Automated Color Image Arrangement Method Using Curvature Computation in Histogram Matching

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Abstract

This paper proposes improvement color image arrangement method by using curvature computation. The previous paper, we have presented the principle of our method using Histogram Matching based on Gaussian Distribution (HMGD), and how to detect input color image peakedness in its histogram. In this paper, we describe about Variance Estimated HMGD (VE-HMGD) as improvement HMGD. We also show how to estimate the histogram variance of original image based on the curvature computation. Moreover we compare processing results between VE-HMGD and HMGD through some experimentation. As the result, we show that VE-HMGD is natural color than HMGD.

Keywords: Automated color arrangement, histogram matching, HMGD, curvature variance estimation

1. Introduction

Automated image processing for enhancement and/or arrangement of color images has been more familiar to us according to the spreading of Digital Camera, Smart Phone, DVD, etc.[1-3]. However, we consider that the research on the automated arrangement method that brings about good sensibility effect (or Kansei effect) is still on the way to practical use.

In the previous papers, we have proposed a novel color image automated arrangement method using an elastic transform (ET) [4, 5]. And we have proposed a principle of Histogram Matching based on Gaussian Distribution (HMGD) on brightness axis as one of the ET method based on histogram and/or histogram matching. Then we also described how to detect input color image peakedness in its histogram by using curvature computation for automated HMGD processing.

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Furthermore, through experiments, we found that HMGD processing is improved the image of Kansei impression if applied to single-peakedness image.

In this paper, first, we explain the principle of HMGD. Second, we explain the method of histogram variance estimation to optimize the shape of the HMGD-reference histogram by using curvature computation.

And then, we illustrate the results of improved HMGD (Variance Estimated HMGD; VE-HMGD) processing which is working better than previous HMGD processing, through some experiments.

2. Principle of HMGD

In this chapter, we describe the principle of HMGD based elastic transform. Let $f(x)$ and $g(y)$ be two probabilistic density functions on real variables x and y, respectively. The probabilistic density function (PDF) is corresponding to histogram of image brightness level which is discretely defined. In addition, let $y=\phi(x)$ be a continuous and monotonous increase function between variables x and y as shown in Fig.1[7-9].

In addition let value of x be the range from 0 to L, therefore, variable y is ranges from 0 to $\phi(L)$. And let P mean the probability. From above definition and Fig. 1, we can derive (1) ~ (3).

$$P(0 \le x \le L) = \int_{x=0}^{x=L} f(x)dx = 1 \tag{1}$$

$$P(0 \le y \le \phi(L)) = \int_{y=0}^{y=\phi(L)} g(y)dy = 1 \tag{2}$$

$$\begin{aligned} f(x)dx &= P(x_0 \le x \le x_0 + dx) \\ &= P(\phi(x_0) \le y \le \phi(x_0 + dx)) \\ &= P(y_0 \le y \le y_0 + dy) = g(y)dy \end{aligned} \tag{3}$$

From (3), we obtain (4) and (5) because $y_0 = \phi(x_0)$ and $y_0 + dx = \phi(x_0 + dx)$.

$$f(x)dx = g(y)dy = g(y)\phi'(x)dx \tag{4}$$

$$f(x) = g(y)\phi'(x) \tag{5}$$

Thus, if we know $y=\phi'(x)$ and $g(y)$, then we have $f(x)$.

Using the above equations, we derive the principle of HMGD.

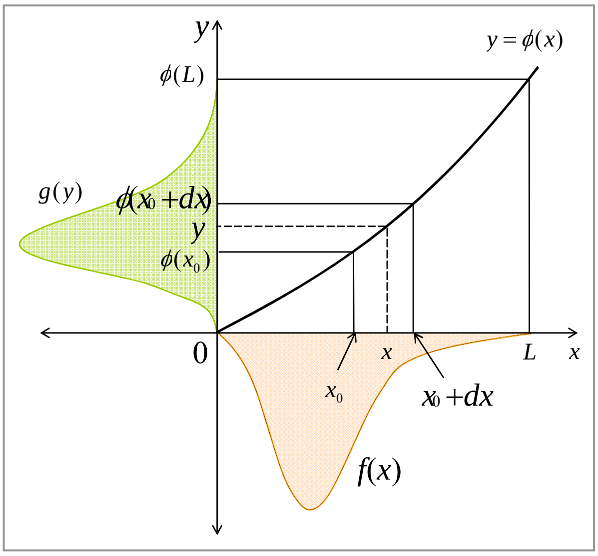


Fig.1. Continuous and monotonous increase function $y=\phi(x)$ and probabilistic density functions $f(x)$ and $g(y)$[7-9].

HMGD processing converts the original image histogram into the Gaussian image histogram as shown in Fig. 2. First, because we aim to match Gaussian distribution image histogram, we have to expand brightness level of original image, due to convert into uniform distribution histogram. Let $\phi(x)$ be defined by (6).

$$y = \phi(x) = L\int_0^x f(x)dx \tag{6}$$

Since $\phi'(x) = Lf(x)$, according to (5) and Fig. 2., we derive (7).

$$f(x) = h(y)\phi'(x) = h(y)Lf(x) \tag{7}$$

Therefore, we can derive (8)

$$h(y)L = 1, \quad h(y) = 1/L \tag{8}$$

From (8), we understand that original image histogram $f(x)$ becomes uniform distribution histogram $h(y)$ and expands brightness level.

Similarly, let $Gauss(z)$ and $\gamma(z)$ be defined by (9) and (10), respectively.

$$Gauss(z) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(z-\mu)^2}{2\sigma^2}\right) \tag{9}$$

$$\gamma(z)=L\int_0^x Gauss(z)dz=\frac{L}{\sigma\sqrt{2\pi}}\int_0^z \exp\left(-\frac{(z-\mu)^2}{2\sigma^2}\right) \quad (10)$$

Then, we obtain (11) because $\phi(x)=\gamma(z)$ from Fig. 2.

$$L\int_0^x f(x)dx=\frac{L}{\sigma\sqrt{2\pi}}\int_0^z \exp\left(-\frac{(z-\mu)^2}{2\sigma^2}\right) \quad (11)$$

And, we can derive (12) from (6) and (10) by differentiating both sides of (11).

$$\frac{d}{dx}L\int_0^x f(x)dx=\frac{d}{dz}\frac{L}{\sigma\sqrt{2\pi}}\int_0^z \exp\left(-\frac{(z-\mu)^2}{2\sigma^2}\right)$$
$$L\phi'(x)=L\gamma'(z) \quad f(x)=Gauss(z) \quad (12)$$

Therefore, we understand that if we take the transform function as (6) and (10), $f(x)$ becomes Gaussian distribution. It corresponds to the HMGD processing, which means that function defined by cumulative histogram transforms the original histogram into the Gaussian histogram.

3. Variance Estimated HMGD (VE-HMGD)

3.1. *Approximate of Curvature Computation*

In this section, we describe how to calculate the curvature approximation using for variance estimation. Let y be a function with respect to x, the definition of the curvature R is given by (13).

$$R=\left(\frac{d^2y}{dx^2}\right)\Big/\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}} \quad (13)$$

$$g(x)=\frac{K}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-a)^2}{2\sigma^2}\right) \quad (14)$$

In (14), K means a coefficient that satisfies (15).

$$\frac{K}{\sigma\sqrt{2\pi}}\int_0^L \exp\left(-\frac{(u-a)^2}{2\sigma^2}\right)du=1 \quad (15)$$

Then, let $y=f(x)$ be a function representing the cumulative histogram. So $f(x)$ can be represented (16). Since $y=f(x)=\int_0^x g(u)du$, $g(x)$ can be described as (17).

$$f(x)=\int_0^x g(u)du=\frac{K}{\sigma\sqrt{2\pi}}\int_0^x \exp\left(-\frac{(u-a)^2}{2\sigma^2}\right)du \quad (16)$$

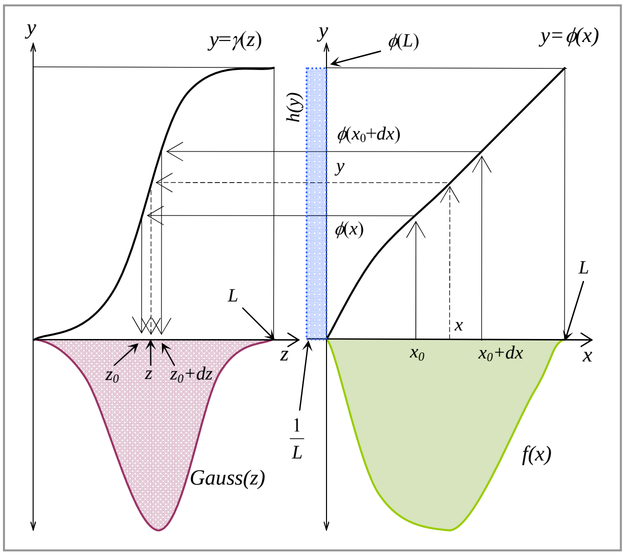


Fig. 2. Conceptual image of HMGD.

$$\frac{dy}{dx}=g(x)=\frac{K}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-a)^2}{2\sigma^2}\right) \quad (17)$$

By the same way,

$$\frac{d^2y}{dx^2}=\frac{dg(x)}{dx}=\frac{K}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{(x-a)^2}{2\sigma^2}}\cdot\left(-\frac{1}{2\sigma^2}\right)\{2(x-a)\}$$

$$=\left(\frac{K}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{(x-a)^2}{2\sigma^2}}\right)\left(-\frac{(x-a)}{\sigma^2}\right)=g(x)\cdot\left(\frac{(a-x)}{\sigma^2}\right) \quad (18)$$

Hence, we obtain the following (19) and we can approximate to (13).

$$R=\frac{\frac{(a-x)}{\sigma^2}g(x)}{\{1+g(x)^2\}^{\frac{3}{2}}}\cong\frac{(a-x)}{\sigma^2}g(x) \quad (19)$$

The curvature R varies the sign according to the value of x;

- $x<a \rightarrow R>0$
- $x=a \rightarrow R=0$
- $x>a \rightarrow R<0$.

3.2. *Variance Estimation of image histogram*

In this section, we describe how to estimate the histogram variance σ^2 of the original image by using curvature computation optimize the shape of the reference histogram, which is used in the VE-HMGD processing. Fig. 3 and Fig. 4 shows a conceptual image of the original histogram and cumulative histogram of variance σ^2 and average a, respectively.

Let $g(a)$ be a Gauss density function with variance σ^2 at average a. From (14),

$$g(a)=\frac{K}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(a-a)^2}{2\sigma^2}\right)=\frac{K}{\sigma\sqrt{2\pi}} \qquad (20)$$

Thus, we can describe $g\left(a\pm\sqrt{2}\sigma\right)$ as follows.

$$g(a\pm\sqrt{2}\sigma)=\frac{K}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(a\pm\sqrt{2}\sigma-a)^2}{2\sigma^2}\right)=g(x)e^{\mp1} \qquad (21)$$

Let $R\left(a\pm\sqrt{2}\sigma\right)$ be curvatures at $a\pm\sqrt{2}\sigma$. From (19), we describe these curvatures in (22).

$$R\left(a\pm\sqrt{2}\sigma\right)=\frac{\dfrac{\left(a-\left(a\pm\sqrt{2}\sigma\right)\right)}{\sigma^2}g(x)}{\left\{1+g(a\pm\sqrt{2}\sigma)^2\right\}^{\frac{3}{2}}} \qquad (22)$$

From (21),

$$R\left(a\pm\sqrt{2}\sigma\right)=\frac{\left(\mp\frac{\sqrt{2}}{\sigma}\right)g(a+\sqrt{2}\sigma)}{\left\{1+g(a+\sqrt{2}\sigma)^2\right\}^{\frac{3}{2}}}$$

$$=\left(\mp\frac{\sqrt{2}}{\sigma}\right)\frac{g(a)e^{-1}}{\left\{1+(g(a)e^{-1})^2\right\}^{\frac{3}{2}}}\equiv\left(\mp\frac{\sqrt{2}}{\sigma}\right)H \qquad (23)$$

In (23), H is a constant that means following (24).

$$H=\frac{g(a)}{e}\Bigg/\left\{1+\left(\frac{g(a)}{e}\right)^2\right\}^{\frac{3}{2}} \qquad (24)$$

Hence, we can obtain reference histogram variance σ^2 by using (23). In the below equation $v=\sqrt{2}\sigma$, and v means distance from average a.

$$R(a-v)-R(a+v)=\frac{2\sqrt{2}}{\sigma}H=\left(\frac{4}{v}\right)H \qquad \sigma^2=\frac{v^2}{2} \qquad (25)$$

3.3. *Process of Variance Estimated HMGD*

In the previous section, we have described the method to estimate the optimum variance in reference histogram

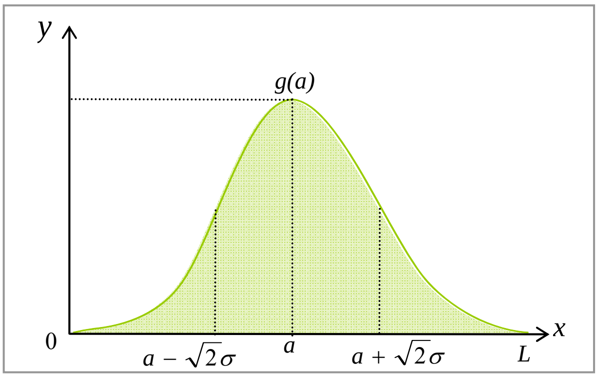


Fig. 3. Conceptual image of the original image histogram of variance σ^2 and average a.

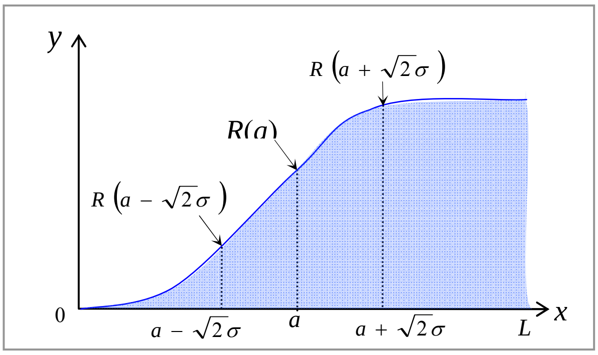


Fig. 4. Conceptual image of the original image cumulative histogram of variance σ^2 and average a.

of HMGD from the histogram of the image, by using curvature computation. In this section, we describe the process of Variance Estimated HMGD (VE-HMGD).

[Step 1] Proceed to Step 3 if input image has single peakedness histogram after performing curvature calculation. Otherwise go to Step 2.

[Step 2] Abort VE-HMGD, and outputting inputted image in Step 1.

[Step 3] Detect the peakedness brightness value of the histogram a. Proceed to Step 5 if the curvature $R(a)$ is obtained. Otherwise go to Step 4.

[Step 4] Set σ=50 (default value), and perform HPA-HMGD[7, 8]. Then proceed to Step 11.

[Step 5] Calculate curvature $R\left(a\pm\sqrt{2}\sigma\right)$ according to (23) and (24). Then, calculate following (26).

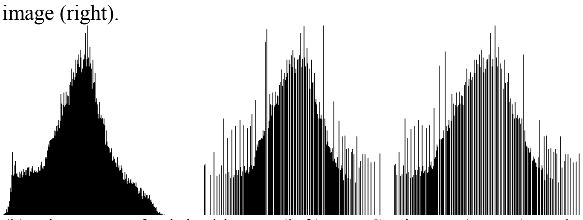
$$\left|\,R\left(a-\sqrt{2}\sigma\right)-R\left(a+\sqrt{2}\sigma\right)\right|-\left|\left(2\sqrt{2}H/\sigma\right)\right|\approx 0 \qquad (26)$$

[Step 6] Proceed to Step 7 if $\sqrt{2}\sigma$ satisfies (26). Otherwise modified $\sqrt{2}\sigma$ and return to Step 6.
[Step 7] Calculate variance σ^2 according to (25).
[Step 8] Calculate the reference histogram by using variance σ^2 and average a. Then perform HMGD.
[Step 9] Output the processed image.

4. Experimentation

Fig. 5 (a) shows example of results of HMGD and VE-HMGD processing which the original image have single peakedness histogram. The set of histogram and cumulative histogram (original image, HMGD image, and VE-HMGD image) are shown in Fig. 5 (b) and (c), respectively. In this case, VE-HMGD have a good feeling impression (or Kansei effect) than HMGD. And we found that shading of VE-HMGD is enhancing than HMGD.


(a) Original image[6, 9] (left), HMGD image[6, 9] (center), and VE-HMGD image (right).


(b) Histogram of original image (left), HMGD image (center), and VE-HMGD image (right).


(c) Cumulative histogram of original image (left), HMGD image (center), and VE-HMGD image (right).

Fig. 5. Example of results by HMGD, VE-HMGD, and the corresponding histograms.

5. Conclusion

Aiming at improvement Histogram Matching based on Gaussian Distribution (HMGD), we have described principal of HMGD, and we have proposed a method for estimate variance of original image histogram. As for concrete method, we have applied the curvature computation to estimation variance.
Furthermore, we suggested the VE-HMGD as improvement of HMGD and described its processing sequence. Through experimentation, we consider that VE-HMGD processing method will be useful and promising than previous HMGD. For further study, we have to verify the effect of VE-HMGD through questionnaire survey.

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