Consensus Problem of Distributed Multi-agent Systems*

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Abstract

The paper focuses on the consensus problem of distributed control systems. It's the basic research subject related with the distributed coordination of multiple robots such as the unmanned robots which has been a very active research one studied widely by the control researchers. The paper mainly discusses the most typical topics which have made large progress in distributed multi-agent coordination in recent years including the system dynamics, the time delay effect, the network topology, the convergence, and so on.

Keywords: multi-agent system, consensus, distributed coordination, network topology, convergence

1. The general description

Consensus refers to the typical group behavior of the multi-agent system that all the agents reach asymptotically a certain common agreement based on a local distributed protocol, with or without predefined common speed and orientation.

To the distributed control of a group of autonomous robots, the main objective is typically to have the whole group of robots working in a cooperative way base on a distributed protocol. The cooperative refers to a close relationship among all robots in the group where information sharing plays a key role. The distributed method has many advantages in achieving cooperative group performances, especially with low operational costs, less system requirements, high robustness, strong adaptivity, and flexible scalability, therefore has been widely interested and recognized [1-2]

Let's express the typical consensus problem based on which we discuss the consensus problem. Consider a group of n agents, each with single-integrator kinematics described by

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$$\dot{x}_i = u_i(t);\ i=1, 2, \ldots, n \qquad (1)$$

where $\dot{x}_i$ and $u_i(t)$ are, respectively, the state and the control input of the *i*th agent. A typical consensus control algorithm is designed as

$$u_i(t) = \sum_{j=1}^{n} a_{ij}(t)[x_j(t) - x_i(t)] \qquad (2)$$

where $a_{ij}(t)$ is the (i, j)th entry of the corresponding adjacency matrix at time t. The main idea with formula (2) is that each agent moves towards the weighted average of the states of its neighbors. Given the switching network pattern due to the continuous motions of the dynamic agents, coupling coefficients $a_{ij}(t)$ in formula (2), so the graph topologies, are generally time-varying. There is the conclusion that consensus is achieved if the underlying directed graph has a directed spanning tree in some jointly fashion in terms of a union of its time-varying graph topologies [1-4].

Consensus does as a fundamental principle for the design of distributed multi-agent coordination algorithms. Therefore, investigating consensus has been a main research direction in the study of distributed multi-agent coordination. To establish the relationship between the study of consensus algorithms and many physical properties inherited in practical systems, it is necessary and meaningful to study consensus by considering many practical factors such as actuation, control, communication, computation, robot dynamics, and so on, which characterize some important features of practical systems [1-6].

Please make sure that shared information is a necessary condition for coordination. Information necessary for cooperation may be shared in various ways. For example, relative position sensors may enable robots to construct state information for other robots [7], knowledge may be communicated among robots using a wireless network [8], or joint knowledge might be preprogrammed into the robots before a mission begins [9]. Based on this concept, information exchange becomes a key problem in coordination control.

2. The system dynamics

Consensus is concerned with the behavior of a group of robots, so it is natural to consider the system dynamics for practical robots in the study of the consensus problem. Although the study of consensus under various system dynamics is due to the existence of complex dynamics in practical systems, it is also interesting to observe that system dynamics play an important role in determining the final consensus state. For example, the well studied consensus of multi-agent systems with single-integrator kinematics often converges to a constant final value. However, consensus for double-integrator dynamics might admit a dynamic final value. These important problems make the study of consensus under various system dynamics become hot.

As a direct extension of the study of the consensus problem for systems with simple dynamics, for example, with single-integrator kinematics or double-integrator dynamics, consensus with general linear dynamics was also studied [10-12], where research is mainly devoted to finding feedback control laws such that consensus (with the output states) can be achieved for general linear systems.

$$\dot{x}_i = Ax_i + Bu_i, \qquad y_i = Cx_i \qquad (3)$$

where A, B, and C are constant matrices with compatible sizes. Apparently, the well studied single-integrator kinematics and double-integrator dynamics are special cases of system (3) for properly choosing A, B, and C.

Consensus for complex systems has been extensively studied too. Here, the term consensus for complex systems is used for the study of consensus problem when the system dynamics are nonlinear or with nonlinear consensus algorithms [13-17].

Although the complex system dynamics are different from the well studied single-integrator kinematics and double-integrator dynamics, the main research problem is same, namely, to drive all agents to some common states through local interactions among agents. Similarly to the consensus algorithms proposed for systems with simple dynamics, the consensus algorithms used for these complex models are also based on a weighted average of the state differences, with some additional terms if necessary. Main research work has been conducted to design proper control algorithms and derive necessary or sufficient conditions such that consensus can be achieved ultimately.

3. The time delay

Time delay exits in almost all practical systems due to several reasons such as limited communication speed, extra time required by the sensor to get the measurement information, computation time required for generating the control inputs and execution time required for the inputs being acted. Generally, time delay reflects an important property inherited in practical systems due to actuation, control, communication, and computation.

Knowing that time delay might degrade the system performance or even destroy the system stability, studies have been conducted to investigate its effect on system performance and stability. A well studied consensus algorithm for system (1) is given in formula (2), where it is now assumed that time delay exists. Two types of time delays, communication delay and input delay, have been considered in some research. Communication delay accounts for the time for transmitting information from source to destination. More precisely, if it takes time Tij for agent i to receive information from agent j, the closed-loop system of (1) with (2) under a fixed network topology becomes

$$\dot{x}_i(t) = \sum_{j=1}^{n} a_{ij}(t)[x_j(t - T_{ij}) - x_i(t)] \quad (4)$$

An explaination of (4) is that at time t, agent i receives information from agent j and uses data $x_j(t - T_{ij})$ instead of $x_j(t)$ due to the time delay. Please note that agent i can get its own information instantly, therefore, input delay can be considered as the sum of computation time and execution time. More precisely, if the input delay for agent i is given by T^p_i ,then the closed-loop system of (1) with (2) becomes

$$\dot{x}_i(t) = \sum_{j=1}^{n} a_{ij}(t)[x_j(t - T_i^p) - x_i(t - T_i^p)] \quad (5)$$

Clearly, (4) refers to the case when only communication delay is considered while (5) refers to the case when only input delay is considered. It should be emphasized that both communication delay and input delay might be time-varying and they might co-exist at the same time.

The main problem involved in consensus with time delay is to study the effects of time delay on the convergence and performance of consensus [18]. The existing study of consensus with time delay mainly focuses on analyzing the stability of consensus algorithms with time delay for various types of system dynamics, including linear and nonlinear dynamics. Generally, consensus with time delay for systems with nonlinear dynamics is more challenging. For most consensus algorithms with time delays, the main research topics is to determine an upper bound of the time delay under which time delay does not affect the consensusability. For communication delay, it is possible to achieve consensus under a relatively large time delay threshold. A notable phenomenon in this case is that the final consensus state is constant. Considering both linear and nonlinear system dynamics in consensus, the main tools for stability analysis of the closed-loop systems include matrix theory [19], Lyapunov functions [20], frequency-domain method [21], passivity [22], and the contraction principle [23].

4. The Network Topology

In multi-agent systems, the network topology among all robots plays a very important role in determining consensus. The objective here is to explicitly identify necessary or sufficient conditions on the network topology such that consensus can be achieved under properly designed algorithms.

It is often reasonable to consider the case when the network topology is deterministic under ideal communication channels. Accordingly, main research on the consensus problem was conducted under a deterministic fixed or switching network topology. That is, the adjacency matrix A(t) is deterministic. In some other times, when considering random communication failures, random packet drops, and communication channel instabilities inherited in physical communication channels, it is necessary and important to study consensus problem in the stochastic setting where a network topology evolves according to some random distributions. That is, the adjacency matrix A(t) is stochastical. By now the current study on consensus over stochastic network topologies has shown some interesting results regarding that how to determine the probability of reaching consensus almost surely and when the network topology itself is stochastic as well as what are the advantages and disadvantages of the stochastic network topology regarding such as robustness and convergence rate when compared with the deterministic network topology.

As is well known, disturbances and uncertainties often exist in networked systems such as channel noise, communication noise, uncertainties in network parameters, etc. In addition to the stochastic network topologies mentioned above, the effect of stochastic disturbances and uncertainties on the consensus problem also needs investigation [24-25]. Study has been mainly devoted to analyzing the performance of consensus algorithms subject to disturbances and to presenting conditions on the uncertainties such that consensus can be achieved. Besides, another interesting direction in dealing with disturbances and uncertainties is to design distributed local filtering algorithms so as to save energy and improve computational efficiency. Distributed local filtering algorithms play an important role and seems more effective than traditional centralized filtering algorithms for multi-agent systems [26-27].

5. The convergence analysis

To the control system, the convergence problem is always an important topic. Of course it is one of the key performance measure for consensus algorithms too.

The existing study mainly focuses on the analysis of the convergence speed under various network topologies and optimization of the convergence speed for certain given network topologies. Considering the fact that consensus under different network topologies may behave different convergence speeds, a natural topic is how to design an optimal network topology with proper adjacency matrix such that optimal convergence speed can be achieved [28-29].

The study of convergence speed for the consensus problem, finite-time consensus, reaching consensus in a finite time, has also been studied. Compared with most existing research on the consensus problem, finite time consensus behaves a disturbance rejection property and robustness against uncertainties. In addition, due to the finite-time convergence, it is often possible to decouple the consensus problem from other control objectives when they are considered simultaneously [30-31].

Please note that the existing research on finite-time consensus mainly focuses on systems with simple dynamics, such as single-integrator kinematics and double-integrator dynamics, in the continuous-time field. Because many practical systems are better and more proper to be described by general linear or nonlinear dynamics, it is natural to study finite-time consensus for systems with general linear or nonlinear dynamics further. Besides, it is meanful to study finite-time consensus in the discrete-time field. Some research on this topic can be found in [32-33], where the objective is to compute the final consensus value for all agents in a finite number of steps.

6. The conclusion

The consensus problem is the basic and very important topic in coordination control of the multi-agent system. It mainly refers to some basic concepts including the system dynamics, the time delay and the system network topolgy. The comvergence is also an very important concept. But the paper doesn’t introduce the consenseu and the convergence related with the subjects such as asynchronous effects, sampled-data framework, quantization, and so on. The existing research on the consensus problem has covered a number of physical properties for practical systems and control performance analysis. However, the study of the consensus problem covering multiple physical properties or control performance analysis has been largely ignored. Therefore more problems discussed in the above need to be taken into consideration simultaneously when studying the consensus problem in the future.

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