Feature Based Estimation for Mapping Robot Environments using Fuzzy Kalman Filter

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Abstract

This paper introduces a fuzzy Kalman filter based approach for mapping robot environments. Takagi-Sugeno fuzzy models for nonlinear system are adopted to represent the vehicle and landmarks state equations. The complete system of the vehicle and landmarks model is decomposed into several linear models. Using the Kalman filter theory, each local model is filtered to find the local estimates. The linear combination of these local estimates gives the global estimate for the complete system. This estimator is simulated using Matlab for the vehicle-landmark system and results prove that the new approach can accurately map the environment.

1 Introduction

Simulataneous localization and map building (SLAM) has been a long term study in the autonomous vehicle research community. The ability to place an autonomous vehicle at an unknown location in an unknown environment and then have building a map, using only relative observations of the environment and moreover to use this map simultaneously to navigate would indeed make such a robot "autonomous." Thus the main advantage of SLAM is that it eliminates the need for artificial infrastructure or *a priori* topological knowledge of the environment. A solution to the SLAM problem would be inestimable value in a range of application where absolute position or precise map information is unobtainable, including amongst others, autonomous planetary exploration, subsea autonomous vehicles, autonomous airborne vehicles, and autonomous all-terrain vehicles in tasks such as a mining and construction.

The solution to the SLAM problem is the estimation of vehicle and landmarks states. How accurately these states are estimated depends on the estimator. Estimation of an unknown variable distorted by noise can be challenged by probabilistic approaches to give a resonable estimation. In SLAM problem, Kalman filteting of the state estimates is widely used due to its popularity as it directly provides both a recursive solution to the navigation problem and a means of computing consistent estimates for the uncertainity in the vehicle and landmark locations on the basis of statistical models for vehicle motion and relative landmark locations. Extended Kalman filter (EKF) has been identified as a very good state estimator for the SLAM problem because it gives very accurate solution to the SLAM. A good EKF algorithm for the SLAM problem has been demostrated by Dissanayake *et al.* [1].

In the history of nonlinear control systems, fuzzy logic control has played a major role in controlling nonlinear systems. Fuzzy logic has been a promising control tool for the nonlinear systems. Fuzzy state estimation is a topic that has receieved very little attention. Fuzzy Kalman filtering [2] is a recently proposed method to extend Kalman filter to the case where the linear system parameters are fuzzy variables withing intervals. As a solution to fuzzy state estimation, Takagi-Sugeno (T-S) fuzzy model based on observation for nonlinear systems has been illustrated in [3]. As the first step of finding full fuzzy Kalman filter algorithm for the SLAM problem, we here introduce feature based mapping of the robot environment using the fuzzy Kalman filter algorithm presented in [3]. Simulation results show that the new approach to mapping with fuzzy logic gives accuarate state estimation with less computational complexity compared to the EKF approach.

Section 2 presents the T-S fuzzy model for nonlinear system and the state estimation. Section 3 presents the state estimator for each local system in T-S model. Section 4 illustrates feature based mapping for vehicle-landmarks system and offers some simulation results and Section 5 mentions some concluding remarks.

2 Kalman Filtering for Nonlinear Systems Presented by Takagi-Sugeno Fuzzy Model

Nonlinear systems can be approximated as locally linear systems in much the same way that nonlinear functions can be approximated as piecewise linear functions. Nonlinear systems can be represented by fuzzy linear models of the following form

if $z_1[k]$ is F_{i1} and...and $z_g[k]$ is F_{ig} then

$$\begin{aligned} \boldsymbol{x}[k+1] &= A_i\boldsymbol{x}[k] + B_i\boldsymbol{u}[k] + G_i\boldsymbol{w}[k] \\ \boldsymbol{y}[k] &= C_i\boldsymbol{x}[k] + \boldsymbol{v}[k] \qquad (i = 1, ..., L) \end{aligned} \tag{1}$$

This is referred to as a Takagi-Sugeno fuzzy model. The z_j are premise variables, k is the time index, F_{ij} are fuzzy sets, $\boldsymbol{x}[k] \in R^n$ is the state vector, $\boldsymbol{u}[k] \in R^m$ is the deterministic input, $\boldsymbol{w}[k]$ is the process noise, $\boldsymbol{y}[k] \in R^r$ is the measured output, and $\boldsymbol{v}[k]$ is the measurement noise. The dynamic behavior of the $\boldsymbol{x}_i[k]$ and $\boldsymbol{y}_i[k]$ signals is presented as follows:

$$\boldsymbol{x}_i[k+1] = A_i\boldsymbol{x}_i[k] + h_i(z[k])B_i\boldsymbol{u}[k] + h_i(z[k])G_i\boldsymbol{w}[k]$$

$$\boldsymbol{y}_i[k] = C_i\boldsymbol{x}_i[k] + h_i(z[k])\boldsymbol{v}[k] \qquad (i = 1, ..., L) \tag{2}$$

Complete proof of Eq. (2) can be found in [3]. Suppose we are given an n-dimentional linear discrete time system of the form:

$$\begin{aligned} \boldsymbol{x}[k+1] &= A\boldsymbol{x}[k] + h[k]B\boldsymbol{u}[k] + h[k]G\boldsymbol{w}[k] \\ \boldsymbol{y}[k] &= C\boldsymbol{x}[k] + h[k]\boldsymbol{v}[k] \end{aligned} \tag{3}$$

where the scalar $h[k] \in [0, 1]$, the process noise $\boldsymbol{w}[k]$ is white with PSD S_w, the measurment noise $\boldsymbol{v}[k]$ is white with PSD S_v, and the process noise and measurement noise are uncorrelated. Although the A, B and C matrices are constant, the system is time-varying because of the time-varying scalar $h[k]$. If the premise variables are functions of the state or control, then the system is also nonlinear because $h[k]$ is a function of the state or control. The state x of the system can be estimated by the Kalman filter, which can be derived by assuming a recursive estimator of the form:

$$\begin{aligned} \hat{\boldsymbol{x}}^+[k] &= M[k]\hat{\boldsymbol{x}}^-[k] + K[k]\boldsymbol{y}[k] \\ \hat{\boldsymbol{x}}^-[k+1] &= A\hat{\boldsymbol{x}}^+[k] + h[k]B\boldsymbol{u}[k] \end{aligned} \tag{4}$$

$M[k]$ and $K[k]$ are related by $M[k] = I - K[k]C$. If $h[k]$ is independent of x, it can be shown that the covariance is propagated as follows:

$$\begin{aligned} P^+[k] = (I - K[k]C)P^-[k](I - K[k]C)^{\mathrm{T}} \\ +h^2[k]K[k]S_vK^{\mathrm{T}}[k] \end{aligned} \tag{5}$$

We can find the optimal value of $K[k]$ by taking the partial derivative of the trace of $P^+[k]$ with respect to $K[k]$ and setting it equal to zero, which gives:

$$(K[k]C - I)P^-[k]C^{\mathrm{T}} + h^2[k]K[k]S_v = 0 \tag{6}$$

3 A State Estimator for the T-S Fuzzy Model

The steady state Kalman filter presented in the preceding section can be used to estimate the states of each of the L dynamic systems given in Eq. (2). This will give us L local steady state estimated as follows:

$$P_i^-[k+1] = A_i(P_i^-[k] - K_i[k]C_iP_i^-[k])A_i^{\mathrm{T}} + G_iS_wG_i^{\mathrm{T}}$$

$$K_i[k] = P_i^-[k]C_i^{\mathrm{T}}(C_iP_i^-[k]C_i^{\mathrm{T}} + S_v)^{-1}$$

$$\hat{\boldsymbol{x}}_i^+[k] = (I - K_i[k]C_i)\hat{\boldsymbol{x}}_i^-[k] + K_i[k]\boldsymbol{y}_i[k]$$

$$\hat{\boldsymbol{x}}_i^-[k+1] = A_i\hat{\boldsymbol{x}}_i^+[k] + h_i[k]B_i\boldsymbol{u}[k] \qquad (i = 1, ..., L) \tag{7}$$

Note that S_w and S_v in the above quations can be repleced with $(1/3)S_v$ and $(1/3)S_w$ respectively for $E(h^2[k]) = 1/3$. Since we know that $\boldsymbol{x}[k] = \sum_{i=1}^{L}\boldsymbol{x}_i[k]$, we can combine the local state estimates in Eq. (7) to estimate the state of the T-S fuzzy model (Eq. (1)) as:

$$\hat{\boldsymbol{x}}[k] = \sum_{i=1}^{L}\hat{\boldsymbol{x}}_i[k] \tag{8}$$

4 Illustration of Feature Based Mapping using Fuzzy Kalman Filter

In the following, the vehicle state is defined by $\boldsymbol{x}_v = [x, y]^{\mathrm{T}}$ where x and y are the coordinates of the center of the rear axel of the vehicle with respect to some global coordinate frame. The landmarks are modeled as point landmarks and represented by a cartesian pair $\boldsymbol{x}_f = [x_i, y_i]^{\mathrm{T}}, i = 1, ..., N$. Both vehicle and landmark states are registered in the same frame of reference.

1) *The Process Model:* Figure 1 shows a schematic diagram of the vehicle in the process of observing a landmark. The following kinematic equations can be used to predict the vehicle state from the orientation of the vehicle ϕ and velocity input V:

$$\begin{aligned} \dot{x} &= V\cos(\phi) \\ \dot{y} &= V\sin(\phi) \end{aligned} \tag{9}$$

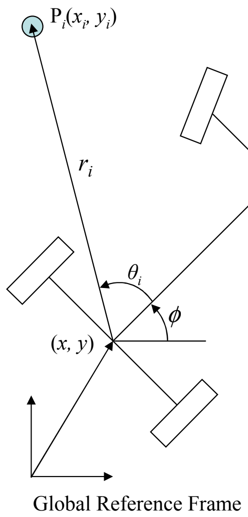

Figure 1: Vehicle and observation kinematics

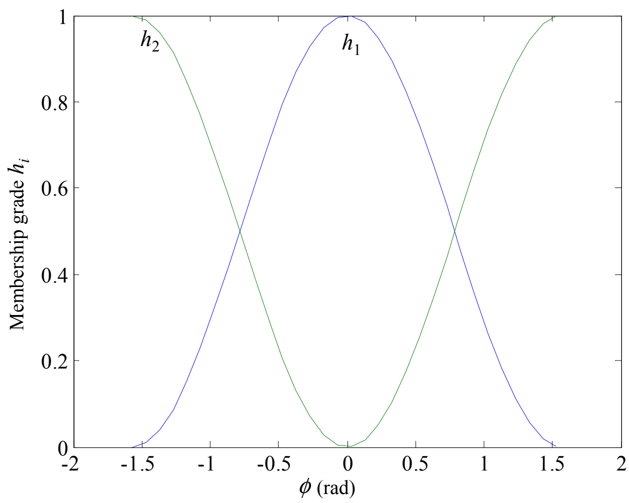

Figure 2: Vehicle-landmarks membership functions

Eq. (9) can be used to obtain a discrete-time vehicle process model in the form

$$\begin{bmatrix} x(k+1) \\ y(k+1) \end{bmatrix} = \begin{bmatrix} x(k) + \Delta T V(k)\cos(\phi(k)) \\ y(k) + \Delta T V(k)\sin(\phi(k)) \end{bmatrix} + \boldsymbol{w}[k] \tag{10}$$

The landmarks in the environment are assumed to be stationary point targets. The landmark process model is thus

$$\begin{bmatrix} x_i(k+1) \\ y_i(k+1) \end{bmatrix} = \begin{bmatrix} x_i(k) \\ y_i(k) \end{bmatrix} \tag{11}$$

for all landmarks $i = 1, ..., N$. Eq. (10) together with Eq. (11) defines the state transition matrix for the vehicle-landmarks system.

2) *The Observation Model*: In general, the range $r_i(k)$ and the bearing $\theta_i(k)$ to a landmark i are recorded by the range and bearing sensors. In this illustration, it is assumed that sensor data are processed to give the horizontal $x_{vf}(k)$ and vertical $y_{vf}(k)$ distances between the vehicle position and a landmark position in the same reference frame as the observations. The range measurements and bearing measurments are taken from the center of rear vehicle axis where the vehicle position (x, y) is taken. Refering to Fig. 1 and the above description, the observation model for a specific landmark can be formulated as

$$\begin{aligned} x_{vf}(k) &= x_i(k) - x(k) + v_x(k) \\ y_{vf}(k) &= y_i(k) - y(k) + v_y(k) \end{aligned} \tag{12}$$

where v_x and v_y are the noise sequences associated with the x_{vf} and y_{vf} respectively and assumed to be equal.

3) *Estimation Process*: Fuzzy Kalman filter described in Sections 3 and 4 is employed to generate the estimates for the fuzzy dynamic models given in Eq. (2).

4.1 Simulation Results

In this section, we are going to show the simulation results for the feature based mapping for the system composite of Eqs. (10), (11) and (12) while assuming the inital estimate and covariance to start estimation process. The process model given by Eqs. (10) and (11), and the observation model given by Eq. (12) can be used to formulate the dynamic system as follows:

$$\begin{aligned} \boldsymbol{x}[k+1] &= \boldsymbol{x}[k] + \begin{bmatrix} \Delta T\cos(\phi(k)) \\ \Delta T\sin(\phi(k)) \\ 0 \\ 0 \end{bmatrix} V(k) + \boldsymbol{w}[k] \\ \boldsymbol{y}[k] &= \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \boldsymbol{x}[k] + \boldsymbol{v}[k] \end{aligned} \tag{13}$$

where ΔT is the sample time. $\boldsymbol{w}$[k] and $\boldsymbol{v}$[k] are the process and observation noise respectively. Now consider the following two subsystems.

The first system is as follows:

$$\begin{aligned} \boldsymbol{x}_1[k+1] &= \boldsymbol{x}_1[k] + h_1 \begin{bmatrix} \frac{\Delta T}{\cos(\phi(k))} \\ 0 \\ 0 \\ 0 \end{bmatrix} V(k) + h_1\boldsymbol{w}[k] \\ \boldsymbol{y}_1[k] &= \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \boldsymbol{x}_1[k] + h_1\boldsymbol{v}[k] \end{aligned} \tag{14}$$

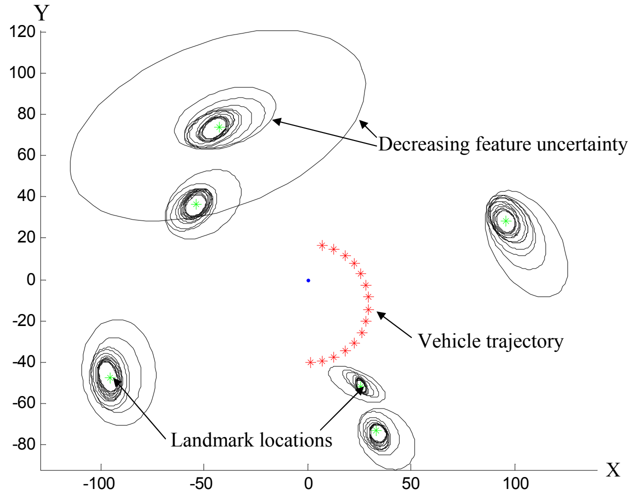

Figure 3: Evolution of map over time

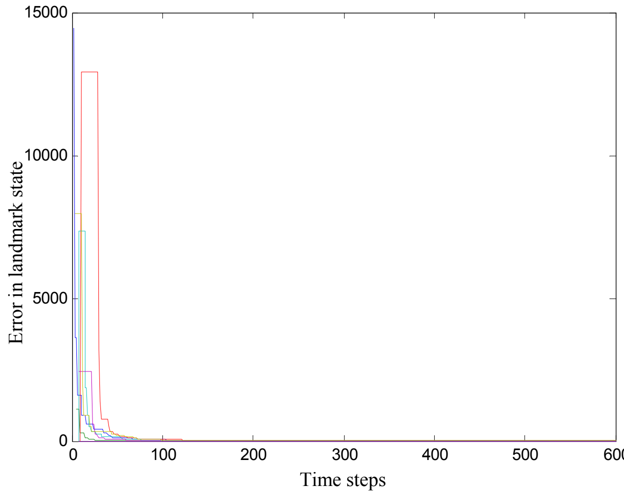

Figure 4: Landmark state estimation errors

where $h_1 = \cos^2(\phi(k))$ is the membership function for the first subsystem. The second subsystem system is given as:

$$\boldsymbol{x}_2[k+1] = \boldsymbol{x}_2[k] + h_2 \begin{bmatrix} 0 \\ \frac{\Delta T}{\sin(\phi(k))} \\ 0 \\ 0 \end{bmatrix} V(k) + h_2 \boldsymbol{w}[k]$$

$$\boldsymbol{y}_2[k] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \boldsymbol{x}_2[k] + h_2 \boldsymbol{v}[k] \quad (15)$$

where $h_2 = \sin^2(\phi(k))$ is the membership function for the second subsystem. Membership grade functions are shown in Fig. 2. It can be seen that $h_1 + h_2 = 1$ and the combination of these two subsystems results in the dynamic system model shown in Eq. (13). The two local state vectors of each subsystem are in the form of the two local state vectors given by Eq. (2) and are estimated according to the Eq. (7) and are combined according to the Eq. (8) to obtain the global state estimate. The system and Kalman filter equations were simulated using Matlab. An enivornment with 6 arbitrarily placed landmarks was simulated with a given vehicle trajectory. Landmark location states were updated using Kalman filter equations for 600 times. Sumulation results are depicted in Figs. 3 and 4. Figure 3 shows the evolution of the map over the time. It can be seen that error ellipses are getting converged to the acutal landmark locations as the map of the landmark locations is being build when the vehicle navigates through the enviroment. Figure 4 shows that the errors in each landmark state decrease over time and reach the minimum value 0. The above mentioned results indicate that the newly presented method for map building performs well and provides state estimates that converge to zero.

5 Conclusion

We have proposed a new approach to state estimation based on Takagi-Sugeno nonlinear fuzzy model. Kalman filter state estimator was modified to give a fuzzy Kalman filter. Kalman filter state estimator equations were designed for each of the local systems of the T-S model and local filters were combined to obtain the global estimator. We showed that the proposed estimator minimizes the expected value of the estimation error and converges to zero over time. Simulation results have been presented for a nonlinear vehicle-landmark system, showing the effectiveness of this scheme of state estimation.

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