Neutral Networks-Based Adaptive Fixed-Time Consensus Tracking Control for Uncertain Multiple AUVs

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Abstract

This paper is concerned with the fixed-time consensus tracking problem for multi-AUV (autonomous underwater vehicle) systems with uncertain parameters and external disturbances. Firstly, a fixed-time terminal sliding mode is proposed, which can avoid the singularity problem. Then, a continuous distributed consensus tracking control law is designed based on Neutral Network approximation technique, which can guarantee the consensus tracking errors converge to the desired regions in fixed time. A simulation example is given to show the effectiveness of proposed methods.

Keywords: Multi-AUV systems; Terminal sliding mode; fixed-time stability; Neutral Networks.

1. Introduction

Distributed cooperative control of multiple AUVs has been paid to much attention due to its potential applications in oceanographic surveys and deep sea inspections [1]. The distributed cooperative control for multi-AUV systems has been investigated by using the backstepping technique [2] and the adaptive control approach [3]. However, the protocols proposed in them can only guarantee the closed-loop system is asymptotically stable. For the distributed cooperative control, one significant requirement is the fast convergence rate. Compared with the asymptotic control approaches, the finite-time control approaches can not only provide fast convergence rate but also provide higher tracking precision and better disturbance rejection ability [4]. Therefore, many finite-time control laws are proposed for various multi-agent systems in the past few years [5]–[7]. However, the settling time can be estimated dependent on the initial conditions of systems in there. In practical applications, we desire that the settling time is estimated independent on the initial conditions of systems. In this paper, we will further investigate the adaptive finite-time consensus tracking problem for multiple AUVs with uncertain dynamics using fixed-time terminal sliding mode.

2. Systems Description

This paper considers the networked multiple AUV system with *n* following AUVs and one virtual leader, and the communications among them are described by a digraph $\bar{G}$. The definitions and descriptions of graph $\bar{G}$ are given in [7] and [9], which is omitted for brevity. Assume that all the following AUVs have fixed attitudes. The translational dynamics of the *i*-th AUV ($i \in V$) are given as [10]:

$$\begin{aligned} \dot{p}_i &= R_i(\Theta_i) v_i \\ M_i \dot{v}_i &= -D_i(v_i) v_i - g_i(\Theta_i) + \tau_i + w_i \end{aligned} \tag{1}$$

where $p_i = [x_i, y_i, z_i]^T, \Theta_i = [\phi_i, \theta_i, \psi_i]^T$ denote position and attitude vectors in the inertial reference frame, respectively, $R_i(\Theta_i)$ is the kinematic transformation matrix, $v_i = [u_i, v_i, \omega_i]^T$ is translational velocity vector in the body-fixed reference frame, M_i is the inertia matrix, $D_i(v_i)$ is the damping matrix, $g_i(\Theta_i)$ is there storing force vector, $\tau_i \in \Re^3$ is the control force vector, and $w_i \in \Re^3$ is the disturbance force vector. M_i, $R_i(\Theta_i)$, $D_i(v_i)$, $g_i(\Theta_i)$ are defined in [10]. In this paper, we assume that $D_i(v_i)$ and $g_i(\Theta_i)$ have uncertain parameters. Note that $R_i R_i^T = I$ .Denote $p_d \in \Re^3$ as the state vector of virtual leader and $\dot{p}_d$, $\ddot{p}_d$ are all assumed to be smooth, bounded and known functions.

Assumption 1. *$\bar{G}$ has a spanning tree, and the leader node is the root node.*

3. Main results

3.1. *Fixed-time terminal sliding mode (FTTSM)*

Denote

$$\begin{aligned} e_{1i} &= \sum_{j=1}^{n} a_{ij}(p_i - p_j) + b_i(p_i - p_d) \\ e_{2i} &= \sum_{j=1}^{n} a_{ij}(\dot{p}_i - \dot{p}_j) + b_i(\dot{p}_i - \dot{p}_d) \end{aligned} \tag{2}$$

Then, we have $e_1 = (H \otimes I)\mathrm{E}_1, e_2 = (H \otimes I)\mathrm{E}_2$, where $e_1 = [e_{11}^T, \ldots, e_{1n}^T]^T$, $e_2 = [e_{21}^T, \ldots, e_{2n}^T]^T$, $\mathrm{E}_1 = [\mathrm{E}_{11}^T, \ldots, \mathrm{E}_{1n}^T]^T$, $\mathrm{E}_2 = [\mathrm{E}_{21}^T, \ldots, \mathrm{E}_{2n}^T]^T$, $\mathrm{E}_{1i} = p_i - p_d$, $\mathrm{E}_{2i} = \dot{p}_i - \dot{p}_d$.

Now, define the FTTSM vector as $s = [s_1^T, \ldots, s_n^T]^T$, where $s_i = [s_{i1}, s_{i2}, s_{i3}]^T \in \Re^3$ is given by

$$s_i = e_{2i} + \alpha_i(e_{1i}) \tag{3}$$

with $\alpha_i(e_{1i}) = [\alpha_i(e_{1i1}), \alpha_i(e_{1i2}), \alpha_i(e_{1i3})]^T$,and

$$\alpha_{i\chi}(e_{1i\chi}) = \begin{cases} \mathrm{sig}\left(\sigma_{1i}\mathrm{sig}(e_{1i\chi})^{m_1} + \sigma_{2i}\mathrm{sig}(e_{1i\chi})^{n_1}\right)^{k_1}, \\ \text{if } \bar{s}_{i\chi} = 0 \text{ or } \bar{s}_{i\chi} \neq 0, |e_{1i\chi}| > \phi \\ l_{1i} e_{1i\chi} + l_{2i}\mathrm{sig}(e_{1i\chi})^2, \text{if } \bar{s}_{i\chi} \leq 0, |e_{1i\chi}| \leq \phi \end{cases} \tag{4}$$

$\chi = 1,2,3, \bar{s}_i = [\bar{s}_{i1}, \bar{s}_{i2}, \bar{s}_{i3}]^T, \bar{s} = \mathrm{sig}(\sigma_{1i}\mathrm{sig}(e_{1i\chi})^{m_1} + \sigma_{2i}\mathrm{sig}(e_{1i\chi})^{n_1})^{k_1}, m_1, n_1, k_1 \in \Re^+, 0 < m_1 k_1 < 1, n_1 k_1 > 1,$ $l_{1i} = (2 - k_1)(\sigma_{1i}\phi^{m_1 - \frac{1}{k_1}} + \sigma_{2i}\phi^{n_1 - \frac{1}{k_1}})^{k_1}, l_{2i} = (k_1 - 1)$ $(\sigma_{1i}\phi^{m_1 - \frac{2}{k_1}} + \sigma_{2i}\phi^{n_1 - \frac{2}{k_1}})^{k_1}, \phi > 0.$

From (3), we can obtain the following equation

$$\dot{s}_i + s_i = \dot{e}_{2i} + \dot{\alpha} + e_{2i} + \alpha_i \tag{5}$$

From the definition of e_{2i}, we further have

$$\dot{e}_{2i} = (d_i + b_i)\ddot{p}_i - \sum_{j=1}^{n} a_{ij}\ddot{p}_j - b_i \ddot{p}_d \tag{6}$$

and from (1), we can obtain

$$\ddot{p}_i = h_i + R_i(\Theta_i) M_i^{-1} \tau_i + R_i(\Theta_i) M_i^{-1} w_i \tag{7}$$

where $h_i = \dot{R}_i v_i - R_i M_i^{-1} D_i v_i - R_i M_i^{-1} g_i$. Assume that $\|(d_i + b_i) R_i(\Theta_i) M_i^{-1} w_i\| \leq \bar{w}_i^*, \bar{w}_i^*$ is an unknown constant. Then, substituting (6) into (5) yields

$$\dot{s}_i + s_i = (d_i + b_i) R_i(\Theta_i) M_i^{-1} \tau_i + (d_i + b_i) R_i(\Theta_i) M_i^{-1} w_i + \Phi_i \tag{8}$$

where $\Phi_i = -\sum_{j=1}^{n} a_{ij}\ddot{p}_j - b_i \ddot{p}_d + \dot{\alpha} + e_{2i} + \alpha_i + (d_i + b_i) h_i$.

3.2. *Control law design*

From the approximation property of RBF Neutral Networks (NNs), we have

$$\Phi_i = W_i^T \Gamma_i\left(Z_i\right) + \varsigma_i \tag{9}$$

where $Z_i = [p_i^T, \dot{p}_i^T, p_j^T, \dot{p}_j^T, \ddot{p}_j^T, p_d^T, \dot{p}_d^T, \ddot{p}_d^T]^T$ and $\|\xi_i\| \le \xi_i^*$, $\xi_i^* > 0$ is a constant. Denote $\hat{\rho}_i$ as the estimate of $\rho_i = \|W_i\|^2$, then the adaptation law is designed as

$$\dot{\hat{\rho}}_i = -2\kappa_i \iota_i \hat{\rho}_i + \frac{\kappa_i}{2h_i^2} s_i^T s_i \Gamma_i^T \Gamma_i \tag{10}$$

where κ_i, ι_i, h_i are designed positive constants.

Theorem 1. *Suppose that Assumption 1 holds for system (1), then we can choose the control law*

$$\tau_i = -\frac{1}{d_i + b_i} M_i R_i^T \left(\mu_{i1} \mathrm{sig}\left(s_i\right)^{m_2} + \mu_{i2} \mathrm{sig}\left(s_i\right)^{n_2} + \frac{1}{2h_i^2} \hat{\rho}_i \Gamma_i^T \Gamma_i s_i \right) \tag{11}$$

where $\mu_{i1} > 0, \mu_{i2} > 0, 0 < m_2 < 1, n_2 > 1$, *such that* s_i *converges into the region*

$$\|s_i\| \le \Delta_s = \min\{2\mu_1^{-\frac{1}{1+m_2}} \left(\frac{\Xi}{1-\Xi_0}\right)^{\frac{1}{1+m_2}}, 2\left(\mu_2\left(\frac{1}{2^{\frac{n_2+1}{2}}-1}\right)^n\right)^{-\frac{1}{1+n_2}} \left(\frac{\Xi}{1-\Xi_0}\right)^{\frac{1}{1+n_2}}\}$$

in fixed time, the local neighborhood state errors $e_{1i\chi}$ *and* $e_{2i\chi}, \chi = 1,2,3$ *converge into the regions* $\Delta_{e_{1i}}$ *and* $\Delta_{e_{2i}}$ *respectively in fixed time, and finally the vectors* E_1 *and* E_2 *converge into regions* Δ_{E_1} *and* Δ_{E_2} *respectively in fixed time, where*

$$\Delta_{e_{1i}} = \max\{\phi, (\frac{(\Delta_s)^{\frac{1}{k_1}}}{\sigma_{1i}})^{\frac{1}{m_1}}, (\frac{(\Delta_s)^{\frac{1}{k_1}}}{\sigma_{2i}})^{\frac{1}{n_1}}\},$$

$$\Delta_{e_{2i}} = \max\{\Delta_s + l_{1i}\Delta_{e_{1i}} + l_{2i}\Delta_{e_{1i}}^2, \Delta_s + (\sigma_{i1}\Delta_{e_{1i}}^{m_1} + \sigma_{2i}\Delta_{e_{1i}}^{n_1})^{k_1}\},$$

$$\mu_1 = \min\{\mu_{1\min} 2^{\frac{m_2+1}{2}}, \varsigma_{\min^{\frac{m_2+1}{2}}} 2^{\frac{m_2+1}{2}}\}, \mu_{1\min} = \min\{\mu_{1i}\},$$

$$\varsigma_{\min} = \min\{\varsigma_i\}, \mu_2 = \min\{\mu_{2\min} 2^{\frac{n_2+1}{2}}, \varsigma_{\min^{\frac{n_2+1}{2}}} 2^{\frac{n_2+1}{2}}\},$$

$$\mu_{2\min} = \min\{\mu_{2i}\}, \varsigma_i = \kappa_i \frac{\rho_i(2o_i - 1)}{2o_i},$$

$$o_i > \frac{1}{2}, 0 < \Xi_0 \le 1, \Delta_{\mathrm{E}_1} = \frac{\sqrt{3\sum_{i=1}^n \Delta_{e_{1i}}^2}}{\sigma_{\min}(H)}, \Delta_{\mathrm{E}_2} = \frac{\sqrt{3\sum_{i=1}^n \Delta_{e_{2i}}^2}}{\sigma_{\min}(H)}.$$

Proof. Denote $\tilde{\rho}_i = \rho_i - \hat{\rho}_i$, and choose the Lyapunov function as

$$V = \frac{1}{2} S^T S + \frac{1}{2} \sum_{i=1}^n \frac{1}{\kappa_i} \tilde{\rho}_i^2 \tag{12}$$

we have

$$\dot{V} \le -\sum_{i=1}^n s_i^T s_i + \sum_{i=1}^n s_i^T \left(d_i + b_i\right) R_i M_i^{-1} \tau_i + \sum_{i=1}^n \overline{w}_i^* \|s_i\| + \sum_{i=1}^n s_i^T \Phi_i \tag{13}$$

From

$$s_i^T \Phi_i \le \frac{1}{2h_i^2} s_i^T s_i \|W_i\|^T \Gamma_i^T \Gamma_i + \frac{1}{2} h_i^2 + \frac{1}{2} s_i^T s + \frac{1}{2} \xi_i^{*,2}$$

$$\overline{w}_i^* \|s_i\| \le \frac{1}{2} s_i^T s + \frac{1}{2} \overline{w}_i^{*,2} \tag{14}$$

Substituting (10), (11), (14) into (13) yields

$$\dot{V} \le -\sum_{i=1}^n \sum_{\chi=1}^3 \mu_{i1} \left|s_{i\chi}\right|^{m_2+1} - \sum_{i=1}^n \sum_{\chi=1}^3 \mu_{i2} \left|s_{i\chi}\right|^{n_2+1} + \sum_{i=1}^n 2\iota_i \tilde{\rho}_i \hat{\rho}_i + \frac{1}{2} \sum_{i=1}^n \left(\overline{w}_i^{*,2} + \xi_i^{*,2}\right) \tag{15}$$

Using the similar proof as in [9], we have

$$\dot{V} \le -\mu_1 V^{\frac{m_2+1}{2}} - \mu_2 \left(\frac{1}{2^{\frac{n_2+1}{2}}-1}\right)^n V^{\frac{n_2+1}{2}} + \sum_{i=1}^n \left(\frac{\varsigma_i}{\kappa_i} \tilde{\rho}_i^2\right)^{\frac{m_2+1}{2}} - \sum_{i=1}^n \frac{\varsigma_i}{\kappa_i} \tilde{\rho}_i^2 + \sum_{i=1}^n \iota_i o_i \rho_i^2 + \frac{1}{2} \sum_{i=1}^n \left(\overline{w}_i^{*,2} + \xi_i^{*,2}\right) \tag{16}$$

Suppose that there exists a compact set Υ such that $\Upsilon = \left\{\tilde{\rho}_i \mid \left|\tilde{\rho}_i\right| \le \Delta\right\}$, then we have

$$\Xi = \begin{cases} \sum_{i=1}^n \iota_i o_i \rho_i^2 + \frac{1}{2} \sum_{i=1}^n \left(\overline{w}_i^{*,2} + \xi_i^{*,2}\right), \text{if } \Delta < \min\left\{\sqrt{\frac{\kappa_i}{\varsigma_i}}\right\} \\ \sum_{i=1}^n \iota_i o_i \rho_i^2 + \sum_{i=1}^n \left(\frac{\varsigma_i}{\kappa_i} \Delta^2\right)^{\frac{m_2+1}{2}} - \sum_{i=1}^n \frac{\varsigma_i}{\kappa_i} \Delta^2 + \\ \frac{1}{2} \sum_{i=1}^n \left(\overline{w}_i^{*,2} + \xi_i^{*,2}\right), \text{if } \Delta \ge \min\left\{\sqrt{\frac{\kappa_i}{\varsigma_i}}\right\} \end{cases} \tag{17}$$

From (16) and (17), we can further obtain

$$\dot{V} \le -\mu_1 V^{\frac{m_2+1}{2}} - \mu_2 \left(\frac{1}{2^{\frac{n_2+1}{2}} - 1} \right)^n V^{\frac{n_2+1}{2}} + \Xi \qquad (18)$$

It can be seen from Lemma 2 in [8] that the system (12) is practical fixed-time stability. Moreover, s_i will converge into the region $\|s_i\| \le \Delta_s$ in fixed settling time. The next proofs are similar with that of [7] and [9], thus are omitted for brevity.

4. Simulations

We consider a direct network with three AUVs and a virtual leader, the matrices L and B are described as:

$$L = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

We assume that all the AUVs have the same structure and the model parameters are $M_i = \mathrm{diag}\{175.4, 140.8, 140.8\}$, $D_i = \{120 + 90|u_i|, 90 + 90|v_i|, 150 + 90|\omega_i|\}, \phi_i = \pi/5, \theta_i = -\pi/10, \psi_i = \pi/12$ [10]. The response curves under control law (11) are shown in Fig. 1. Note that the control law (11) can ensure the closed-loop system has desired robustness.

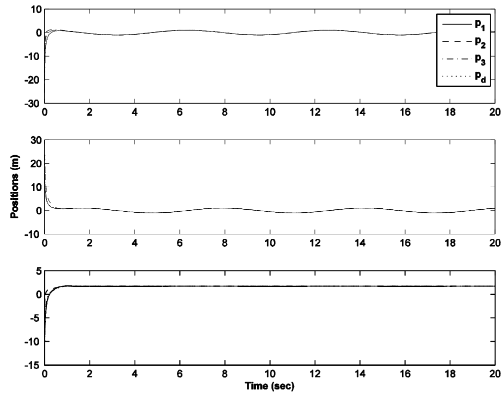

Fig. 1. Response curves of p_d and p_i $(i=1,2,3)$ under control law (11).

5. Conclusions

This paper studied the fixed-time consensus tracking control of Multiple AUVs. A FTTSM based adaptive chattering-free control law was designed, which could guarantee the closed-loop system had desired fixed-time tracking performance.

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