Design of Sliding Mode Controller for Droplet Position in EWOD Microfluidic System

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Abstract

In microfluidic lab-on-chip devices, electrowetting on dielectric (EWOD) are widely used for various applications. To manipulate the micro droplet to achieve the desired path and accurate target position by using electrowetting technique are one of the common applications. In this paper, the motion of droplet is modeled as a single rigid body driven by both linear and nonlinear forces. In order to evaluate the potential of controller, the sliding mode controller is applied to this nonlinear microfluidic system. The effect of bounded disturbances is included in the designed controller. Simulation results provided the feasibility of the sliding mode controller for EWOD microfluidic manipulation under the effect of bounded disturbances.

Keywords: Sliding mode control, Electro-wetting on Dielectric device, Lab-on-chip, Microfluidics.

1. Introduction

Electro-wetting on Dielectric (EWOD) is a microfluidic transport method utilizing the disturbance behavior of electrical field on the free surface energy and wet contact angle of a droplet. As shown in Fig. 1, the contact angle and droplet shape can be changed by applying voltages on one-side of a droplet, thus inducing the droplet to move. The technique is well-known for its precise manipulation of droplet movement. Currently, many lab-on-chip devices, equipped with EWOD micro-droplet transport are widely used for various applications such as in DNA sequencing [1], protein analysis and detection [2], Disease diagnosis [3, 6] molecular biology processes [4], detection of triglyceride in human fluid [5] and concentration detection of L-amino acid [7]. With its immense capability to precisely transport tiny droplets with little power consumption, EWOD technique has also been used in micro pimps [8] and micro-conveyors [9]. Pamula *et al.* [10] created a micro-cooling system for IC circuit cooling, in which EWOD required less pumping power than typical mechanical pumps. In dealing with complicated analysis, diagnosis and detection processes, more complex array-typed EWOD devices have been created to support sophisticated lab-on-chip platforms. Multiple droplet manipulation becomes essential and enquires more precise control algorithms. Bhattacharjee and Najjaran [11] mentioned about the need of feedback control of a single droplet position in a digital micro fluidic system (DMS) and presented simulation results of the feedback control system. Also, the work by Oprins *et al.* [12] proposed the modeling and control of droplet motion of the electro-wetting system. Based on the proposed models presented in [11] and [12], various nonlinear control techniques can be applied to control droplet position on top of a EWOD lab-on-chip platform.

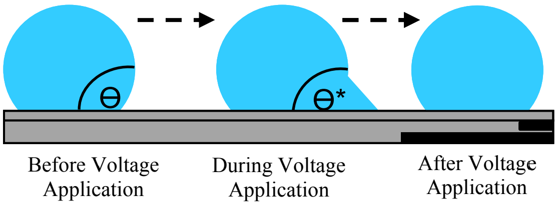


Fig. 1. Droplet movement by altering contact angle with applied voltage

This paper focuses on the design of sliding mode tracking controller to control the droplet position in the EWOD microfluidic system under the disturbance. The control system is simulated to evaluate the ability of this control technique.

2. EWOD Microfluidic System

2.1. *Equation of Motion*

In this study, the dynamic response of droplet is formulated as a simplified one-dimensional model based on the proposed by [11] and [12]. As shown in Fig. 2, the droplet is modeled as a single rigid body moving with velocity *U*. The motion of droplet is driven by the driving force (F_{dr}) actuated by electrostatic force and its threshold (F_{thresh}).The details of the threshold of electrostatic force can be determined from [13-15]. There are 3 sources of frictional force exerting against the motion of droplet including viscous dissipation due to internal fluid friction (F_d), viscous drag due to moving through surrounding fluid (F_f) and frictional force due to the contact line (F_c). According to [11], the equation of droplet motion in EWOD configuration can be represented by

$$m\frac{d^2x}{dt^2} = F_{dr} - F_{thresh} - F_d - F_f - F_c \tag{1}$$

where *m* and *x* are the mass and the displacement of the micro droplet, respectively. Assuming 5% overlap of adjacent electrodes [11], the droplet radius, *a* can be defined as $a = L/2 + 0.05L$ where *L* is the electrode width and *l* is the gap between adjacent electrodes. As mentioned in [12], viscous drag due to moving through surrounding fluid (F_f) can be neglected because air is the surrounding fluid. Thus, the motion of droplet in EWOD microfluidic system can be mathematically expressed as

$$m\frac{d^2x}{dt^2} = F_{dr} - F_{thresh} - F_d - F_c \tag{2}$$

As presented in [14], the viscous dissipation due to internal fluid friction (F_d) for the open EWOD configuration can be described as

$$F_d = \left(\frac{5\mu_d U}{2H}\right)\left(\pi a^2\right) \tag{3}$$

where μ_d and *H* denote the droplet viscosity and height respectively.

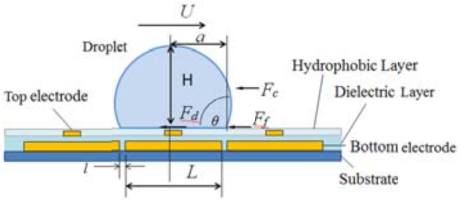


Fig. 2. The micro droplet motion on the EWOD plate

As presented in [11] and [12], frictional force due to the contact line (F_c) can be expressed as

$$F_c = \varsigma U^n \left(2\pi a\right) \tag{4}$$

where ς is the coefficient of contact-line friction based on kinetic-molecular theory. *n* is in the range from 0 to 2.

2.2. *State Space Representation*

The equation of motion in (2) can be presented in the form of first order nonlinear differential equation or the state equation and the output equation

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f\left(\bar{x},t\right) + g(\bar{x},t)u(t) + d(t) \\ y &= x_1 \end{aligned} \tag{5}$$

where $g\left(\bar{x},t\right) = \dfrac{1}{m}$

$$f\left(\bar{x},t\right) = \frac{1}{m}[-\left(\frac{5\mu_d x_2}{2H}\right)\left(\pi a^2\right) - \zeta x_2{}^n(2\pi a)] \cdot$$

The state variable x_1 and x_2 represent displacement, *x* and velocity of the micro droplet, $\dot{x}$. Also, $\bar{x} = \left[x_1 \; x_2\right]^T$ is denoted as state vector. The control input, $u(t)$ is F_{elect} . A $d(\bar{x},t)$ is bounded disturbance and $\left|d(\bar{x},t)\right| \le M$ where M is a positive constant. The output variable is denoted by y . $f(\bar{x},t)$ is a nonlinear smooth function.

3. Controller Design

The sliding mode tracking controller was designed to track two types of desired signals: 1) step signal 2) sinusoidal signal. The step signal represents the case when micro droplet moves from the current electrode to the adjacent electrode. The sinusoidal signal was used to test the ability of the controller to track the periodic motion of the droplet between two electrode plates.

According to (5), the sliding mode controller design with the selected reaching law as in [16] which can be shown as follows. First, the switching surface is defined as

$$s = e + c\dot{e} \tag{6}$$

where $c > 0$. e and $\dot{e}$ are denoted as tracking error and its derivative. Second, the Lyapunov function is defined as

$$V = \frac{1}{2}s^2 \tag{7}$$

From (6) and (7), the derivative of switching surface $\dot{s}$

$$\dot{s} = \dot{e} + c\ddot{e} = (\ddot{x}_r - \ddot{x}) + c(\dot{x}_r - \dot{x}) = (\ddot{x}_r - \dot{x}_2) + c(\dot{x}_r - \dot{x}_1)$$

$$\dot{s} = (\ddot{x}_r - (f(\bar{x},t) + g(\bar{x},t)u(t) + d(t))) + c(\dot{x}_r - x_2) \tag{8}$$

where x_r is the reference signal or desired displacement of the droplet. The exponential rate reaching law [16] or constant plus proportional rate reaching law [17] is selected

$$\dot{s} = -ks - \varepsilon\,\mathrm{sgn}(s) \tag{9}$$

where $\varepsilon > 0$ and $k > 0$. Equating (8) and (9), the control input is determined as shown in (11).

$$\begin{aligned} -ks - \varepsilon\,\mathrm{sgn}(s) = {} & [\ddot{x}_r - (f(\bar{x},t) + g(\bar{x},t)u(t) + d(t))] \\ & + c(\dot{x}_r - x_2) \end{aligned} \tag{10}$$

$$\begin{aligned} u(t) = {} & \frac{1}{g(\bar{x},t)}[ks + \varepsilon\,\mathrm{sgn}(s) + (\ddot{x}_r - f(\bar{x},t) - d(t) \\ & - c(\dot{x}_r - x_2))] \end{aligned} \tag{11}$$

Based on the control input in (11), the stability of the control system can be verified as follows

$$\dot{V} = s\dot{s} \tag{12}$$

$$\begin{aligned} \dot{V} = {} & s\{\ddot{x}_r - (f(\bar{x},t) + g(\bar{x},t)\frac{1}{g(\bar{x},t)}[ks + \varepsilon\,\mathrm{sgn}(s) + (\ddot{x}_r - f(\bar{x},t) \\ & - d(t) + c(\dot{x}_r - x_2))] + d(t) + c(\dot{x}_r - x_2))\} \end{aligned} \tag{13}$$

$$\dot{V} = s(-d - ks - \varepsilon\,\mathrm{sgn}(s)) \quad = -sd - ks^2 - \varepsilon s\,\mathrm{sgn}(s)) \tag{14}$$

If ε is selected such that $\varepsilon \geq M$, then the derivative of Lyapunov function satisfies the following condition.

$$\dot{V} = -sd - ks^2 - \varepsilon|s| \leq 0 \tag{15}$$

According to (15), It is clear that the designed control input is able to output signal to track the reference signal under disturbance.

4. Simulation Results and Discussions

Interesting points for tracking control problems in this research consists of two types of reference signals which are: 1) step function and 2) sinusoidal signal. The magnitude of the step function and sinusoidal function can be defined by the distance between centers of the first and the second electrode plate.

4.1. *Simulation Example*

The example of a EWOD microfluidic system is controlled by the designed controller in section 3 as shown in Fig. 3. The simulations of the control system for both cases are carried out in MATLAB program. The integration method is Runge-Kutta with sampling time of 0.00001 seconds. The water droplet is used in this simulation, and the system is operated at 20° C. The system parameters are presented as follows.

$L = 1600 \times 10^{-6}\mathrm{m}, l = 70 \times 10^{-6}\mathrm{m}, \mu_d = 1.01 \times 10^{-3}\ \mathrm{Pa.s}$,
$\varsigma = 0.08\ \mathrm{Ns/m^2}, \rho_d = 1000\ \mathrm{kg/m^3}, H = 0.0013\ \mathrm{m}$
$a = 8.8 \times 10^{-4}\mathrm{m}, m = 2.5681 \times 10^{-6}\mathrm{kg}, n = 2$,
and $d(t) = 2000 \times 10^{-6}\sin(100t)$. $F_{thresh} = 2.6541 \times 10^{-5}\,N$

The values of ς , ρ_d , and C are from [11] and [12]. L and l are from [18]. Assuming contact angel equal to $\theta = 110^\circ$, H is determined from L based on [14]. μ_d is selected at the temperature equal to 20° C. m is calculated from the density, ρ_d, and volume of the droplet defined by a and H. According to [15], the value of F_{thresh} is the value of capillary force and is approximated based on the value of the contact angle, $\theta = 110^\circ$, the electrode width L, and the surface tension between water and air $\gamma = 72mN/m$ [14], and the hysteresis angle $\alpha = 7^\circ$ [14].

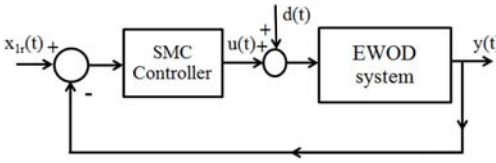


Fig. 3.The diagram of the feedback control system.

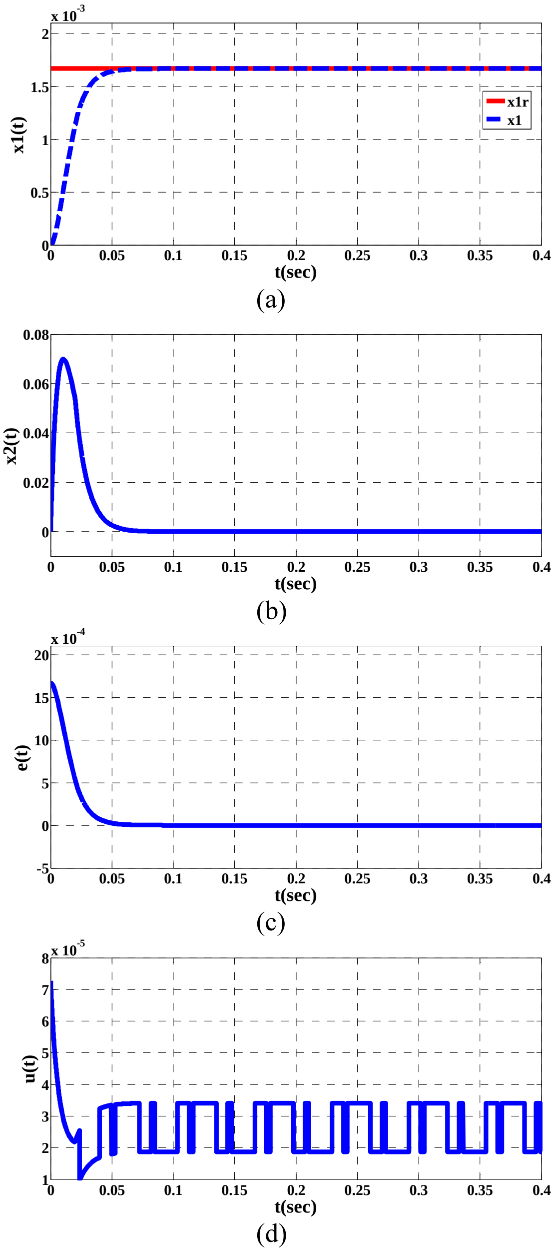
4.2. *Step Reference Signal*

The control EWOD microfluidic system has a step function as reference signal presented as

$$x_{1r}(t) = \begin{cases} 1670 \times 10^{-6}, t \geq 0 \\ 0 \qquad\quad\ , t < 0 \end{cases} \tag{19}$$

with the initial condition $\bar{x}(0) = [0 \quad 0]^T$. (20)

The control parameters are selected as follows, c=100, k=90and ε=3.The simulation results of droplet position and velocity are shown in Figs. 4(a) and 4(b). Position tracking error of the droplet is shown in Fig. 4(c). The control input signal is shown in Fig 4(d). Therefore, it is clear that the sliding mode controller is feasible to track the step reference signal accurately under the effect from the disturbance.



(a)

(b)

(c)

(d)

Fig. 4.Simulation results of tracking step reference signal

4.3. *Sinusoidal Reference Signal*

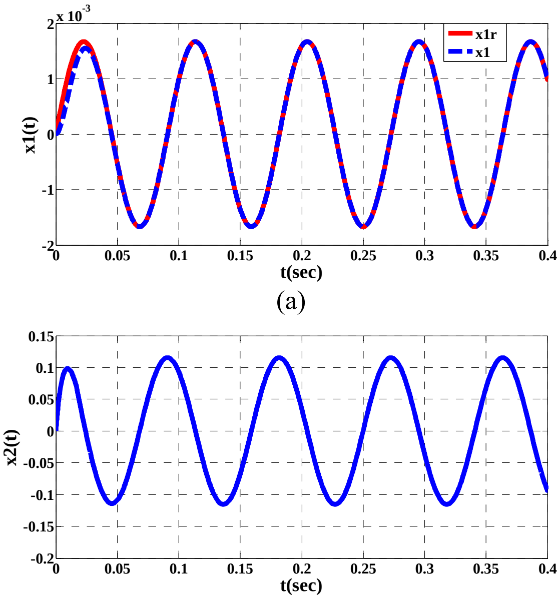
The sinusoidal function is the reference signal of the control EWOD microfluidic system expressed as

$$x_{1r}(t) = 1670\times10^{-6}\sin(\omega t) \qquad (21)$$

where the frequency $\omega = 22\pi\ rad/s$

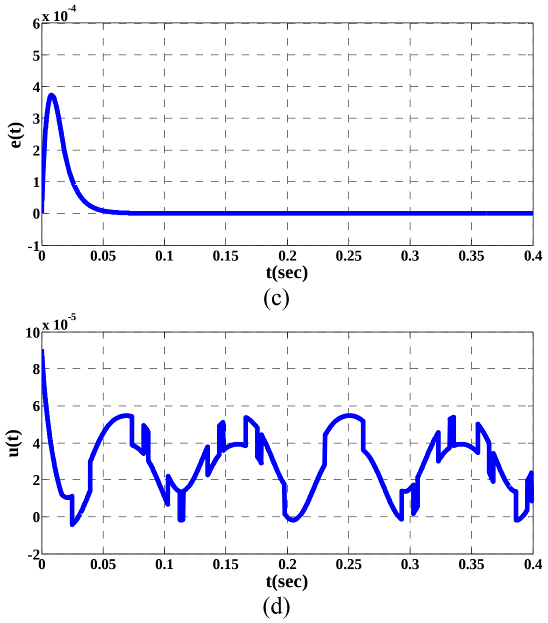
with the initial condition $\bar{x}(0) = [0 \quad 0]^T$. (22)

The control parameters, c, k and ε are selected as the same values in the previous case. The simulation results of droplet position and velocity as well as position tracking error of the droplet are shown in Fig. 5 (a), (b), and (c), respectively. The control input signal is shown in Fig. 5 (d). The controller with selected parameters is able to track the sinusoidal reference signal, so that the droplet can move along desire path as shown in Fig 5(a). The plot of velocity versus time is shown in Fig 5(b). At certain range of time for example $t \geq 0.091$ s., the plot in figure 4 (b) is similar to the first derivative of the displacement in figure 4 (a) as considering from the amplitude and the phase difference of the plots from both figures. Thus, this plot is reasonable. The plot of position tracking error versus time in Fig 5(c) shows that the sliding mode control provides low position tracking error. Thus, the sliding mode controller is feasible to track the sinusoidal reference signal accurately even though the system is affected by the bounded disturbance. Therefore, the sliding mode controller is an appropriate technique for the position control of the micro droplet in the EWOD microfluidic system.



(a)

(b)



(c)

(d)

Fig. 5.Simulation results of tracking sinusoidal reference signal

5. Conclusions

The sliding mode control with exponential rate reaching law is feasible to control the microfluidic droplet motion in the EWOD microfluidic system under the effect of bounded disturbance signal. For the step reference signal, the controller provides the step response with no overshoot and tracks the reference signal accurately under the disturbance. In the case of the sinusoidal reference signal, the designed sliding mode controller can also provide accurate tracking.

6. References

[1] K. Brady, “Electrowetting for DNA sequencing on chip”, 2004 NNIN REU Research Accomplishments, pp. 26-27, 2004.

[2] V. Srinivasan, V.K. Pamula, P. Paik, and R.B. Fair, “Protein stamping for MALDI mass spectrometry using an electrowetting-based microfluidic platform”, in *Proc. of SPIE-The International Society for Optical Engineering*, vol. 5591, 2004, pp. 26-32.

[3] V. Srinivasan, V.K. Pamula, M.G. Pollack, and R.B. Fair, “Clinical diagnostics on human whole blood, plasma, serum, urine, saliva, sweat, and tears on a digital microfluidic platform”, in *Proc. the 7th International Conference on Miniaturized Chemical and Biochemical Analysis Systems,* Squaw Valley, California, U.S.A., 2003, pp1287-1290.

[4] M.G. Pollack, P.Y. Paik,, A.D. Shenderov, V.K. Pamula, F.S. Dietrich, and R.B. Fair, “Investigation of electrowetting-based microfluidics for real-time PCR applications”, in *Proc. the 7th International Conference on Miniaturized Chemical and Biochemical Analysis Systems,* Squaw Valley, California, U.S.A., 2003, pp.619-622.

[5] C.S. Yu, M.Y. Lin, H.T. Hu, Y.C. Hu, and H.Y. Chou, “A droplet-based device for triglycerides colorimetric measurement”, *Tamkang Journal of Science and Engineering*, vol. 8, no. 3, pp. 211-216, 2005.

[6] V. Srinivasan, V. Pamula, M. Pollack, and R. Fair, “A digital microfluidic biosensor for multianalyte detection”, in *Proc. of IEEE The Sixteenth Annual International Conference on Micro Electro Mechanical Machine (MEM-03Kyoto)*, Kyoto, Japan, 2003, pp. 327-330.

[7] W. Satoh, H. Hosono, K. Morimoto and H. Suzuki, “Micro analysis system with an integrated microfluidic system based on electrowetting”, *in Proc. of the fourth IEEE Conference on Sensors(IEEE SENSORS 2005)*, Irvine, CA, U.S.A., Oct. 30-Nov. 3, 2005, pp. 37-40.

[8] K.S. Yun, I.J. Cho, J.U. Bu, G.H. Kim, Y.S. Jeon, C.J.(CJ) Kim, and E. Yoon, “A micropump driven by continuous electrowetting actuation for low voltage and low power operations”,in *Proc. of the 14th IEEE international conference on Micro Electro Mechanical Systems (MEMS 2001)*, Interlaken, Switzerland, pp. 487-490.

[9] I. Moon, and J. Kim “Using EWOD (electrowetting-on-dielectric) actuation in a micro conveyor system”, *Sensor and Actuators A: Physical*, vol. 130-131, pp. 537-544, Aug. 2006.

[10] V. K. Pamula and K. Chakrabarty, “Cooling of integrated circuits using droplet-based microfluidics”, in *Proc. of the 13 th ACM Great Lakes Symposium on VLSI (GLSVLSI’03)*, Washington, DC, U.S.A., April 28-29, 2003, pp.1-4.

[11] B. Bhattacharjee, and H. Najjaran, “Simulation of droplet position control in digital microfluidic systems”, *Journal of Dynamic Systems, Measurement, and Control*, vol. 132, iss. 1, pp. 014501-1 - 014501-3, Jan. 2010.

[12] H. Oprins, B. Vandevelde, and M. Baelmans, “Modeling and control of electrowetting induced droplet motion”, *Micromachines*, (3)1, pp. 150-167, 2012.

[13] H. Ren, R.B., Fair, M.g., Pollack, and E.J. Shaughnessy, “Dynamics of electrod-wetting droplet transport ”, Sensor and Actuators B 87 (2002), pp.201-206.

[14] J. Berthier, Microdrops and digital Microfluidics, Norwich New York:William Andrew, 2008.

[15] J. Berthier, P. Dubois, P. Clementz, P. Claustre, C. Peponet, Y. Fouillet, “Actuation potential and capillary forces in electrowetting based microsystem”, Sensor and Actuators A 134 (2007) 471-479.

[16] J. Liu, and X. Wang, “Advanced sliding mode control for mechanical systems” Beijing, Tsinghua University Press and Berlin Heidelberg, Springer-Verlag, 2011

[17] W. Gao and J.C. Hung,”Variable Structure Control of Nonlinear Systems: A New Approach”, IEEE Transasction on Industrial Electronics, Vol. 40, No.1 February, 1993, pp. 45-55.

[18] C. Phongsomboon, “Mobility control of a droplet on top of a flat plate”, M.S. thesis, Dept. Mechanical Engineering, King Mongkut University of Technology Thonburi,Bangkok, Thailand, 2012. (in Thai)

[19] R. W. Fox, P. J., Pritchard, and A. T., Macdonald, “Introduction to fluid mechanics”, 7th edition, John Wiley and Sons, 2010.