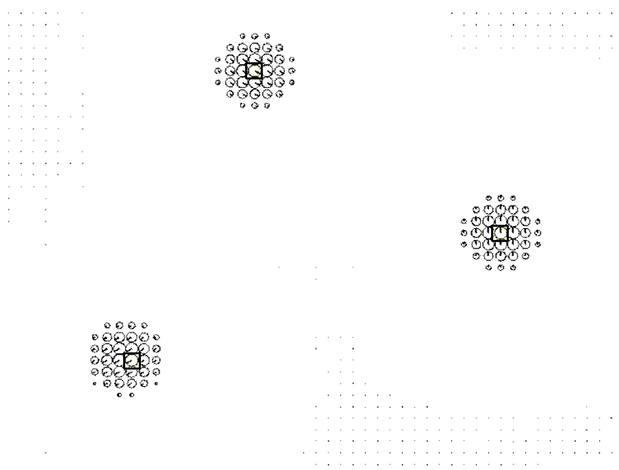

Fig. 5: The non-linear oscillatory field given the external input which had three maximal points. Three localized oscillatory excitation areas were kept stably around the same number of maximal points.

peaks from the external input by the localized oscillatory areas.

In this case, the oscillatory field entrained in-phase within each localized oscillatory excitation area, and entrained the phase difference $2\pi/3$ between the different localized oscillatory excitation areas.

4 Conclusion

We analyzed the dynamics of the non-linear oscillatory field consisting of RICs, or SL oscillators.

Our computer simulation showed that the non-linear oscillatory field with two kinds of Mexican-hat-type connection could keep three localized oscillatory excitation areas stably around the same number of maximal points of the external input. Thus, our model has the ability of peak detection of the function, which Amari model also has. Moreover, the non-linear oscillatori field could also realize in-phase phase-locking within each localized oscillatory excitation area, but maximize the phase difference between different localized oscillatory excitation areas.

This non-linear oscillatory field provides theoritical base for the oscillatory SOM algorithm, as Amari (1977) model of the nerve field provided mathematical base for the SOM algorithm [14].

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