Consideration on the Recognizability of Three-Dimensional Patterns

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Abstract

Due to the advances in computer vision, robotics, and so forth, it has become increasingly apparent that the study of three-dimensional pattern processing should be very important. Thus, the study of three-dimensional automata as the computational model of three-dimensional information processing has been significant. During the past about thirty years, automata on a three-dimensional tape have been obtained. On the other hand, it is well-known that whether or not the pattern on a two- or three-dimensional rectangular tape is connected can be decided by a deterministic one-marker finite automata. As far as we know, however, it is unknown whether a similar result holds for recognition of the connectedness of patterns on three-dimensional arbitrarily shaped tape. In this paper, we deal with the recognizability of three-dimensional patterns, and consider the recognizability of three-dimensional connected tapes by alternating Turing machines and arbitrarily shaped tapes by k marker finite automata.

Keywords: connectedness, finite automaton, marker, pattern, recognizability, three-dimension, Turing machine

1. Introduction

Recently, there have been many interesting investigations on digital geometry [2]. These works form the theoretical foundation of digital image processing. Among them, the problem of connectedness is one of the most interesting topics. For instance, Yamamoto, Morita and Sugata showed that a three-dimensional nondeterministic one-marker automaton can recognize connected tapes. In the case of $L(m)$ space-bounded five-way three-dimensional deterministic Turing machine, they proved that space $m^2\log m$ is necessary and sufficient amount for recognizing connected tapes of size $m \times m \times m$ [3]. Nakamura and Rosenfeld showed that three-dimensional connected tapes are not recognizable by any three-dimensional deterministic or nondeterministic finite automaton. By the way, it is well known that two-dimensional digital pictures have 4- and 8-connectedness, and three-dimensional digital pictures have 6- and 26-connectedness. It is also known that various topological properties can be defined by making use of these connectedness. For example, Nakamura and Aizawa proposed a new topological property of three-dimensional digital pictures — the interlocking component which is a chainlike connectivity. They showed that three-dimensional deterministic one-marker automaton cannot detect interlocking components in a three-dimensional digital picture [3]. Moreover, in [3], Sakamoto, et al. proposed various new three-dimensional automata, and studied several their properties. In general, however, to recognize three-dimensional connectedness seems to be much more difficult than the two-dimensional case, because of intrinsic characteristics of three-dimensional pictures.

In this paper, we consider about recognizability of three-dimensional patterns. First, we deal with the recognizability of three-dimensional connected cubic tapes by three-dimensional alternating automata. Next, we consider whether or not the pattern on a three-dimensional arbitrarily shaped tape is connected can be decided by a deterministic multi-marker finite automaton.

2. Preliminaries

Definition 2.1. Let Σ be a finite set of symbols. A three-dimensional tape over Σ is a three-dimensional rectangular array of elements of Σ. The set of all three-dimensional tapes over Σ is denoted by $\Sigma^{(3)}$. Given a tape $x \in \Sigma^{(3)}$, for each integer j ($1 \leq j \leq 3$), we let $l_j(x)$ be the length of x along the jth axis. The set of all $x \in \Sigma^{(3)}$ with $l_1(x)=n_1$, $l_2(x)=n_2$, and $l_3(x)=n_3$ is denoted by $\Sigma^{(n_1,n_2,n_3)}$. When $1 \leq i_j \leq l_j(x)$ for each j ($1 \leq j \leq 3$), let $x(i_1,i_2,i_3)$ denote the symbol in x with coordinates (i_1,i_2,i_3). Furthermore, we define $x[(i_1,i_2,i_3),(i'_1,i'_2,i'_3)]$, when $1 \leq i_j \leq i'_j \leq l_j(x)$ for each integer j ($1 \leq j \leq 3$), as the three-dimensional input tape y satisfying the following (1) and (2): (1) for each j ($1 \leq j \leq 3$), $l_j(y) = i'_j - i_j + 1$; (2) for each r_1, r_2, r_3 ($1 \leq r_1 \leq l_1(y)$, $1 \leq r_2 \leq l_2(y)$, $1 \leq r_3 \leq l_3(y)$), $y(r_1,r_2,r_3)=x(r_1+i_1-1, r_2+i_2-1, r_3+i_3-1)$. (We call $x[(i_1,i_2,i_3),(i'_1,i'_2,i'_3)]$ the $[(i_1,i_2,i_3),(i'_1,i'_2,i'_3)]$-segment of x.) For each $x \in \Sigma^{(n_1,n_2,n_3)}$ and for each $1 \leq i_1 \leq n_1$, $1 \leq i_2 \leq n_2$, $1 \leq i_3 \leq n_3$, $x[(i_1,1,1),(i_1,n_2,n_3)]$, $x[(1,i_2,1),(n_1,i_2,n_3)]$, $x[(1,1,i_3),(n_1,n_2,i_3)]$, $x[(i_1,1,i_3),(i_1,n_2,i_3)]$, and $x[(1,i_2,i_3),(n_1,i_2,i_3)]$ are called the i_1th (2-3) plane of x, the i_2th (1-3) plane of x, the i_3th (1-2) plane of x, the i_1th row on the i_3th (1-2) plane of x, and the i_2th column on the i_3th (1-2) plane of x, are denoted by $x^{(2\text{-}3)}_{i_1}$, $x^{(1\text{-}3)}_{i_2}$, $x^{(1\text{-}2)}_{i_3}$, $x[i_1,*,i_3]$, and $x[*,i_2,i_3]$, respectively[3].

Definition 2.2. A three-dimensional alternating Turing machine (denoted by 3-ATM) is a 10-tuple M = (Q, q_0, U, E, S, F, Σ, Γ, δ), where (1) Q = U ∪ E ∪ S is a finite set of states, (2) $q_0 \in Q$ is the initial state, (3) U is the set of universal states, (4) E is the set of existential states, (5) F ⊆ Q is the set of accepting states, (6) Σ is a finite input alphabet (# ∉ Σ is the boundary symbol), (7) Γ is a finite storage tape alphabet containing the special blank symbol B, (8) $\delta \subseteq (Q\times(\Sigma\cup\{\#\})\times\Gamma)\times(Q\times(\Gamma-\{B\})\times$ {east, west, south, north, up, down, no move} × {left, right, no move}) is the next move relation. As shown in Fig.2, M has a read-only cubic input tape with boundary symbols #'s (# ∉ Σ) and one semi-infinite storage tape, initially filled with the blank symbols. M begins in state q_0. A position is assigned to each cell of the input tape and the storage tape, as shown in Fig.1. A step of M consists of reading one symbol from each tape, writing a symbol on the storage tape, moving the input and storage tape heads in specified directions, and entering a new state, according to the next move relation δ [3].

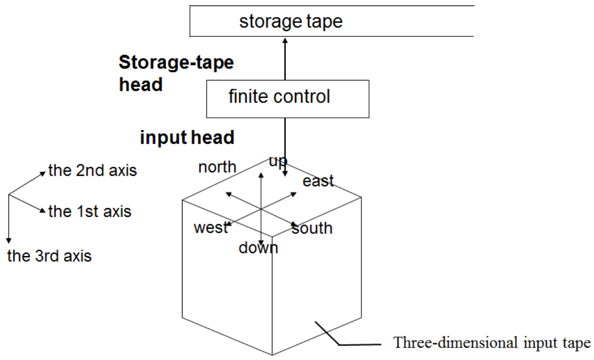

Fig.1:Three-dimensional alternating Turing machine.

Definition 2.3. A three-dimensional k marker finite automaton M(k) is defined by the six-tuple M = (Q, q_0, F, Σ, {+,−}, δ), where (1) Q is a finite set of states; (2) $q_0 \in Q$ is the initial state; (3) F ⊆ Q is the set of accepting states; (4) Σ is a finite input alphabet (# ∉ Σ is the boundary symbol); (5) {+,−} is the pair of signs of presence and absence of the marker; and (6) δ: $(Q\times\{+,-\})\times((\Sigma\cup\{\#\})\times\{+,-\}) \rightarrow 2^{(Q\times\{+,-\})\times((\Sigma\cup\{\#\})\times\{+,-\})\times\{\text{east,west,south,north,up,down,no move}\}}$ is the next-movefunction, satisfying the following: For any $q, q' \in Q$, any $a, a' \in \Sigma$, any $u, u', v, v' \in \{+,-\}$, and any d ∈ {east, west, south, north, up, down, no move}, if $((q',u'),(a',v'),d) \in \delta((q,u),(a,v))$ then $a=a'$, and $(u,v,u',v') \in \{(+,-,+,-),(+,-,-,+),(-,+,-,+),(-,+,+,-),(-,-,-,-)\}$. We call a pair (q,u) in Q×{+,−} an extended state, representing the situation that M holds or does not hold the marker in the finite control according to the sign u = + or u = −, respectively. A pair (a,v) in Σ×{+,−} represents an input tape cell on which the marker exists or does not exsit according to the sign v = + or v = −, respectively. Therefore, the restrictions on δ above imply the following conditions. (A) When holding the marker, M can put it down or keep on holding. (B) When not holding the marker, and (i) if the marker exists on the current cell, M can pick it up or leave it there, or (ii) if the marker does not exist on the current cell, M cannot create a new marker any more (see Fig.2) [3].

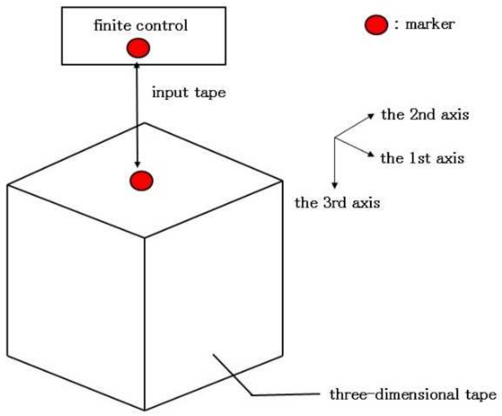

Fig.2:Three-dimensional k marker finite automaton.

3. Recognizability of Connected Tapes by Three-Dimensional Alternating Turing machines

Definition 3.1. Let x be in {0,1}(3). A maximal subset, P of N3 satisfying the following conditions is called a 1-component of x. (1)For any (i1, i2, i3)∈P, we have 1≤i1 ≤l1(x), 1≤i2 ≤l2(x),1≤i3 ≤l3(x), and x(i1,i2,i3)=1. (2)For any (i1, i2, i3), (i0 1, i0 2, i0 3)∈P, there exists a sequence (i1,0, i2,0, i3,0), (i1,1, i2,1, i3,1),···, (i1,n, i2,n, i3,n) of elements in P such that (i1,0, i2,0, i3,0) = (i1, i2, i3), (i1,n, i2,n, i3,n) = (i0 1, i0 2, i0 3), and|i1,j −i1,j−1|+|i2,j −i2,j−1|+|i3,j −i3,j−1|≤1 (1≤j ≤ n). A tape x ∈{0,1}3 is called connected if there exists exactly one 1-compnent of x. We denote the set of all the cubic connected tapes by Tc. It is shown in [3] that a 3-ATM can accept Tc. From this fact and from the fact *L*[FV 3-AFA] ⊇*L*[3-AFA] by using a technique similar to that in [15], the following theorem holds. (3-AFA means 3-ATM without the storage tape and the storage-tape head, FV3-AFA means 3-AFA which cannot move up.)

Theorem 3.1. Tc ∈*L*[FV 3-AFA]. It is shown in [3] that log*m* space is necessary and sufficient for FV 3-ATM's to accept Tc. We below show the necessary and sufficient space for FV 3-SUTM's to accept ¯Tc (=the complement of Tc).

Theorem 3.2. m^2 space is necessary and sufficient for FV 3-ATM's to accept ¯Tc.

Proof: (The proof of sufficiency) It is shown in [3] that Tc is accepted by a deterministic one-way parallel/sequential array acceptor (DOWPS), and it is shown that *L*[DOWPS] = *L*[TR2-DTM(m)] (TR2-DTM(m) means m space-bounded three-way two-dimensional deterministic Turing machine). From these facts and the fact that *L*[TR2-DTM(m)] is closed under complementation, it follows that ¯ Tc is in *L*[TR2DTM(m)], and thus in *L*[TR2-SUTM(m)]. (TR2-SUTM(m) means m space-bounded three-way two-dimensional synchronized alternating Turing machine with only universal states). By applying the same idea of such a two-dimensional case, we can easily get the fact that ¯ Tc is in *L*[FV 3-SUTM(m2)]. (The proof of necessity) Suppose that there is an FV 3-SUTM(*L*(m)) M accepting ¯ Tc, where *L*(m) = o(m^2). We assume without loss of generality that M enters an accepting state only on the bottom boundary. Let T0 c ={x ∈ {0,1}(4m+1,4m+1,4m+1) |m≥1 & ∀i1(1≤i1 ≤m + 1)∀i2(1≤i2 ≤2m + 1) [x [(2i2 −1, 1, 2i1 −1), (2i1 −1, 4m−2i1 + 3, 2i1 −1)], x[(2i2 −1, 1, 4m−2i1 + 3), (2i2 −1, 4m−2i1 + 3, 4m−2i1 + 3)]}, x[(2i1 −1, 4m−2i1 + 3, 2i1 −1), (2i2 −1, 4m−2i1 + 3, 4m−2i1 + 3)]∈{1}(3)] &∀i2(1≤i2 ≤2m) [x [(2i2, 1, 2m + 1), (2i2, 2m + 1, 2m + 1)]∈{1}(3)] &∀i1(1≤i1 ≤2m)∀i2(1≤i2 ≤2m + 1) [x(2i2 −1, 1, 2i1) = x(2i2 −1,1,4m−2i1 + 2)] & (the other part of x consists of 0's)}, where we define ¯0 = 1 and ¯1 = 0. 4).Clearly T0 c ⊆ Tc. Let s and t be the numbers of states (of the finite control) and storage tape symbols of M, respectively. For each m(m≥1), let V(m) ={x∈T0 c |l1(x) = l2(x) = l3(x) = 4m+1}. For each x in V(m), let S(x) and C(x) be sets of configurations of M defined as follows. S(x) = {((i1, i2, 2m + 1), (q, α, k)) | there exists a computation path IM(x) `* M (x, ((i1, i2, 2m), (q0, α0, k0)))`M (x, ((i1, i2, 2m + 1), (q, α, k))) of M on x (that is, (x, ((i1, i2, 2m+1), (q, α, k))) is an ID of M just after the point where the input head left the (2m+1)th plane of x)}, C(x) ={{ρ1,ρ2}|ρ1 and ρ2 are configurations in S(x) such that (i)in case of ρ1 = ρ2, there exists a sequential computation of M which starts with ID(x,ρ1) and either terminates in a rejecting ID, or enters an infinite loop, and (ii)in case of ρ1 ≠ ρ2, there exist two sequential computations of M which start with ID's(x, ρ1) and (x, ρ2), respectively, and terminate in sync ID's with different sync elements}. (Note that, for each x in V(m), C(x) is not empty, since x is not in ¯ Tc, and so not accepted by M.) Then the following proposition must hold.

Proposition 3.1. For any two different tapes x, y∈V(m), C(x)∩C(y)= φ.

[**Proof:** For otherwise, suppose that x6= y(x, y ∈ V(m)), C(x)∩C(y)6= φ, and {ρ1, ρ2} ∈C(x)∩C(y). Let z (with l1(x)=l2(x)=l3(x)=4m + 1) be the tape such that (i)z [(1, 1, 1), (4m + 1, 4m + 1, 2m + 1)] = x[(1,1,1), (4m + 1, 4m + 1, 2m + 1)], and (ii)z [(1, 1, 2m + 2), (4m + 1, 4m + 1, 4m + 1)] = y[(1, 1, 2m + 2), (4m + 1, 4m + 1,4m + 1)]. Since {ρ1,ρ2}∈ C(x), there exist computation paths I_M(z) ├* M (z,ρ1) and IM(z) ├* M (z,ρ2). Since{ρ1,ρ2}∈C(y), in case of ρ1=ρ2, there exists a sequential computation of M which starts with the ID (z,ρ1) and either terminates in a rejecting ID, or enters an infinite loop, and in case of ρ1 6= ρ2, there exist two sequential computations of M which start with ID's (z, ρ1) and (z, ρ2), respectively, and terminate in sync ID's with different sync elements. This means that z is not accepted by M. This contradicts the fact, that z is in $\overline{T}c$ = T(M). □]

Proof of Theorem 3.2(continued): Let p(m) denote the number of pairs of possible configurations of M just after the point where the input head left the (2m+1)th planes of tapes in V(m). Then p(m) $={}_{K}C_{2}+K$ where K = s(4m + 3)^2 L(4m + 1)$t^{L(4m+1)}$. On the other hand, |V(m)| = 2m(2m+1). Since L(m) = o(m), we have|V(m)|≥ p(m) for large m. Therefore, it follows that for large m there must be two different tapes x, y in V(m) such that C(x) ∩ C(y) $\neq \phi$. This contradicts Proposition 6.1 and completes the proof of necessity. □

4. Recognizability of Three-Dimensional Arbitrarily Shaped Tapes by k Marker Finite Automata

Let Σ(3) be a set of points in the three-dimensional Euclidean space with integer coordinates. Each point in Σ(3) is called a vertex, Each unit-length segment connecting two vertices is called an edge. Each region of unit area enclosed by twelve edges is called a voxel. Each voxel can have an input symbol '0' or '1', or a boundary symbol '#'. A voxel is called 0-voxel (1-voxel, or #-voxel) if it has symbol 0 (1, or #). Two-voxels are 6-adjacent (or 27-adjacent) if they share a common edge (or a common vertex) [3]. A 6-adjacent (or 27-adjacent) path is a sequence of voxels c(1),c(2),...,c(i) such that for each 1 ≤ j ≤ i−1, c(j) and c(j + 1) are 6-adjacent (or 27-adjacent). A three-dimensional arbitrarily shaped pathwise-connected tape (p−tape) T is a set of 0, 1-voxels surrounded by #-voxels, where any two 0,1-voxels in T are connected by a 6-adjacent path with only 0,1-voxels in T. (Note that T can contain some 'holes' in its inside.) the pattern P on a p-tape T is the set of all 1voxels that appear there. For the pattern P on a p-tape, a 1-component C is any maximal set of 1-voxels such that any 1-voxels in C are connected by a 6-adjacent path with only 1-voxels in C. A pattern P is connected if and only if any two 1-voxels are connected by a 6-adjacent path with only 1-voxels in P. That is, P is connected if and only if there exists only one 1-component. A k-marker finite automaton M(k) consists of a finite control with a read-only input head and k (labelled) markers operating on a p-tape T. M(k) is started on a 0,1-voxel in its start state with carrying its markers . The markers can be placed on or collected back to the finite control from only the voxel the input head is currently scanning. In each step, M(k) can change its internal state, place a marker 'carried' by the finite control (or collect back a marker (if it exists) to the finite control) on (or from) the voxel the input head is currently scanning, and move the input head to a 6-adjacent cell, according to the current state, the symbol and the presence of marker on the voxel currently scanned by the input head. M(k) is called deterministic if its next-move function is deterministic, otherwise it is called nondeterministic. We assume that M(k) can visit any #-voxel which is 6-adjacent to some 0,1-voxel in T, but can never fall off the tape T beyond these #-cells.

By using the same technique as in the proof of Theorem 3.1 in [1], we get the result.

Theorem 4.1. whether or not the pattern on a p-tape is connected can be decided by a deterministic three marker finite automaton.

It is shown in [1] that there is no deterministic one marker finite automaton which is able to search all mazes (i.e., p-tapes). Moreover, it is shown in [3] that the set of all three-dimensional connected tapes is not recognizable by any three-dimensional nondeterministic multi-inkdot finite automaton (an inkdot machine is a conventional machine capable of dropping an inkdot on a given input tape for a landmark, but unable to further pick it up[3]). This result means that whether or not the pattern on a p-tape is connected cannot be decided by any deterministic one marker finite automaton.

5. Conclusion

In this paper, we considered about recognizability of three-dimensional patterns by some three-dimensional automata. It is an open problem whether the set of all the three-dimensional connected tapes is not accepted by any three-dimensional nondeterministic Turing machine with spaces of size smaller than log*m*, and by any three-dimensional alternating one marker finite automaton with only universal states.

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