A fractional-order hyper-chaotic system and its circuit implementation

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Abstract

In this paper, the commensurate3.6-order Qi hyper-chaotic system is investigated. Based on the predictor-corrector method, we obtain phase portraits, bifurcation diagrams, Lyapunov exponent spectra of this system, and find that the system can present a four-wing hyperchaotic attractor. In addition, a circuit is designed for the fractional-order hyperchaotic system and the circuit implementation result show the existence of the four-wing hyperchaotic attractor, which verifies the correct of the theoretic analysis and provides the technical support for its application in engineering.

Keywords: Fractional-order; Qi hyper-chaotic system; Lyapunov exponent; Circuit design;Implementation.

1. Introduction

Since Lorenz discovered the famous Lorenz chaotic attractor in 1963,[1]a great deal of new chaotic systems have been proposed, such as Chen system,[2] Lü system,[3] and so on.[4-5]which have made very significant influences in the research on nonlinear science. Recently, the research on the chaotic dynamics are gradually expanding to the engineering applications on the basis of mathematical theory and physical theory, such as secure communication,[6] image encryption,[7] and other fields.[8-9]

There also are some specific characteristics in the fractional-order system, that is, the fractional-order chaotic system not only can enhance the nonlinear and complexity of the systems, but also possess a very strong historical memory to reflect historical information of the system. Therefore, The fractional-order equation could describe the physical phenomenon more accurately,[10] in recent years, the fractional differential operator was introduced into the integer-order chaotic system to construct the fractional-order chaotic system.[11-13] It is also been verified that the fractional-order chaotic systems have more complex dynamics characteristics than integer-order chaotic systems. And thus the research on the fractional-order chaotic system has attracted more and more focus from application point of view. In addition, as the fractional-order hyper-chaotic system has two positive Lyapunov exponent , it could be more suitable for the application in secure communication and image encryption.

Therefore, it is necessary to investigate the analog circuit design and implementation of the fractional-order hyper-chaotic systems, which provides the technical basis for its further applications in engineering. In this paper, the commensurate fractional-order Qi hyper-chaotic system is investigated and an analog circuit is designed to implement the hyper-chaotic system.

2. Numerical analysis of fractional-order Qi hyper-chaotic system

Qi hyper-chaotic system has been proposed in Ref. [14], and it is constructed by adding a linear feedback control to the chaotic system in Ref.[5]. On this basis , we will analyze the commensurate fractional-order Qi hyper-chaotic system which is described as follow:

$$\begin{cases} \frac{d^q x}{dt^q} = a(y-x) + eyz, \\ \frac{d^q y}{dt^q} = cx + dy - xz - w \\ \frac{d^q z}{dt^q} = -bz + xy \\ \frac{d^q w}{dt^q} = fy \end{cases} \qquad (1)$$

Where $0 < q < 1, a, b, c, d, e, f \in R$, and when $q = 0.9$, $a = 15$, $b = 40$, $c = -5$, $d = 20$, $e = 5$, $f = 50$, system(1) is 3.6-order system, and it is chaotic .

2.1. *Phase portraits*

When $q = 0.9$, $a = 15$, $b = 40$, $c = -5$, $d = 20$, $e = 5$, $f = 50$,the phase portraits of 3.6-order Qi hyper-chaotic system is shown in Fig.1, It can be seen that this system presents four-wing chaotic attractor.

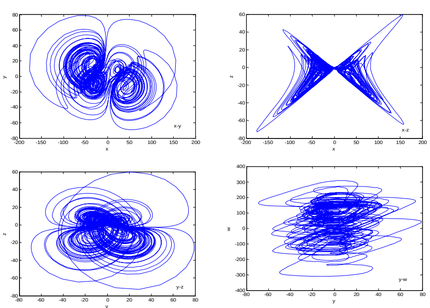
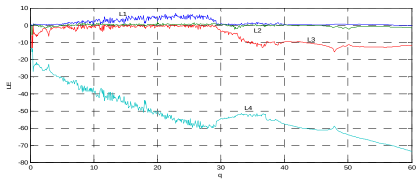


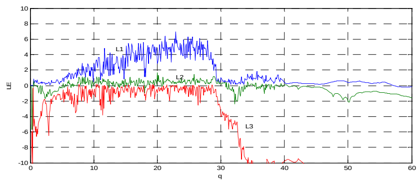
Fig.1 (color online) Phase portraits of system (1)

2.2. *Lyapunov exponent & bifurcation diagrams*

The characteristics of some system motion can be characterize by its Lyapunov exponent, and for a hyper chaotic system, there must have at least two positive Lyapunov exponents. Here, through numerical simulations, the effect on the Lyapunov exponent and the bifurcation of system(1) are analyzed when system parameter a changes.The Lyapunov exponent spectra and bifurcation diagrams of system(1) with $a \in [0,60]$ are shown in Fig.2 and Fig.3, respectively.



(a)



(b) intercept $[-10,10]$ on the y axis

Fig.2 (color online) Lyapunov exponent diagrams of system (1)

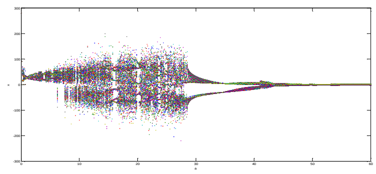


Fig.3 (color online) Bifurcation diagram of system (1) on x axis

In Fig. 2 and Fig. 3, it can be seen that the Lyapunov exponent on the parameter a is in agreement with the bifurcation diagram. As $L4$ always less than -10 in Fig.2, in order to observe the other three exponent expediently, so we intercept $[-10,10]$ on the y axis as shown in Fig.2(b). When $a = 15$, four Lyapunov exponents of system (1) are $L1 = 2.199, L2 = 0.4077, L3 = 0, L4 = -44.64$ respectively.It can be seen that this system is hyper-chaotic.

3. Circuit implementation of fractional-order Qi hyper-chaotic system

Through carrying out a series of numerical simulation and analysis of system (1), further for circuit simulation and realization of the system. Based on the method of approximation conversion from time domain to frequency domain, we utilize the approximation of $1/s^{0.9}$ with discrepancy 2dBto design the analog circuit, the circuit unit of $1/s^{0.9}$ is shown in Fig.4. Where $R_1 = 62.84M\Omega, R_2 = 250K\Omega, R_3 = 2.5K\Omega$ $C1 = 1.232\mu F, C2 = 1.84\mu F, C3 = 1.1\mu F$.

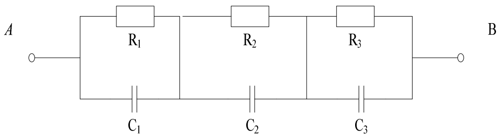


Fig. 4 Unit circuit of $1/s^{0.9}$

The fractional-order hyper-chaotic circuit is designed. In this circuit, LF353P is selected to be the amplifier and AD633JN (the output gain is 0.1) is selected to be the multiplier. In order to make the change range of state variables can work in the voltage range of the operational amplifier, The parameters are scaled according to the phase portraits of the system, let *x=50x', y=25y', z=25z', w=100w'*, the system(1) is modified for circuit implementation, which is described as follow:

$$\begin{cases} \frac{d^q x'}{dt^q} = -ax' + 0.5ay' + 12.5\,ey'z', \\ \frac{d^q y'}{dt^q} = 2cx' + dy' - 50\,x'z' - 4w' \\ \frac{d^q z'}{dt^q} = -bz' + 50\,x'y' \\ \frac{d^q w'}{dt^q} = 0.25\,fy' \end{cases} \tag{2}$$

Then the time scale is amplified by letting $t = \tau_0 \tau, \tau_0 = 200$. Thus the circuit design is shown in Fig.5.Where the module of $f\,0.9$ is the circuit unit of $1/s^{0.9}$ as shown in Fig.4.

According to the circuit diagram in Fig.5, the actual circuit is built to implement the fractional-order hyper-chaotic system(1), and the circuit experimental results are shown in Fig.6 through the TDS2014B digital oscilloscope, which is consistent with the numerical simulation result as shown in Fig.1, which verifies the existence of hyper-chaotic attractor of the fractional-order hyper-chaotic system on the physical level.

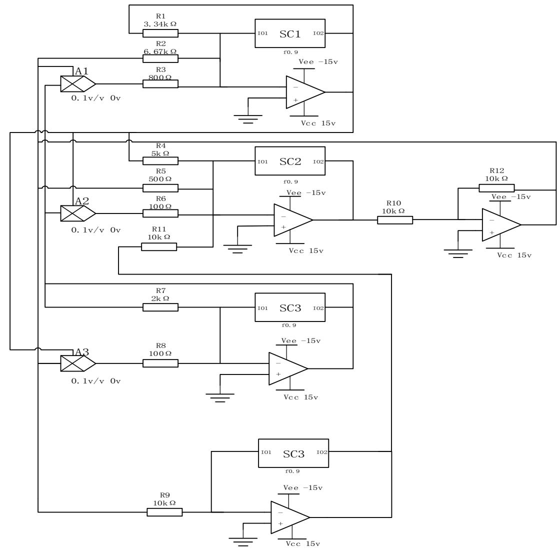
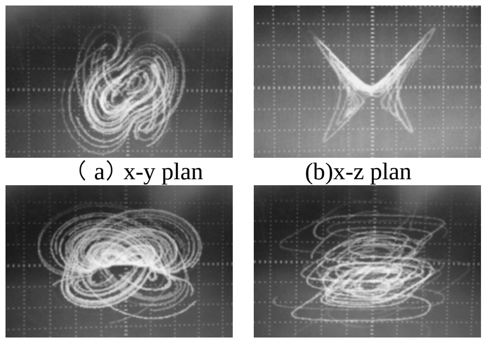


Fig. 5 Circuit diagram of system(1)



(a) x-y plan (b)x-z plan

(c) y-z plan (d) y-w plan

Fig.6 Phase portraits of hyper-chaotic system(1) through the oscilloscope TDS2014B

4. Conclusion

In this paper, the fractional-order Qi hyper-chaotic system is investigated. By analyzing phase portraits, bifurcation diagrams, Lyapunov exponent spectra of fractional-order Qi hyper-chaotic system, it is found that the system has four-wing hyper-chaotic attractors in certain range of parameters. Based on the numerical simulation of the fractional-order hyper-chaotic system. An analog circuit is designed to implement the fractional-order chaotic system, and circuit experiment results are coincide with the numerical simulation results, which provides technical basis and support for the further application of the fractional-order hyper-chaotic system in engineering.

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