# Effect of interaction between rules on rule dynamics in a multi-group minority game

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*Abstract*: Rules, such as laws, institutions, and norms, are shaped by interaction among social members, which are affected by the rules. Since they have such mutually regulating relationship, the rules can be changed dynamically. This paper discusses how interaction between rules shaped in different groups has an influence on the rule dynamics. In order to investigate it, we propose a multi-group minority game model in which each agent plays it in different groups independently with contact to other group's members through the game. Our simulation showed that the frequency of synchronization in the rule dynamics is more than a model without interaction. Further, we found that, in a model with interaction by exchanging agents in different groups, the frequency is increased till the exchange number of agents between different groups is increased to half of the members, but this effect is lost by decreasing the interaction opportunity and increasing the exchange number from more than half of the members.

*Keywords*: Rule Dynamics, Multi-Group Minority Game, Multi-Agent System, Internal Dynamics, Micro-Macro Loop, Simple Recurrent Network with Self-Influential Connection

### **I. INTRODUCTION**

In our society, there are many rules such as laws, institutions, and norms. Such rules shown in a macro level are shaped by interaction among social members at a micro level from bottom up. The rules have an influence on the members. The members affected by the rules might change their ways of thinking and understanding about the world. Consequently, the rules might also be changed, and totally-new rules might emerge. This mutually regulating relationship between a micro and a macro levels can be referred to as the micro-macro loop [1]. In this paper, we describe such dynamical change of rule as rule dynamics.

For example, the institutions have been discussed among many economists. Veblen [2] defines an institution as "settled habits of thought common to the generality of men." According to North [3], institutions can be defined as "the formal rules (common law, etc), the informal constraints (norms, etc), and the enforcement characteristics of each." Both of Veblen and North emphasize an importance of interaction between institutions and individuals (or organizations consisted of individuals) in their discussions on the institutional change.

Based on these arguments, in our previous works [4-5], we investigated about relationship among the micro-macro loop, internal dynamics, and changes of macro structures like the rules by using our proposed multi-agent system. This system is consisted of many agents with the internal dynamics. An autonomous change of internal state, on which the agent's

action depends, refers to as the internal dynamics. As the results in our previous work, we found that internal dynamics and micro-macro loop are necessary to form and maintain an endogenous dynamics of social structure. Further, our simulation showed that the macro structure shaped by all agents' actions is internalized in the agents through the learning process.

In the previous simulation system, only one group was used, and coexistence and competition of many macro structures could not be observed simultaneously in the one group. However, there are many groups, many rules, and interaction among them in actual world. Moreover, there are hardly researches which take internal dynamics, micro-macro loop and interaction between different rules into consideration.

The purpose of this study is to investigate how interaction between rules shaped in different groups has an influence on the rule dynamics. In order to discuss it, we propose a multigroup minority game in which many agents in different groups play the minority game independently and interact with other members.

## **II. MULTI-GROUP MINORITY GAME**

The minority game (MG) [6] is a simple game which can be regarded as a simple stock market model. In this game, n(odds)-players must select one out of two hands (e.g., -1 or 1) independently. And then, those who are in the minority side win. Here, the MG is employed as social interaction. The schematic view of the basic MG in our simulation is illustrated in Fig.1.

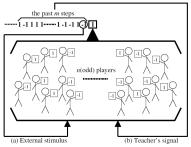


Fig.1. The schematic view of the basic MG in our simulation. (a) Each agent receives past minority hand shaped by all agents' actions. (b) All agents learn a time series of the minority hands for past m steps.

We propose a multi-group minority game (MGMG), where there are many groups playing the MG independently, and at the same time interacting with other members of different groups. There are three types of interacting ways as follows:

- 1. All agents in each group play the MGMG independently. That is to say, they have no interaction (Fig.2a).
- Each agent receives not only own past minority hand but also other group's past minority hand at each step (Fig.2b).
- 3. The agents in different groups play together with other group's members by periodically exchanging agents between groups (Fig.2c).

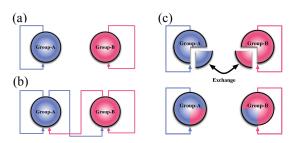


Fig.2. Three types of interactions in our proposed MGMG. (a) No interaction. (b) Each agent receives not only their group's past minority hand but also other group's past minority hand at each step. (c) Each agent interacts with other group's members by exchanging agents at fixed intervals.

We adopt a model of dynamical cognitive agent with internal dynamics represented by a simple recurrent network with self-influential connection (SRN-SIC) proposed in our previous work [4] as illustrated in Fig.3. The SRN-SIC is an Elman-type network [7] modified by adding recurrent connections between the output and the input layers so that the agent can determine his/her own action based on its past action.

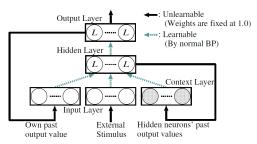


Fig.3. The SRN-SIC proposed in our previous work as an architecture of agent with internal dynamics. This is a particular Elman-type network with additional recurrent connections between the output and the input layers. The symbol L indicates a differentiable nonlinear function to output a real number between -1.0 and 1.0. Not all connections are shown.

In our previous work, we observed various ordered patterns such as fixed point and periodic motions at macro level. These ordered patterns can be generated by some rules of dynamical systems. Therefore, when an ordered pattern is shown in a time series of minority hands, we can interpret that a rule emerges through interaction among the agents. Besides, we confirmed that the agents can internalize the rules in their SRN-SIC and act based on the internalized rule. That is to say, by using the MGMG and the SRN-SIC, we can observe dynamics of interaction between certain rules shaped in different groups.

### **III. SIMULATION RESULTS**

The basic parameters of our simulation are set up as follows:

The population size of a group is set to 101. The number of group is set to 2. The settings of the SRN-SIC are omitted for space constraint (For more details refer to [4]). The simulation is conducted for 1,000 turns, where 10,000 steps are referred to as 1 turn, namely, for 10,000,000 steps. The learning of each agent is processed by using the error-backpropagation every 1 turn.

# 1. Interaction by receiving another group's minority hand on rule dynamics

First, we compare the results of the simulation without interaction to that of the simulation with interaction in which

all agents receive not only their minority hand but also the one shaped by another group.

Figure 4a shows the difference of the number of all rule dynamics that the period number is the same at the identical turn. However, all rule dynamics with more than 3,500 periods are regarded as the same aperiodic dynamics, since last 7,000 steps of 1 turn are used to analyze the period number of the dynamics. As can be seen from Fig.4a, the number of the rule dynamics with the same period in the simulation having interaction is more than that of the dynamics in the simulation without interaction.

Figure 4b depicts the difference of the number of times that both of different groups' rule dynamics in each simulation become only aperiodic at the same turn. As shown in Fig.4a, there is a major difference between the result of the simulation without interaction and the simulation with interaction.

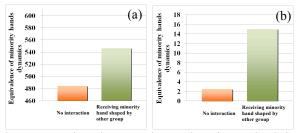


Fig.4. Comparison between the results of our simulation without interaction and with interaction in number of the rule dynamics with the same period between different groups. (a) The difference of the number of all rule dynamics that the period number is the same at the identical turn. (b) The difference of the number of times that both of different groups' rule dynamics in each simulation become only aperiodic at the same turn.

Next, we confirm whether the patterns of the rule dynamics shaped by different groups in the simulation with interaction are completely the same or not. Analysis reveals that most patterns of rule dynamics except for period-2 dynamics are different, although the period number of the dynamics is the same. Even if the patterns between the rule dynamics are the same, most of them are out of phase. It is rarely seen the cases that the dynamics shaped in different groups are completely the same. These rule dynamics are exemplified in Fig.5<sup>1</sup>.

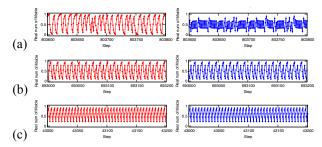


Fig.5. Three examples of the rule dynamics with the same period numbers. (a) Both period numbers are 610 but the patterns are absolutely different. (b) Both period numbers are 12 with the same patterns but out of phase. (c) This is a rare case that the dynamics are exactly the same. Each period number in this case is 5.

# 2. Interaction by exchanging the agents between different groups

As the other simulation with interaction, we conduct the MGMG in which some agents exchange between different groups in order to investigate what kind of effect it has on the rule dynamics and what type of change it produces in the one.

Figure 6 shows how the equivalence of minority hands dynamics is affected by changing the number of exchanging the agents between different groups and its interval.

As the number of agents exchanged between different groups is increased, it increases the number of times that each period number of the rule dynamics shaped in different groups is the same at the identical turn is increased. When the exchange number is 50, the number of times peaks and is more than the one shown in the simulation with the interaction receiving another group's minority hand, as can be seen from comparison between Fig. 4a and 6.

As the exchange number is increased from greater than half of the members, and as the exchange interval is extended from 10,000 to 1,000,000 steps, however, when the number of times that the rule dynamics shaped in different groups are synchronized, the number of times is decreased. On the surface, it also seems that extending the exchange interval has an effect of decreasing the synchronization of the rule dynamics. But in fact, by changing the length of steps and making the number of interacting opportunity the same, we found there is not a dime's worth of difference between the equivalence observed in our simulations using different exchange intervals. Therefore, it suggests that learning the rule dynamics at different group emphasizes synchronization of the rule dynamics.

<sup>&</sup>lt;sup>1</sup> The graphs in Fig.5 are drawn after converted the time series of the minority hands to the real number between 0.0 and 1.0.

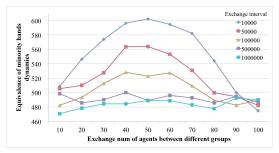


Fig.6. Effect of exchange number of agents and exchange interval on equivalence of minority hands dynamics. As the exchange number is increased, the equivalence of minority hands dynamics is also increased til the exchange number is 50. Howerver, this effect is lost by increasing the exchange number from more than half the members.

#### **IV. DISCUSSION**

Let us consider the difference causes of results shown in three types of simulations.

In the simulation in which agents receive other group's minority hand, the agents can know not only the rule dynamics formed by themselves but also the one shaped by different group. Therefore, both of rules are internalized in their SRN-SIC through the learning process, and synchronization between the rule dynamics is induced by all agents' behaviors based on their internalized rules. However, the agents cannot learn the other's rule dynamics appropriately, because teacher's signal using the learning is only their own minority hand. Thus, unstable states expressed as complex aperiodic dynamics in different groups would emerge.

Why is the equivalence shown in the simulation with interaction exchanging the agents in different groups higher than the one observed in the simulation with interaction receiving the another group's minority hand? In the simulation having the replacement, the agents can learn both of different rule dynamics shaped in each group by alternating between the groups and form such dynamics as the members of the same group temporarily. That is to say, it would appear that both rules are internalized and commoditized in the agents at the mutual groups. Therefore, it is important that the exchanged agents learn the opponent's rule dynamics to improve the equivalence of rule dynamics shaped in different groups.

### V. CONCLUSION

In order to investigate how interaction between rules shaped in different groups has an influence on rule dynamics, we have proposed a multi-group minority game (MGMG) and conducted three types of the MGMG simulation without interaction, with interaction by receiving other's minority hand, and by exchanging the agents in different groups.

The simulation results showed that the frequency of synchronization of the rule dynamics shaped in different groups is increased by interaction that the agents receive not only their own minority hand but also other's minority hand. Further, we have found that, as the exchange number of the agents is increased to about half of the group member, the equivalence of rule dynamics is also improved.

Our simulation results suggested that internalizing and commoditizing several different rules in the individuals through direct learning of the rules shaped by others play an important role to emerge stable societies and rule dynamics.

#### ACKNOWLEDGEMENT

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